

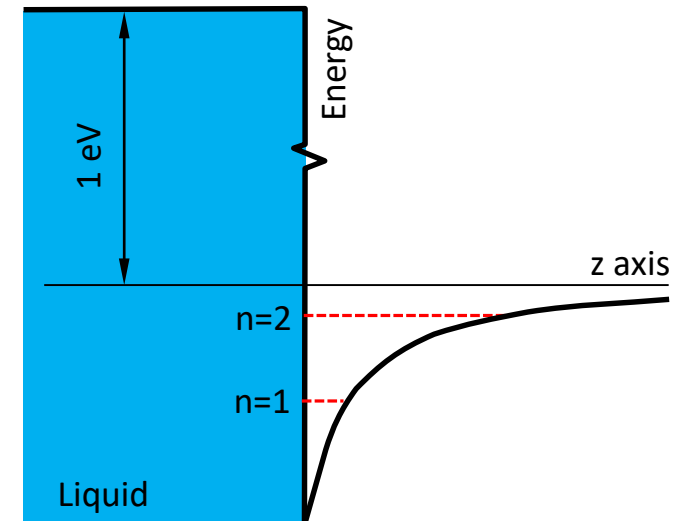
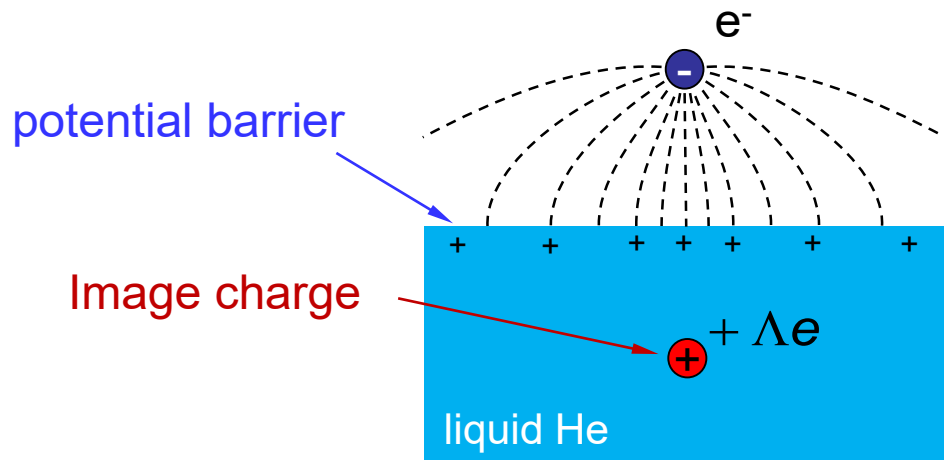
# Nonlinear transport of Wigner crystal on liquid helium in microchannel devices

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Denis Konstantinov, Quantum Dynamics Unit  
OIST Graduate University



# Surface States of Electrons on Helium



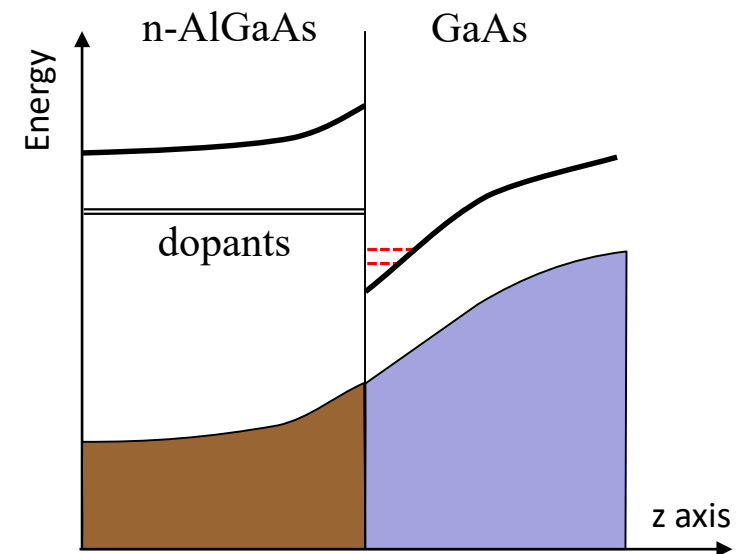
Weak image potential:

$$V = \frac{(\epsilon - 1)}{4(\epsilon + 1)} \left( \frac{e^2}{z} \right) < 0.01 \times \left( \frac{e^2}{z} \right)$$

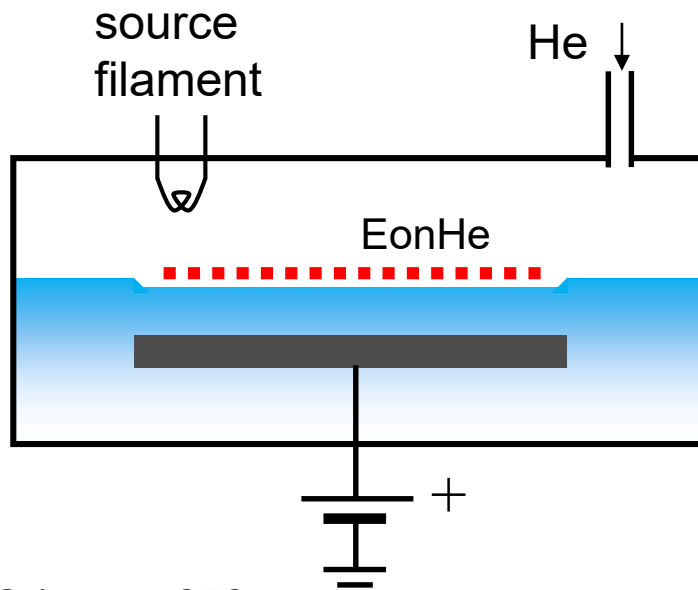
Small Rydberg energy:

$$R_y = \frac{m\Lambda^2 e^2}{2\hbar^2} \approx 1 \text{ meV} = 10 \text{ K}$$

Form 2D electron systems at  $T < 1 \text{ K}$

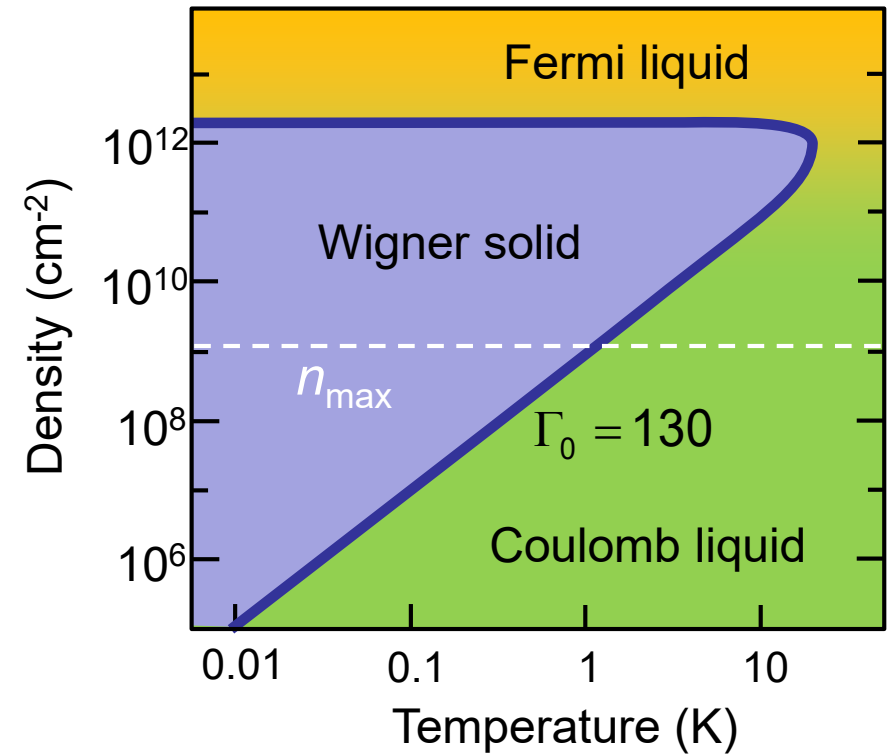
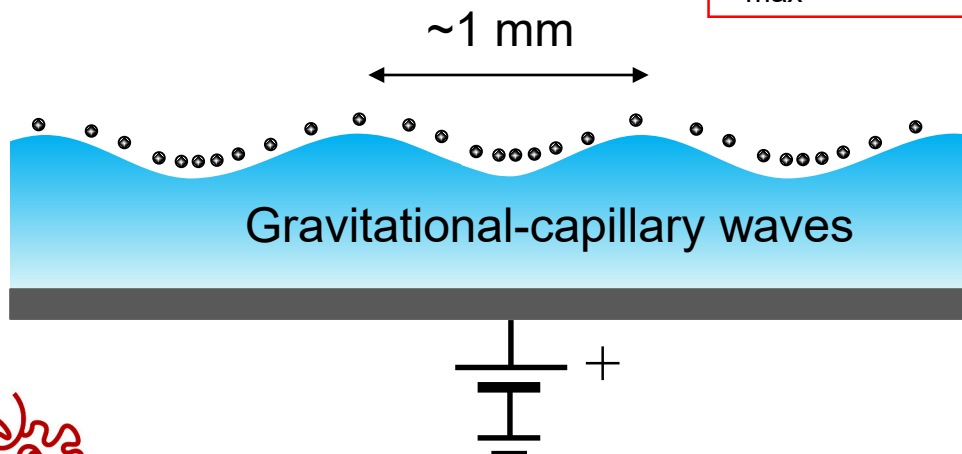


# Electron Density and Phase Diagram



Brown and Grimes, 1972

$$n_{\max} \approx 2 \times 10^9 \text{ cm}^{-2}$$

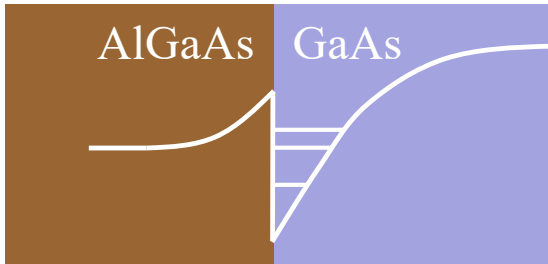


$$\Gamma = \frac{V_c}{K} = e^2 \sqrt{\pi n_s} \frac{1}{k_B T}$$

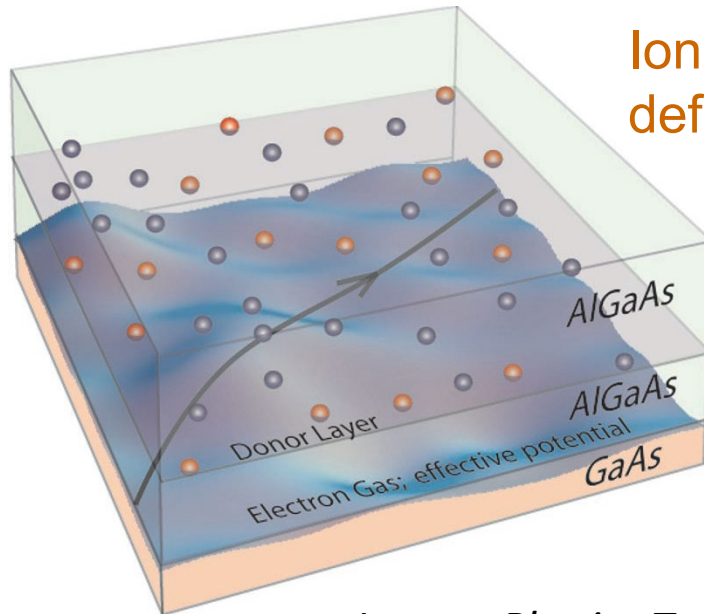
*Thermal fluctuations*



# Scattering of electrons



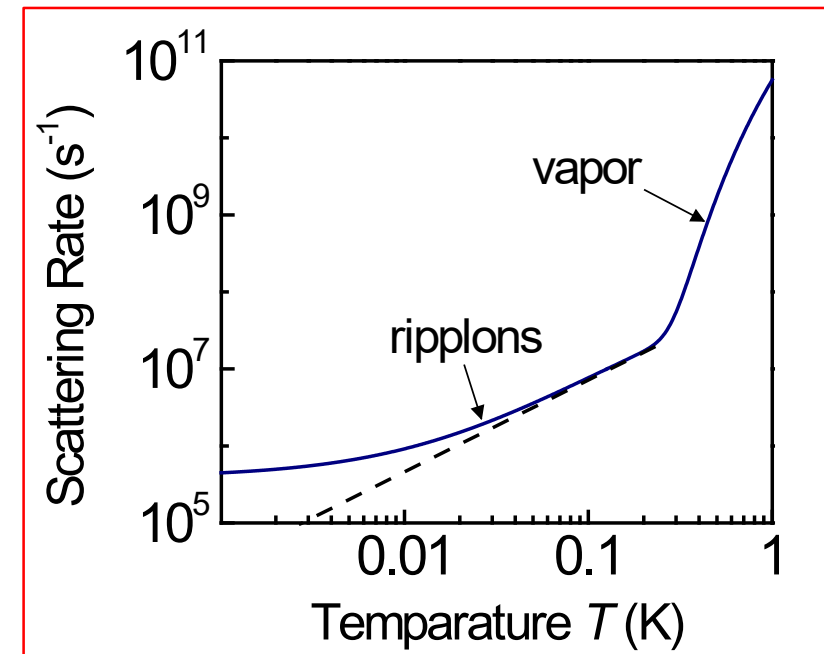
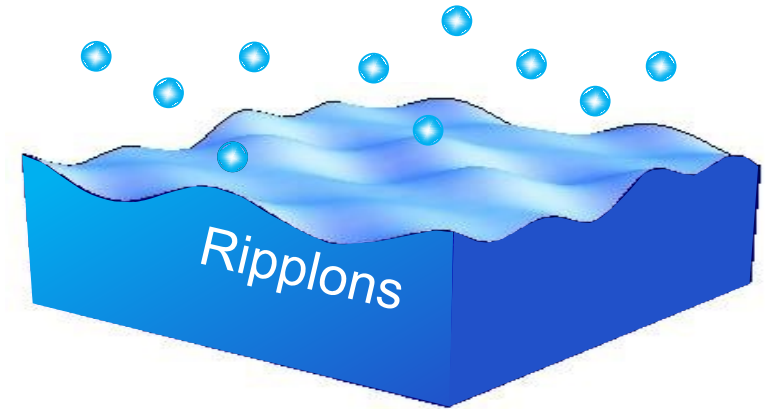
Potential landscape



Ionized dopants,  
defects, etc.

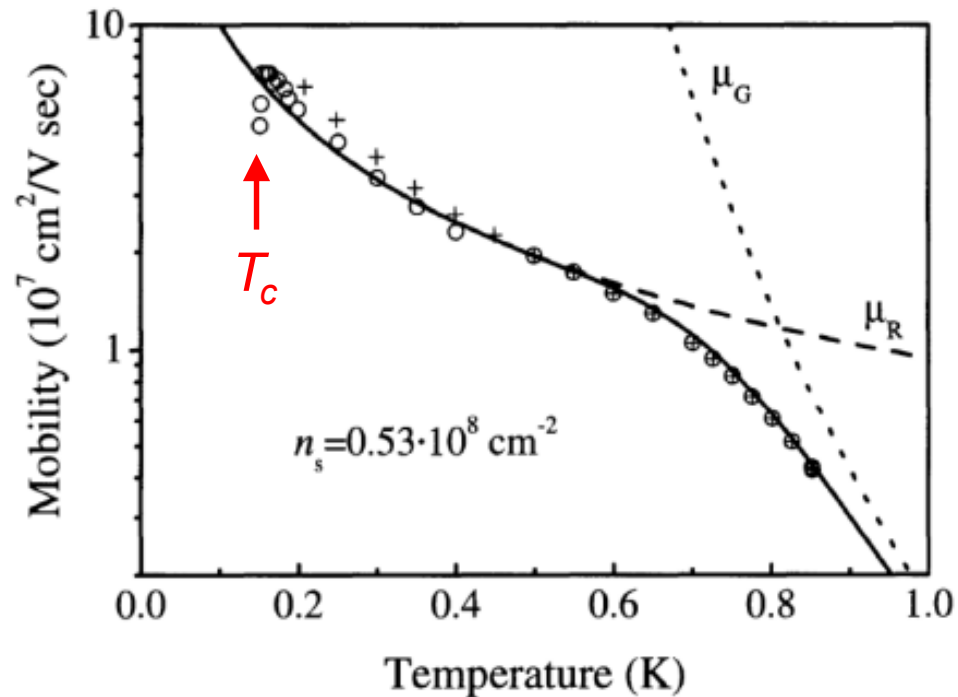
Image: *Physics Today* 56, 47 (2003)

Vapor atoms

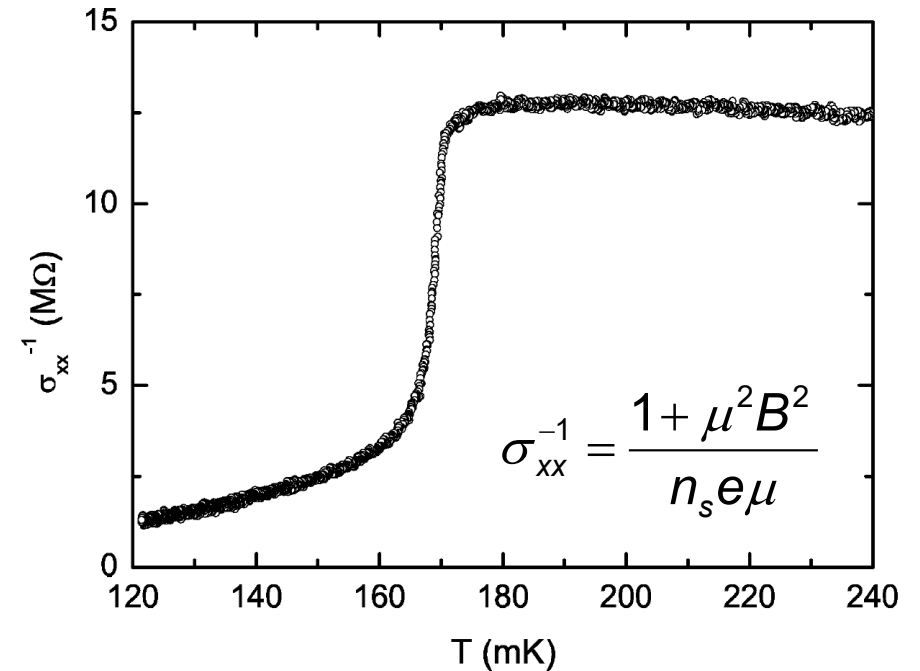


# Electron Mobility

Mobility exceeding 100 M!



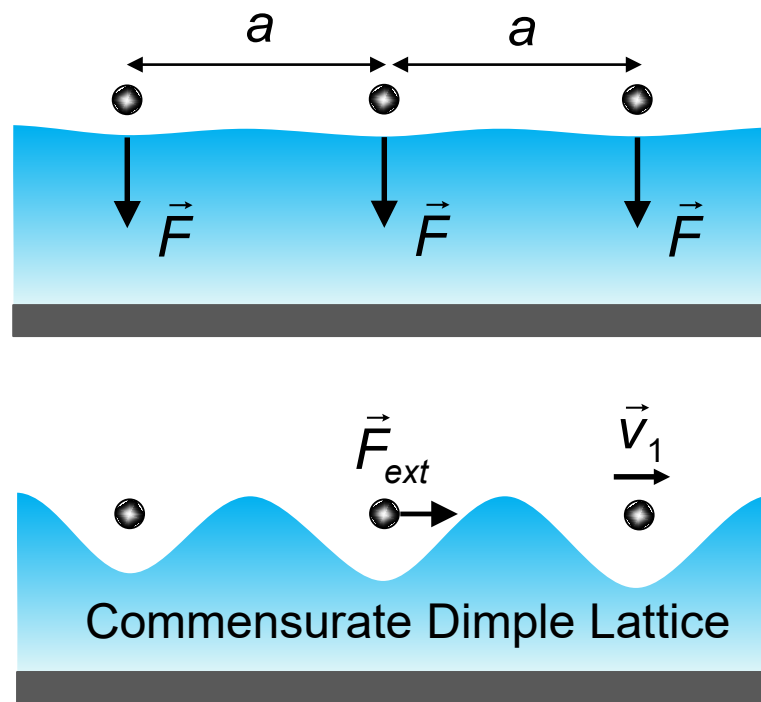
*Mehrotra et al., 1984*



*DK and Kono, 2008*

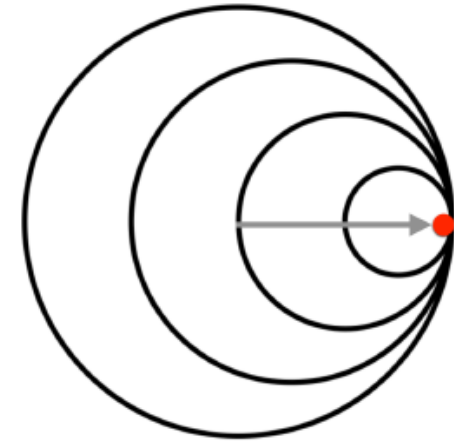


# Nonlinear transport of Wigner crystal (Bragg-Cherenkov scattering)

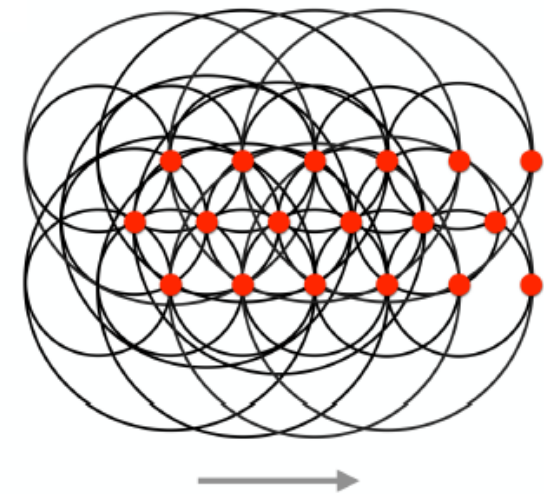


$$v_1 \equiv v_{ph}(\mathbf{G}_1) = \sqrt{\frac{\alpha \mathbf{G}_1}{\rho}}$$

Emission of ripples by moving  $e^-$ :



Constructive interference of ripples:



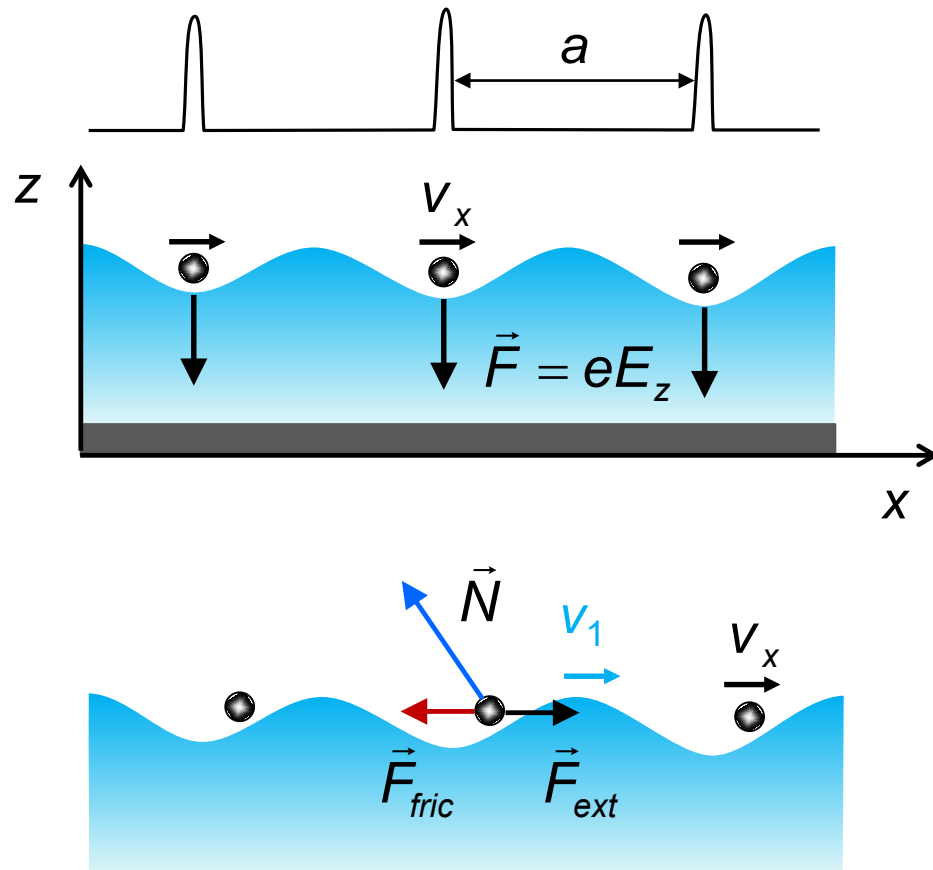
$$\mathbf{G}_n = \frac{2\pi}{a} n$$



# Bragg-Cherenkov emission of riplons

Dykman and Rubo, 1997

Vinen, 1999



Force exerted on liquid surface:

$$F = eE_z \sum_{n=-\infty}^{+\infty} \delta(x - an - v_x t) \approx \frac{eE_z}{a} e^{i(G_1 x - G_1 v_x t)}$$

BC at liquid surface:

$$-\frac{\partial F}{\partial t} + \rho \frac{\partial^2 \phi}{\partial t^2} - \alpha \frac{\partial}{\partial z} \left( \frac{\partial^2 \phi}{\partial x^2} \right) = 0$$

Deformation of liquid surface:

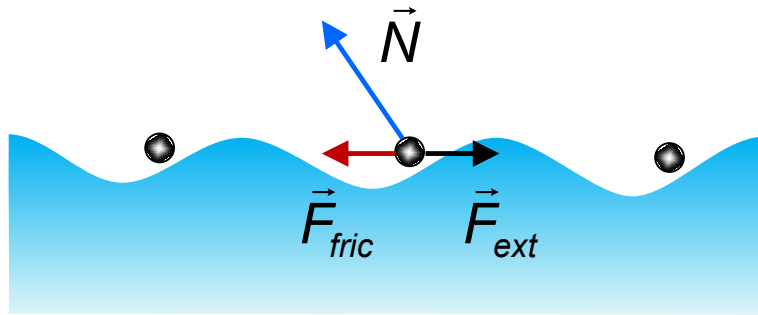
$$\zeta(x) = \frac{eE_z}{\rho G_1} \left( \frac{1}{v_x^2 - v_1^2} \right) e^{i(G_1 x - G_1 v_x t)} \xrightarrow{v_x \rightarrow v_1} \infty$$

Include damping:

$$\zeta(x) = \frac{eE_z}{\rho G_1} \left( \frac{1}{v_x^2 - v_1^2 + i v_d v_x} \right) e^{i(G_1 x - G_1 v_x t)}$$



# Nonlinear transport of Wigner crystal



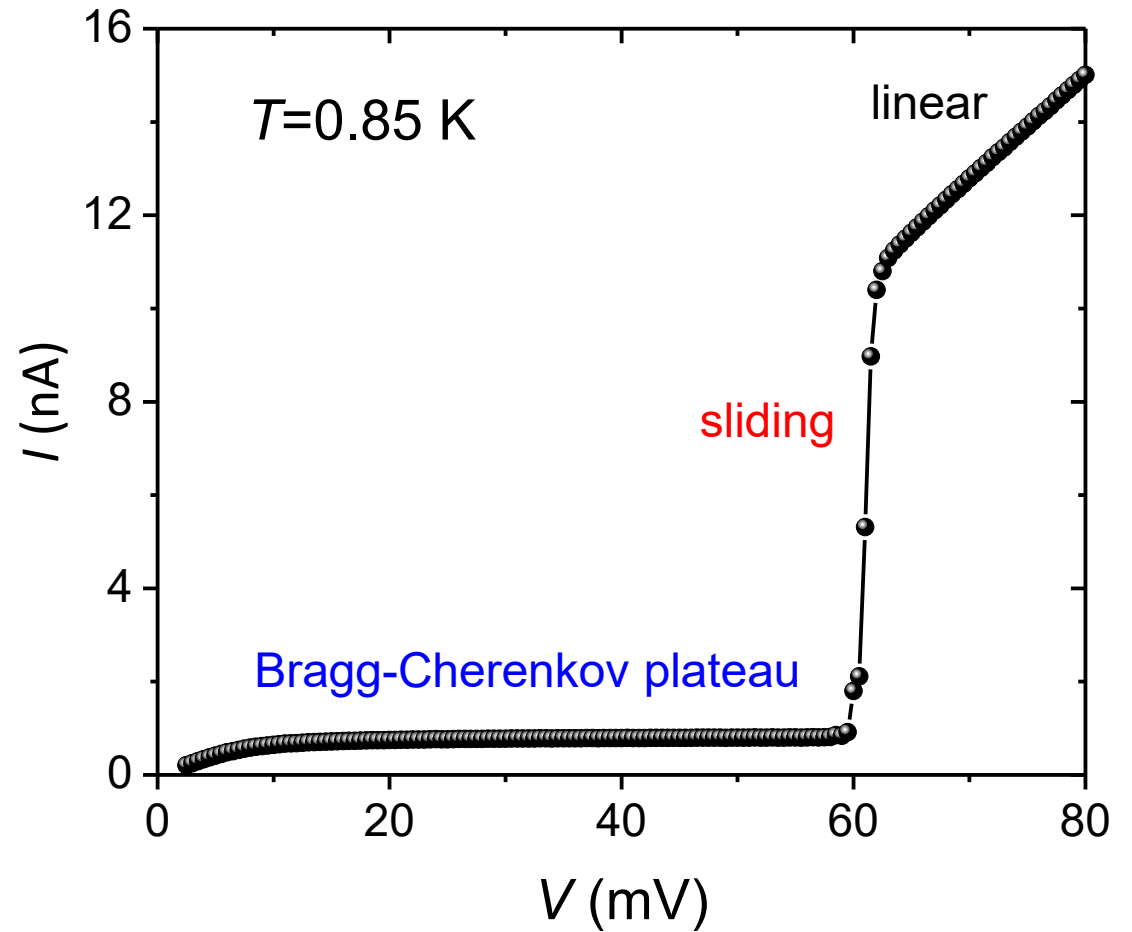
Vinen, 1999

Friction force on Wigner crystal:

$$F_{fric} = \frac{e^2 E_z^2}{\rho a} \frac{v_d v_x}{\left( (v_x^2 - v_1^2)^2 + v_d^2 v_x^2 \right)}$$

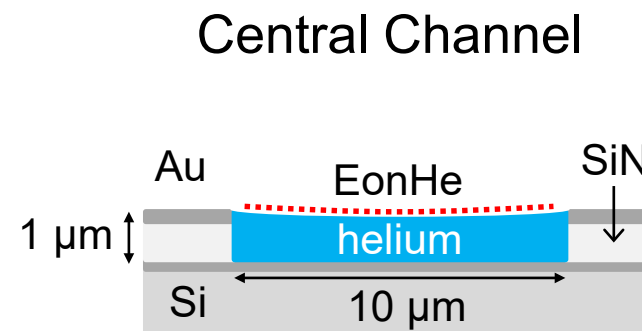
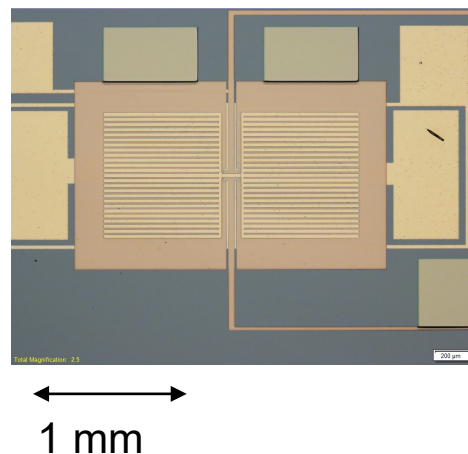
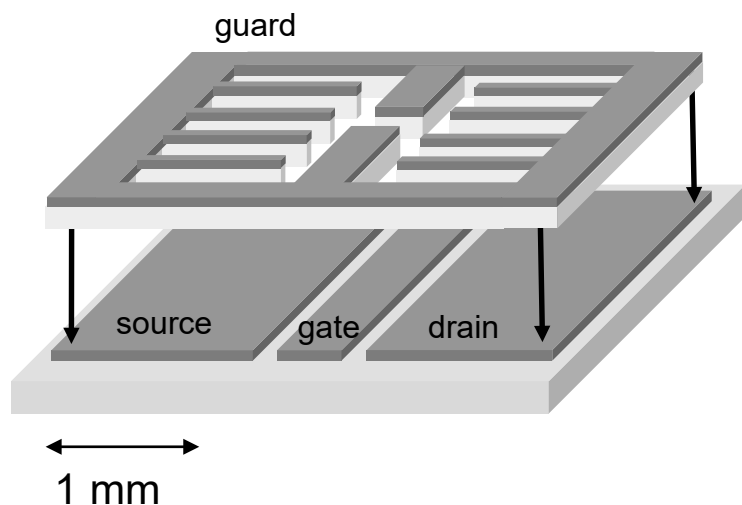
$$F_{fric}^{(max)} = \frac{e^2 E_z^2}{\rho a v_d v_1} < F_{ext}$$

I-V curve

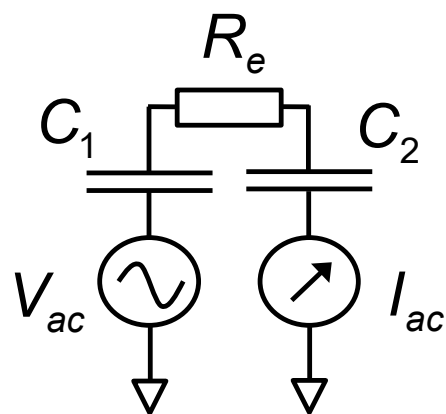
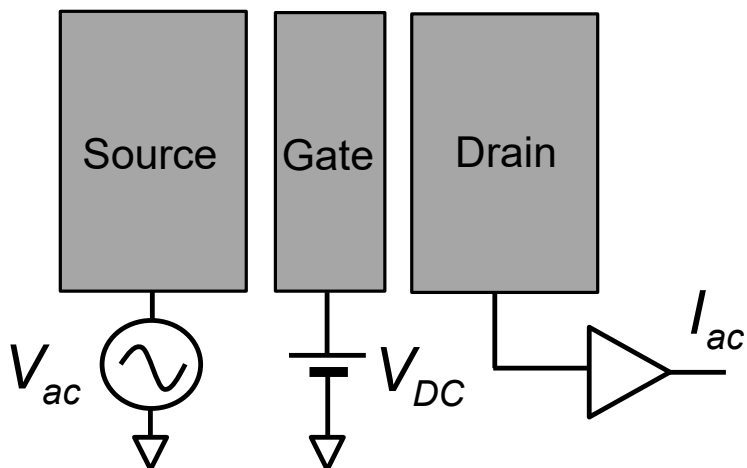




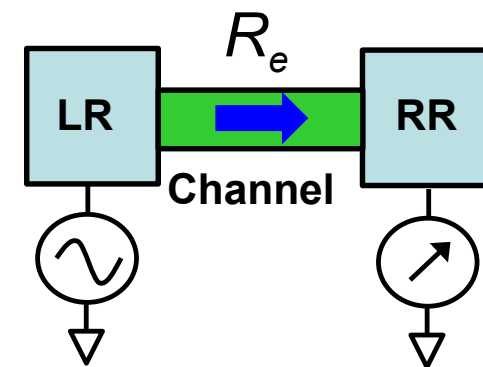
# Microchannel devices



## Helium FET



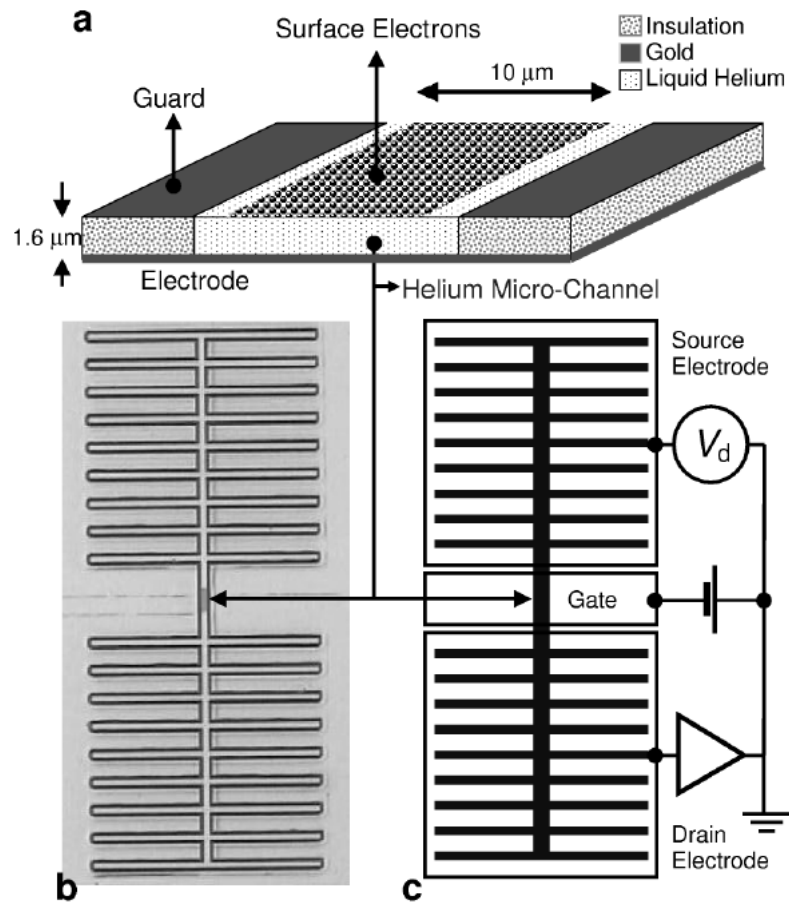
Sommer-Tanner



2-terminal device

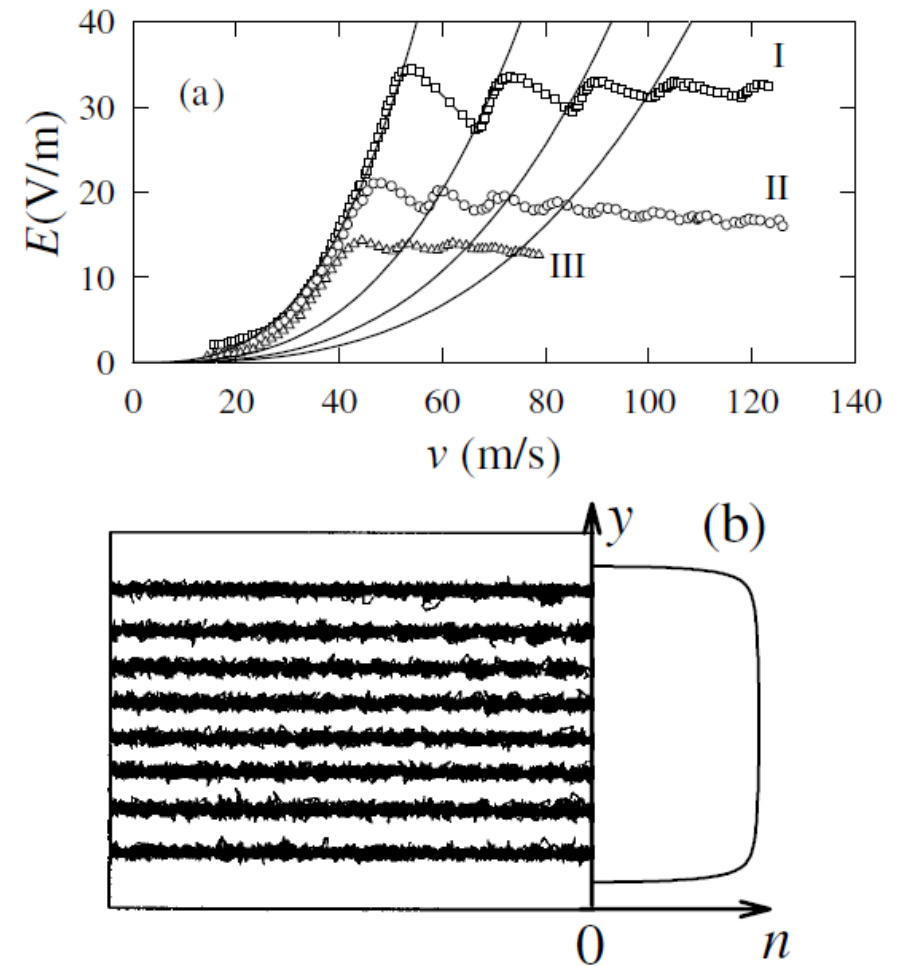


# The first microchannel device



*Glasson et al., PRL 2001*

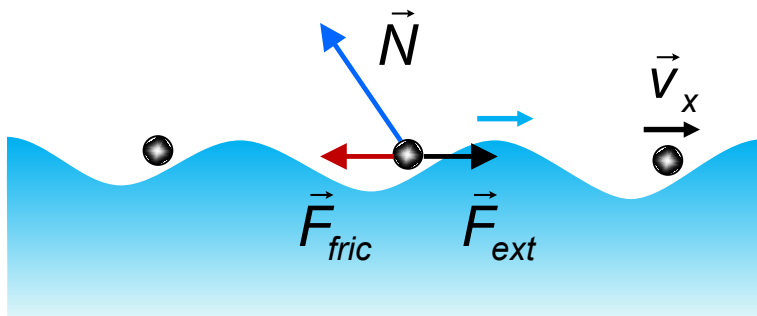
## Force-velocity (field-current) curve



Stripe phase of Wigner crystal



# Damping in Vinen's model



Vinen, 1999

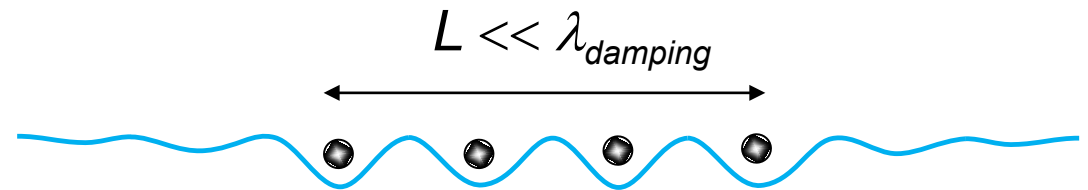
Friction force on Wigner crystal:

$$F_{fric} = \frac{e^2 E_z^2}{\rho a} \frac{v_d v_x}{\left( (v_x^2 - v_1^2)^2 + v_d^2 v_x^2 \right)}$$

$$F_{fric}^{(max)} = \frac{n_s e^2 E_z^2}{\rho v_d v_1}$$

Damping parameter  $v_d$

- Damping of ripplons due to dissipation
- Finite size of Wigner crystal



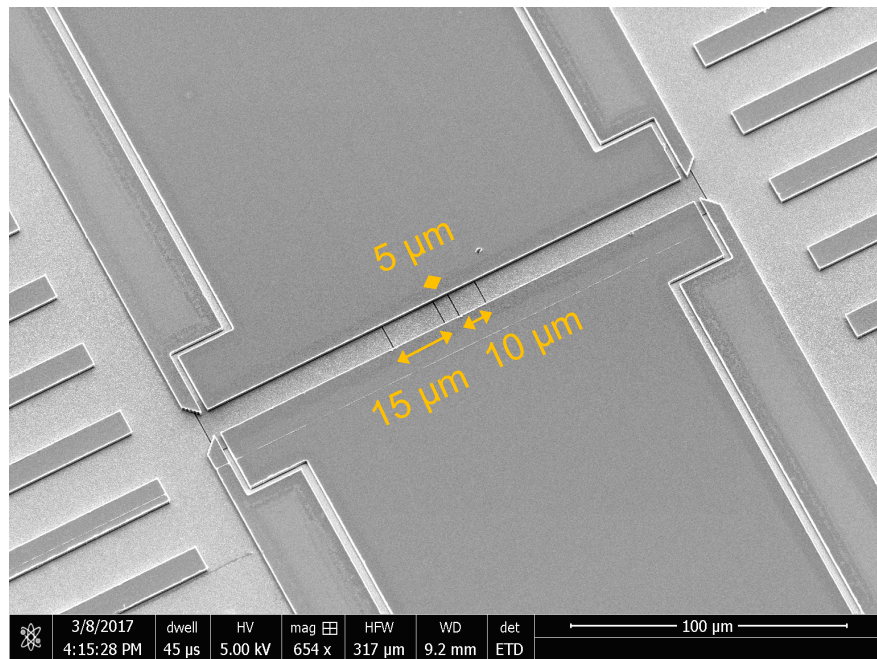
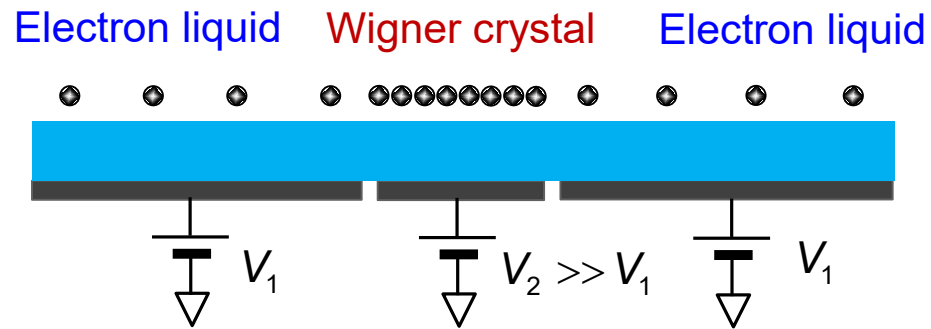
$$F = eE_z \sum_{n=0}^{+N} \delta(x - an - v_x t) \approx$$

$$\approx \frac{NeE_z}{\pi} \sum_{m=-\infty}^{+\infty} e^{i(qx - mqv_x t)} \frac{\sin[L(q - mG_1)/2]}{L(q - mG_1)}$$

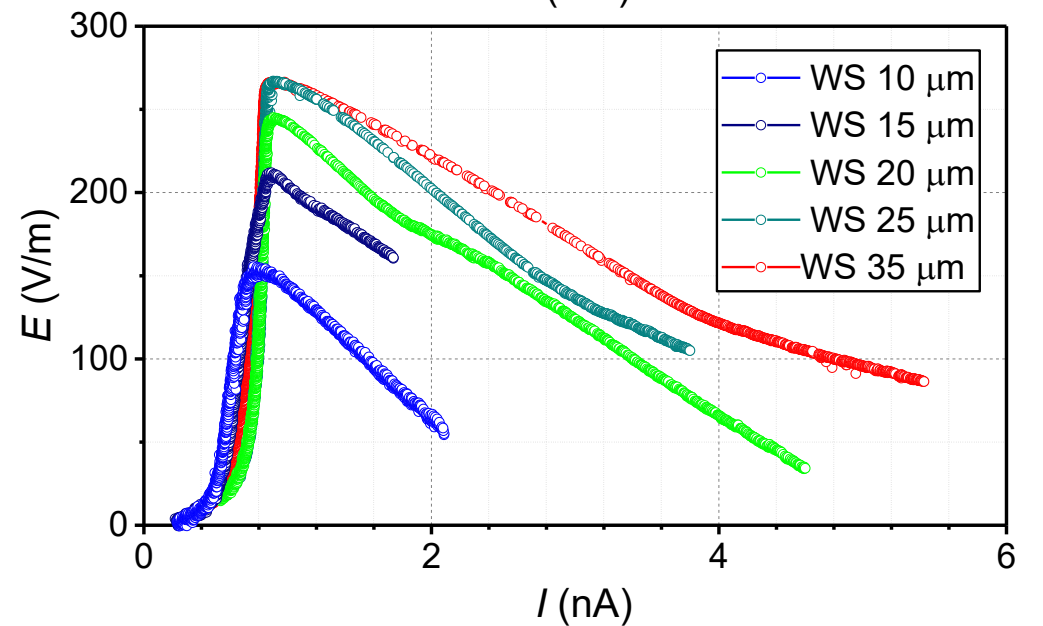
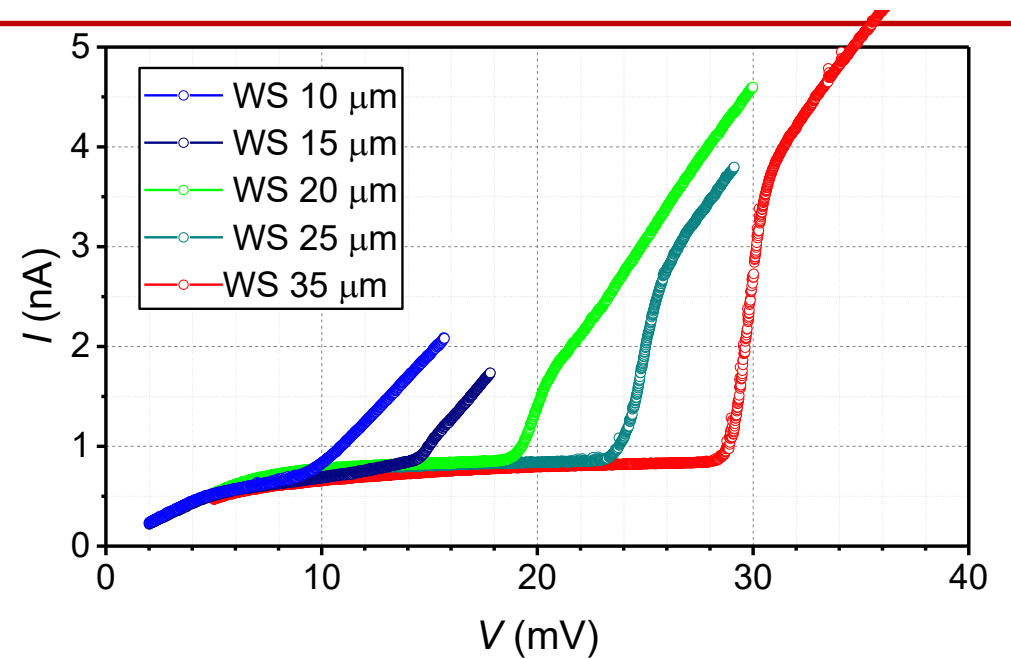
$$F_{fric}^{(max)} = \frac{n_s e^2 E_z^2}{\rho v_d v_1} \left( 1 - e^{-\frac{LG_1}{2} \left( \frac{v_d}{v_1} \right)} \right)$$



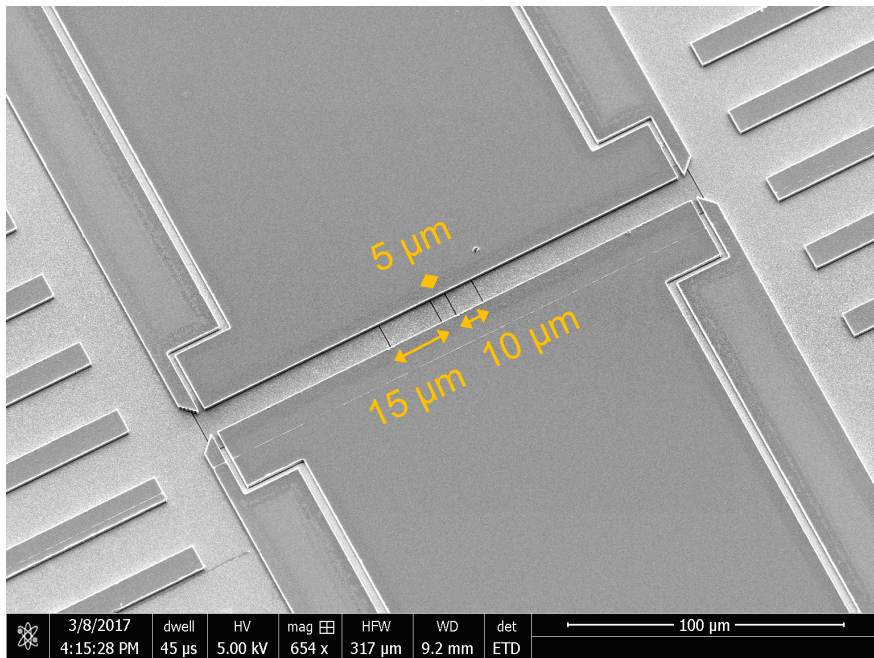
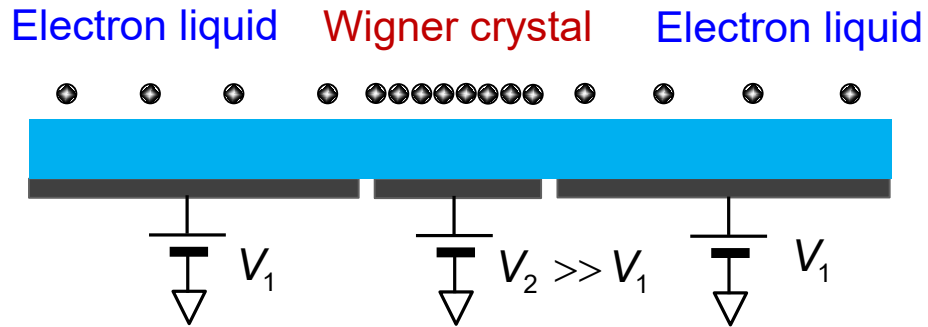
# Wigner crystal of finite size



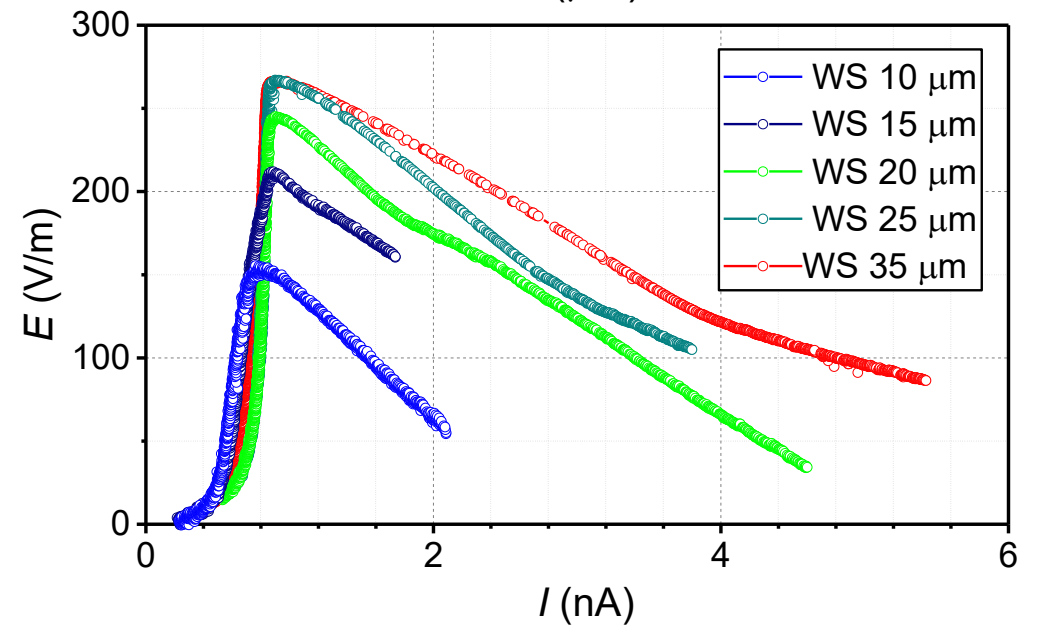
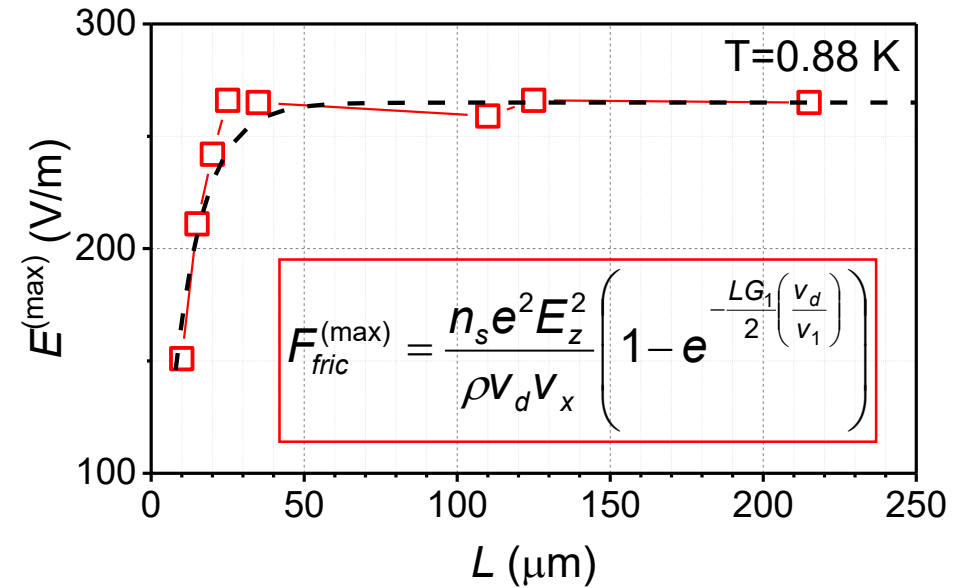
Lin et al., PRB **98**, 085412 (2018)



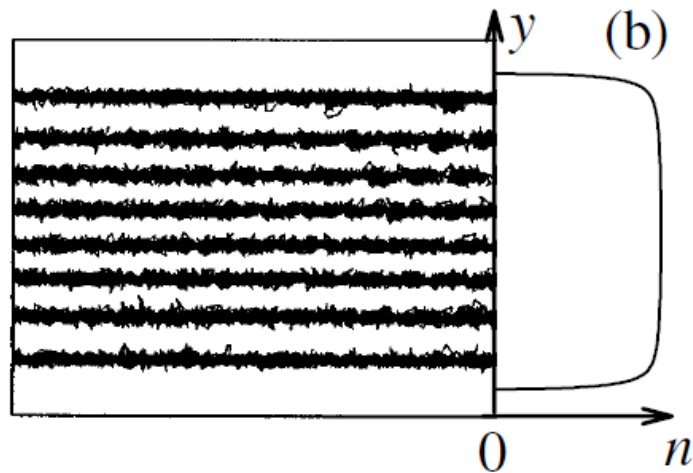
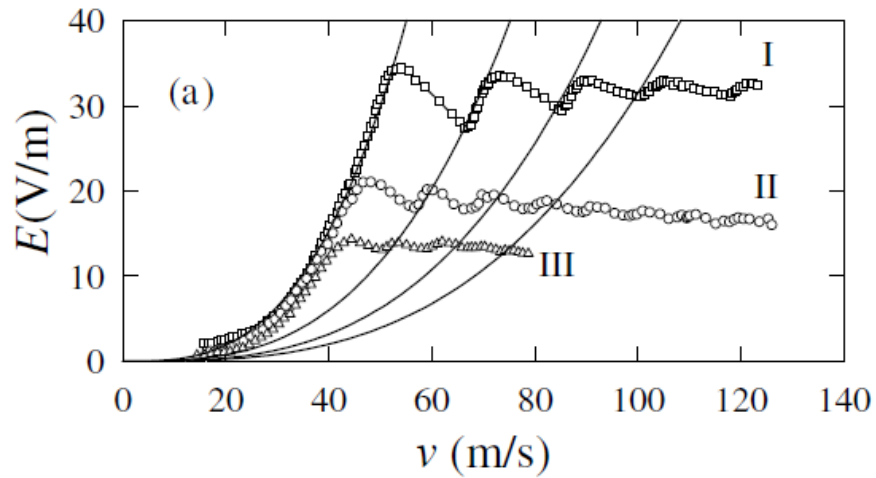
# Wigner crystal of finite size



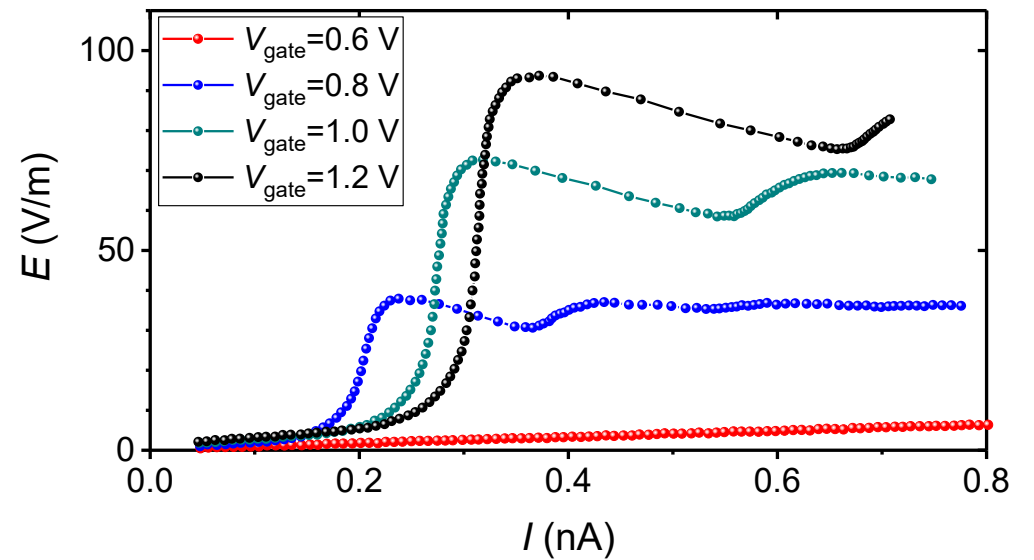
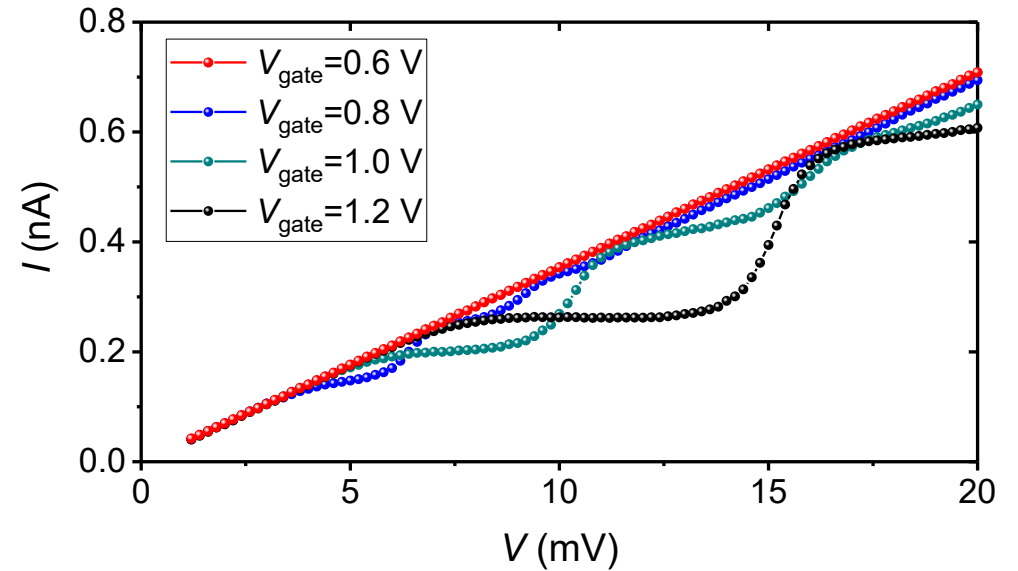
Lin et al., PRB **98**, 085412 (2018)



# Stripe phase Wigner crystal

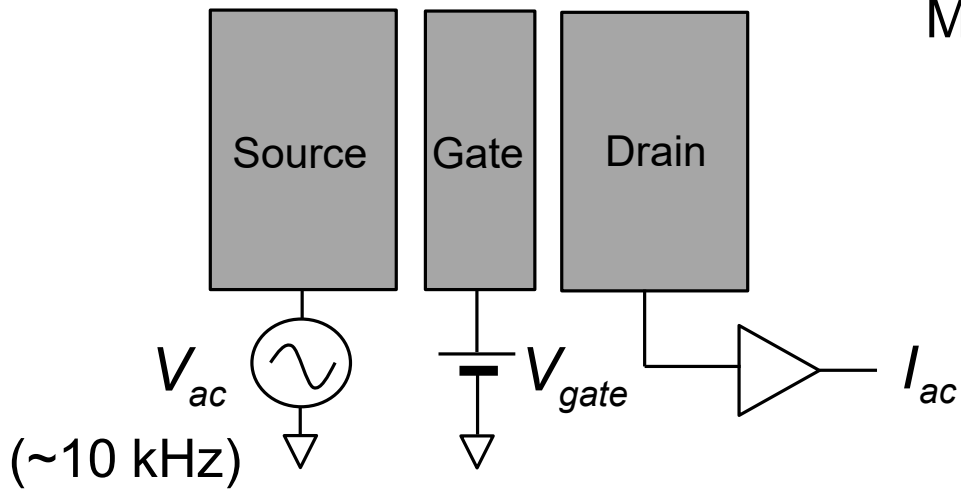


*Glasson et al., PRL 2001*



# Time-resolved measurements

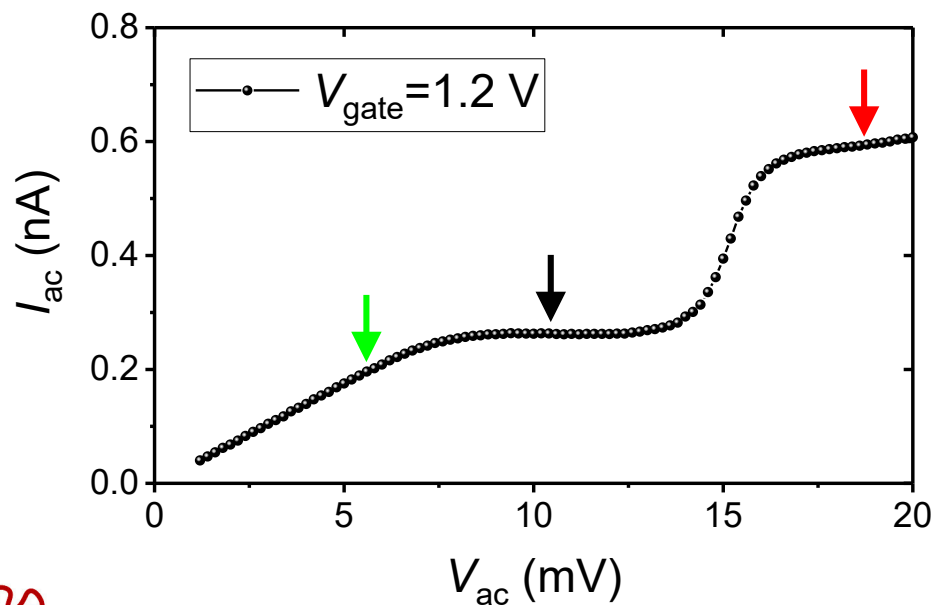
## Helium FET



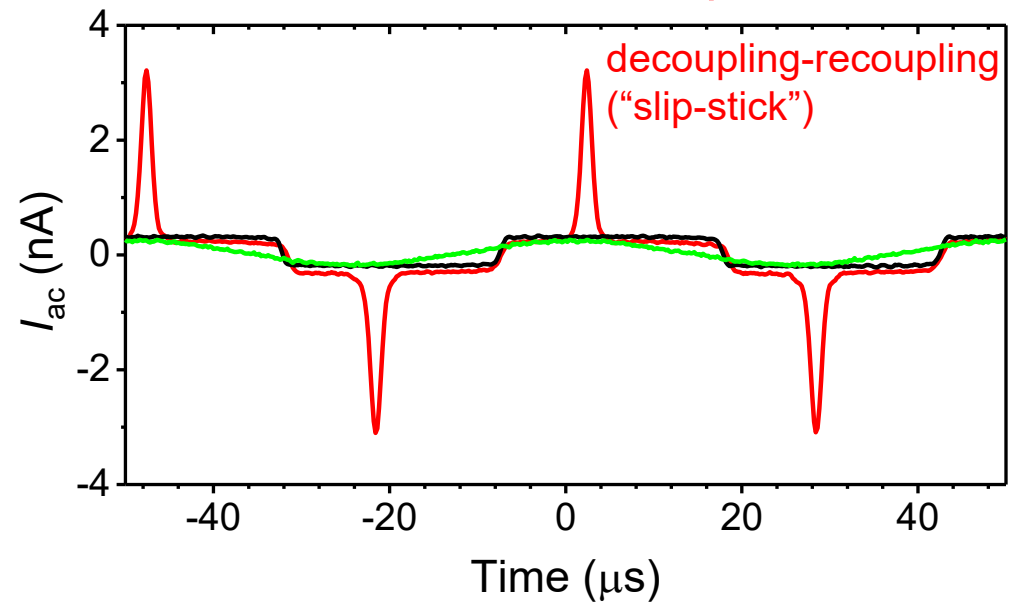
Measured current  $I_{ac}$

- Time-averaged using locking-in
- Time-resolved using fast oscilloscope

*Rees et al., PRL 2016*

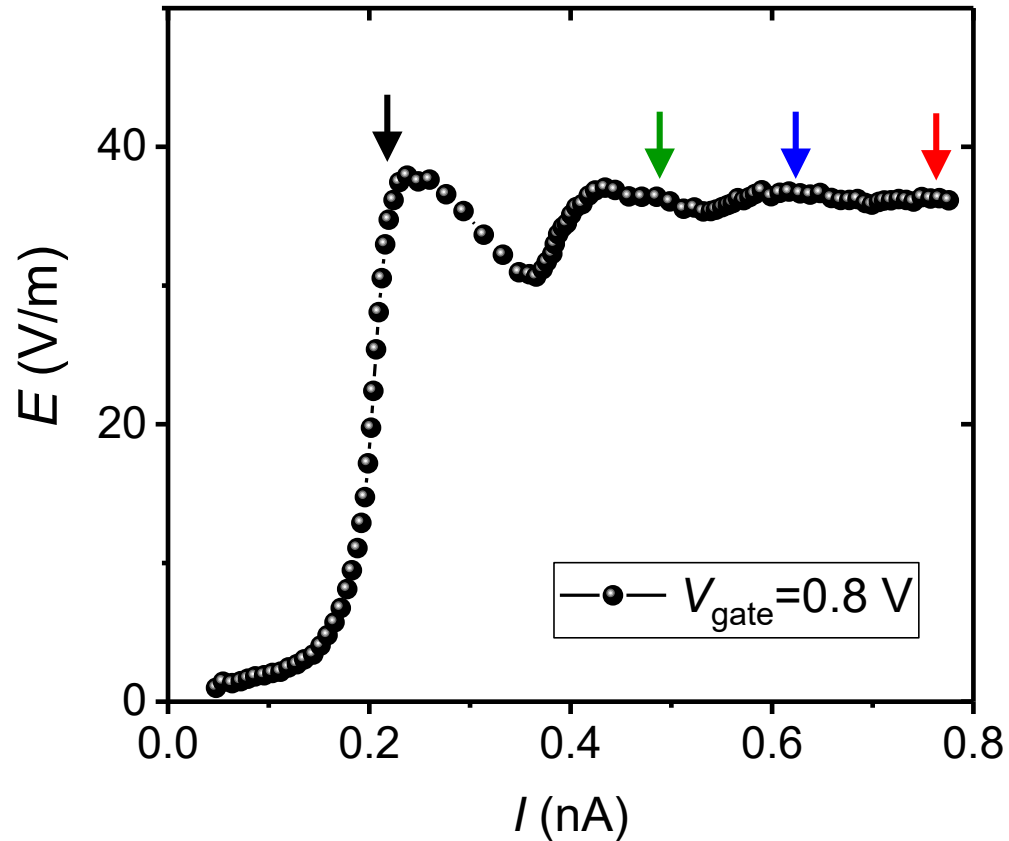


## Time-resolved response

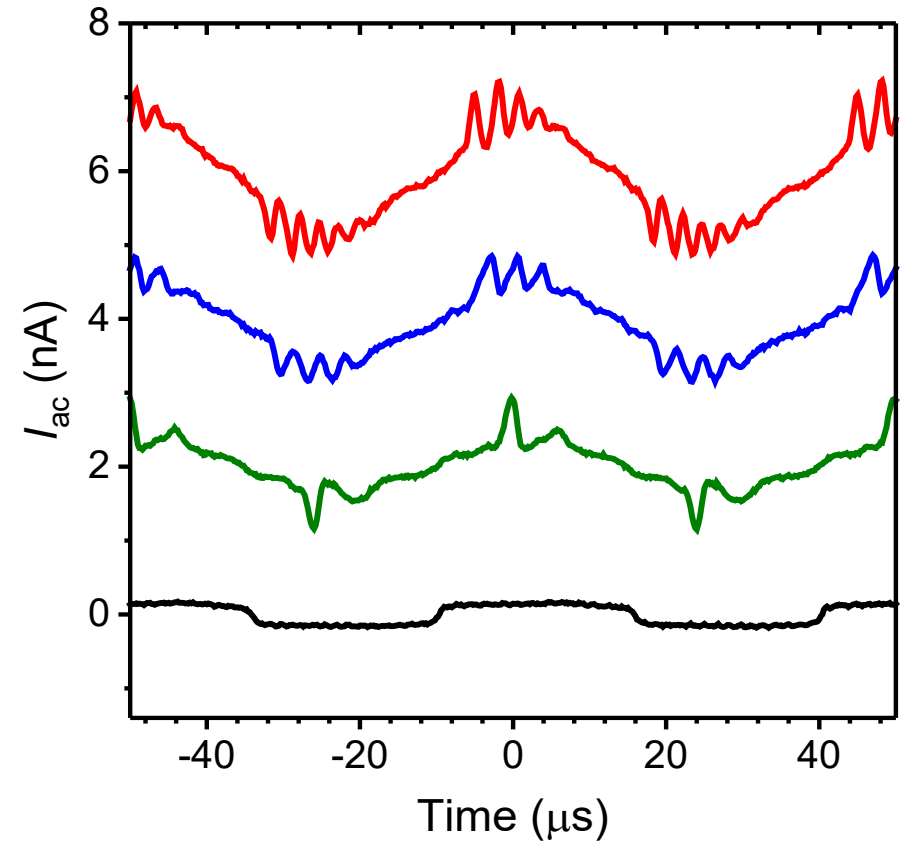


# Repetitive Slip-Stick Transitions

Time-averaged response



Time-resolved response

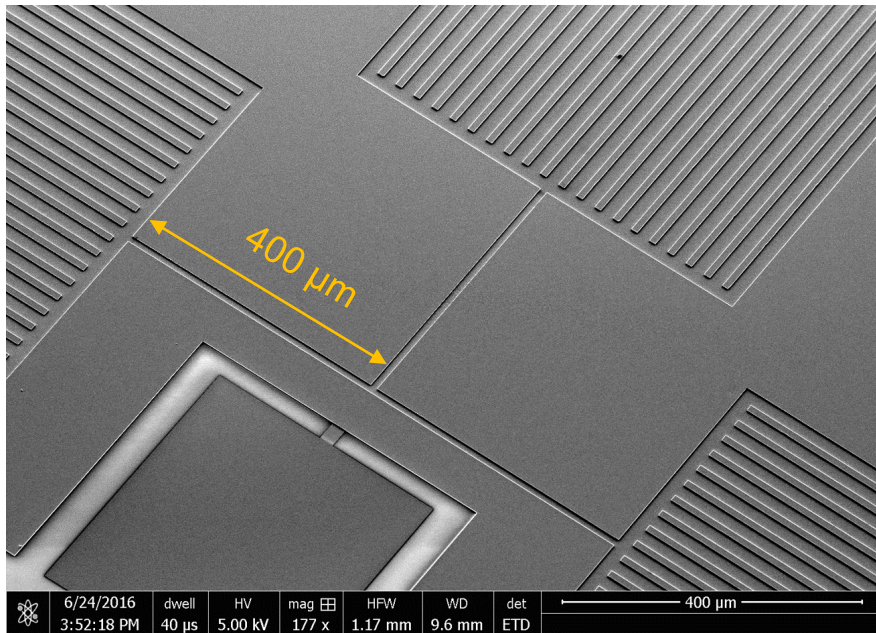
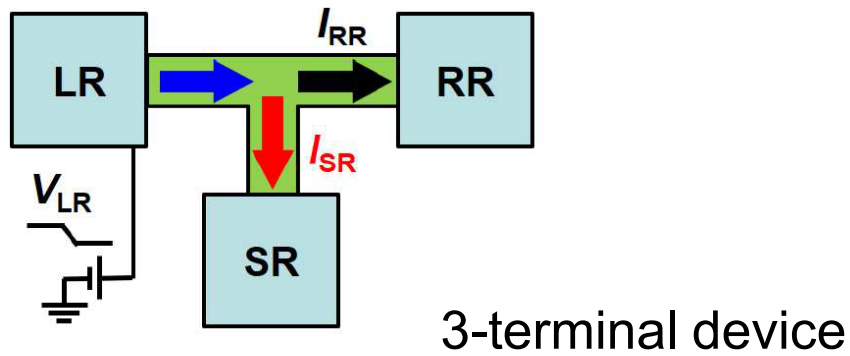


Zou et al., *PRB* 104, 045427 (2021)

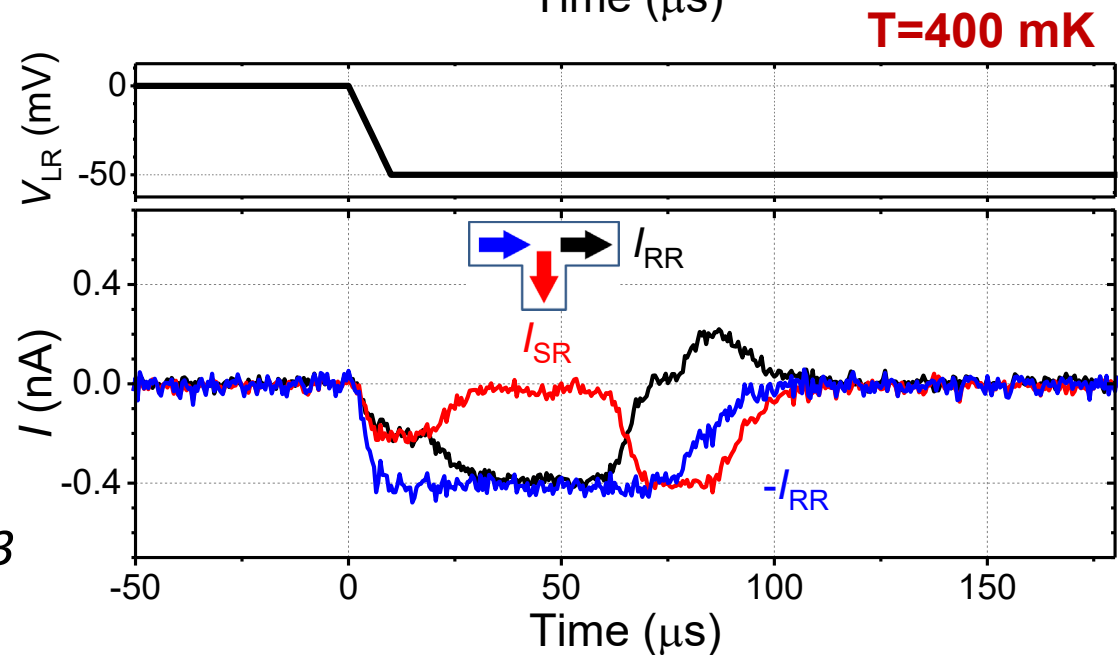
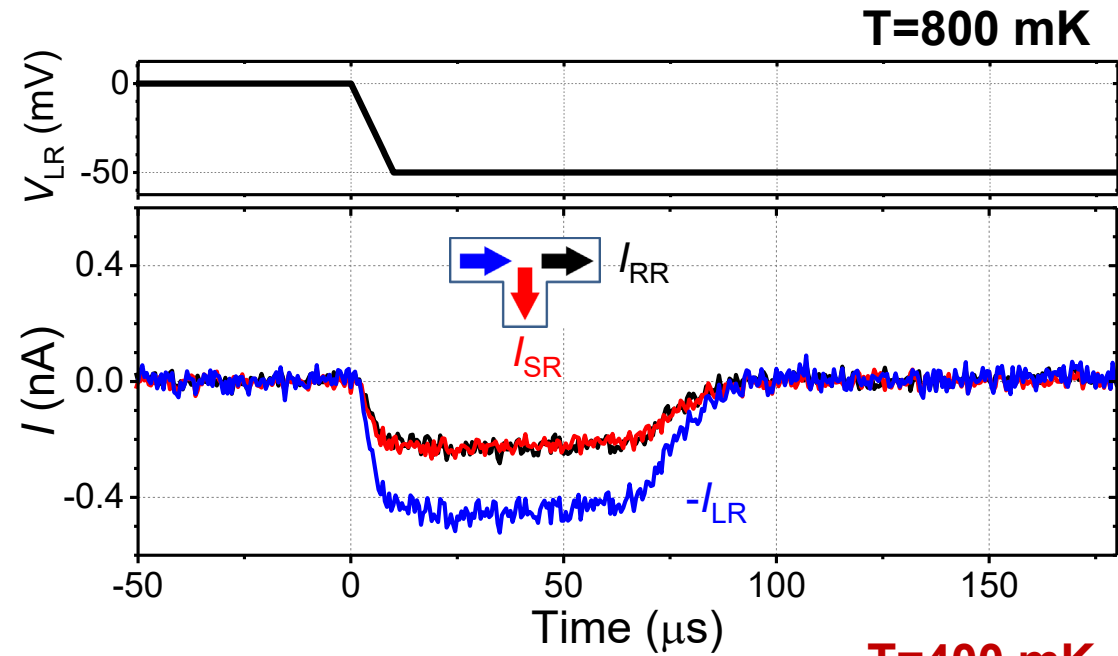




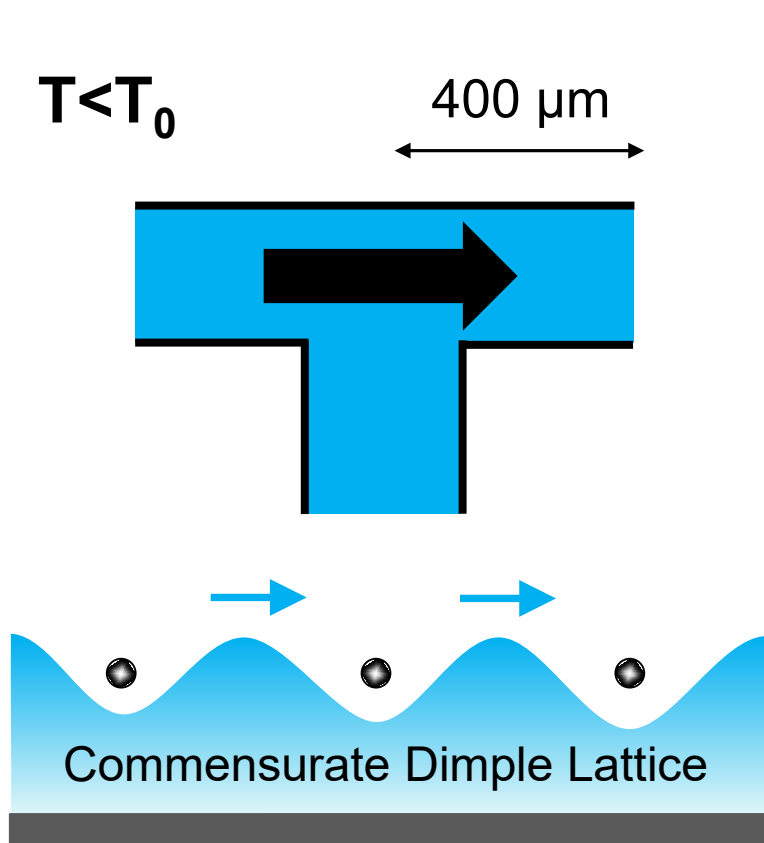
# Transport through 3-terminal device



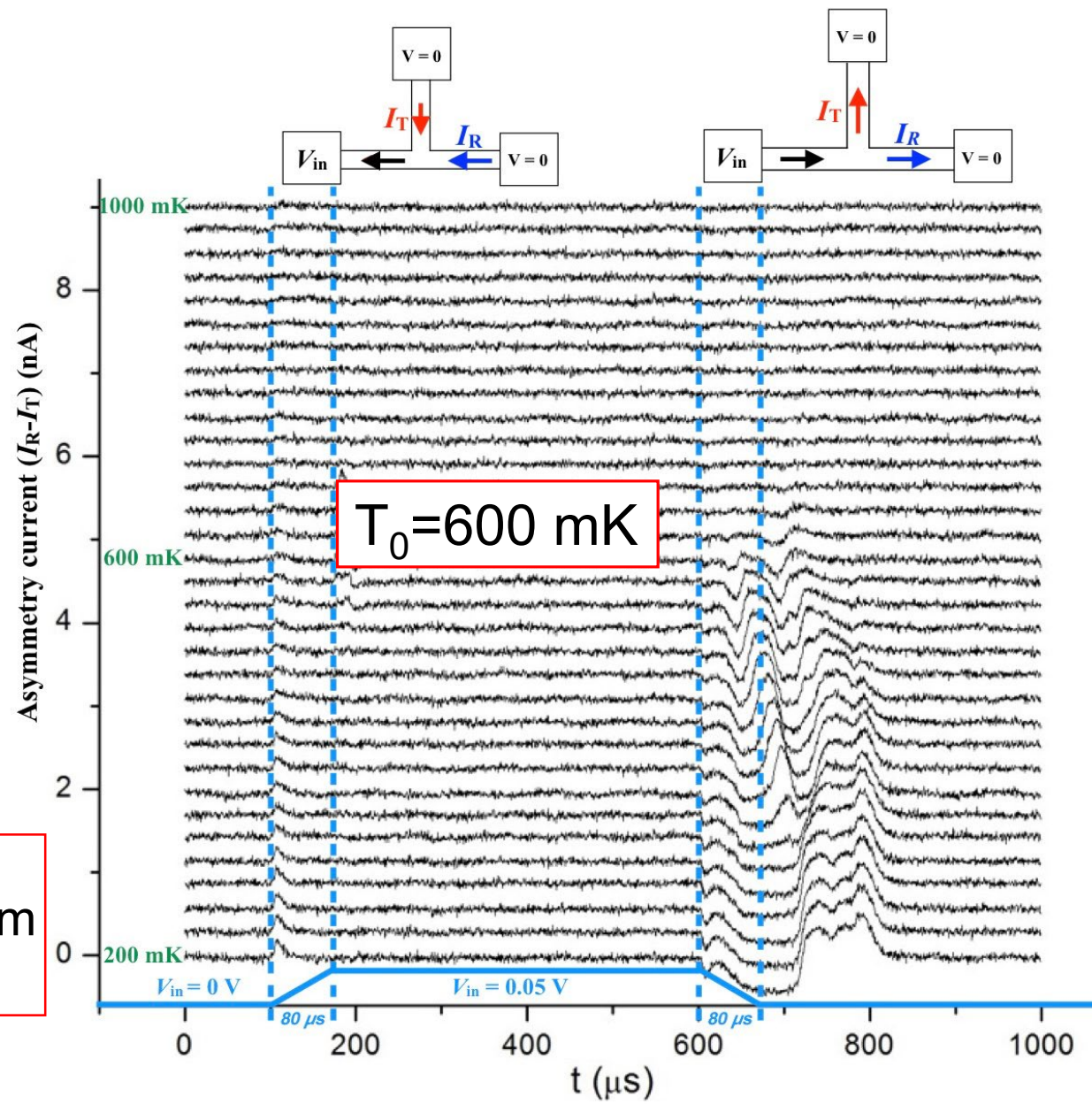
*Badrutdinov et al., PRL 124, 126803 (2020)*



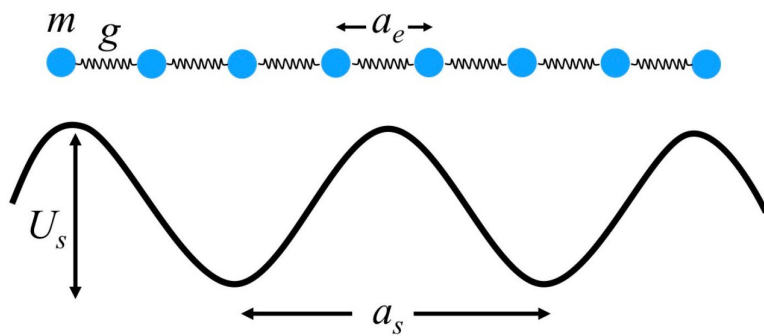
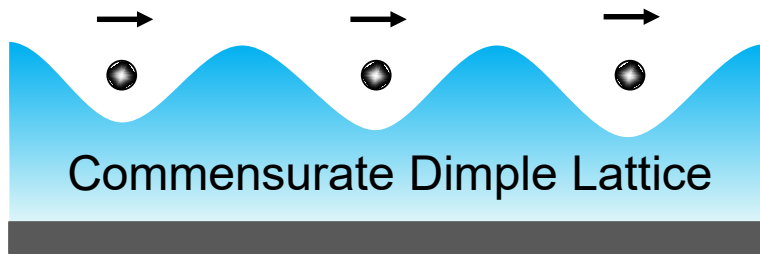
# Unidirectional (polaronic) transport



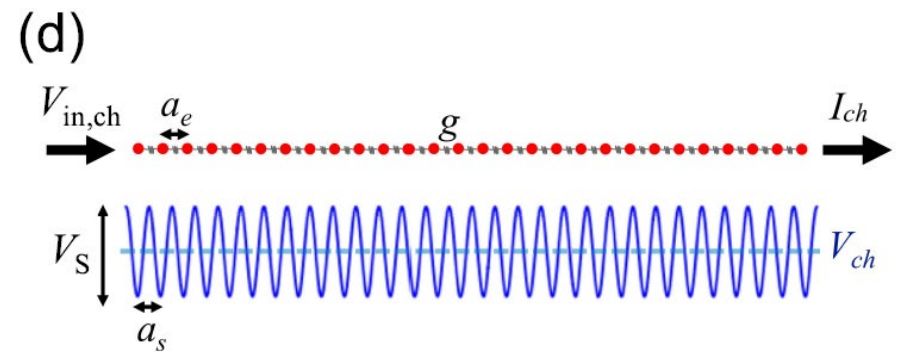
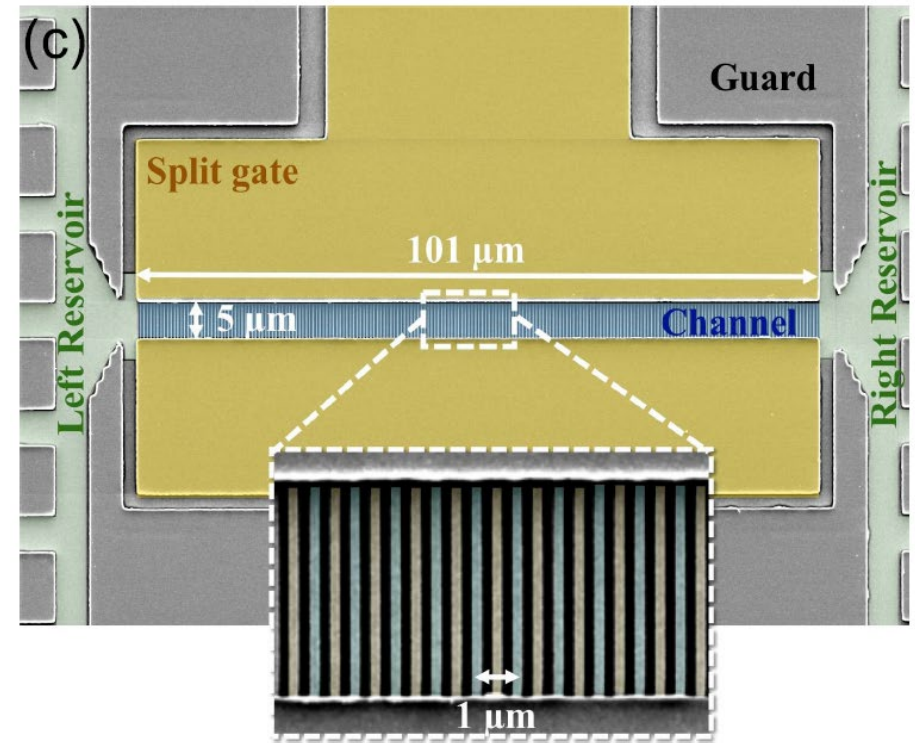
$$\lambda_{\text{damping}} = \frac{\pi^2 \hbar G_1^2}{90 \rho \omega_{G_1}} \left( \frac{k_B T}{\hbar s} \right)^4 \Bigg|_{600 \text{ mK}} = 230 \mu\text{m}$$



# Prospects



Frenkel-Kontorova Model (FKM)



Lin et al., *JLTP* **195**, 289 (2018)



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## PhD Students

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Shan Zoe



<https://www.groups.oist.jp/qdu>

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Oleksiy Smorodin  
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<https://www.oist.jp>

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OIST



KAKENHI MEXT

