SPECIAL RELATIVITY – FINAL EXAM

Exercise 1. *A particle with mass m and negative electric charge $-q$ travels with spatial momentum p. It passes by a very heavy particle with positive charge $+Q$. The distance of closest approach (also called the "impact parameter") is b.*

1. *Find the angle θ by which the first particle's trajectory is deflected, assuming that this angle is small. Hint: you can perform a calculation based entirely on the non-deflected trajectory.*

2. *How does θ scale with p in the limits $p \ll m$ and $p \gg m$?*

Exercise 2. *Let's explore the space of antisymmetric matrices $F^{\mu\nu} = F^{[\mu\nu]}$ on $\mathbb{R}^{3,1}$. Consider the following classes of such matrices:*

1. *$F^{\mu\nu} = a^{[\mu}b^{\nu]}$, where a^μ is timelike.*

2. *$F^{\mu\nu} = a^{[\mu}b^{\nu]}$, where a^μ and b^μ are spacelike and orthogonal to each other.*

3. *$F^{\mu\nu} = a^{[\mu}b^{\nu]}$, where a^μ is lightlike, and b^μ is orthogonal to it.*

How many degrees of freedom are in each class? Do these cover the degrees of freedom of a general antisymmetric $F^{\mu\nu}$? In class 3, what is the signature of b^μ?

Exercise 3. *Show that the Maxwell equations $\partial_\nu F^{\mu\nu} = J^\mu$ in $\mathbb{R}^{3,1}$ admit the following solution for the electromagnetic potential A_μ:*

$$A^\mu(x) = \int d^4x'\, G(x - x')\, J^\mu(x') \ , \tag{1}$$

where $G(x)$ is the retarded propagator:

$$G(x^\mu) = \frac{1}{2\pi}\, \delta(x_\mu x^\mu)\, \theta(t) \ , \tag{2}$$

and we assume that everything falls off sufficiently quickly at infinity. Hint: we already demonstrated $\Box G(x) = -\delta^4(x)$ in the lectures. Another hint: what can you say about $\partial_\mu A^\mu$?

Exercise 4. *Consider two spinors ψ^α and χ^α in $\mathbb{R}^{2,1}$. Let us work out the geometric meaning of the spinor sum $\psi^\alpha + \chi^\alpha$.*

1. *Recall that the squares* $\psi^\alpha\psi^\beta, \chi^\alpha\chi^\beta$ *of* ψ^α, χ^α *describe lightlike vectors, which we'll denote as* a^μ, b^μ*. What is the vector* c^μ *described by* $\psi^{(\alpha}\chi^{\beta)}$*? Hint: consider its scalar products with* a^μ, b^μ*, as well as with itself.*

2. *What is the signature of* c^μ*?*

3. *What is the lightlike vector described by the square of* $\psi^\alpha + \chi^\alpha$*?*