Strong coupling of an electron ensemble on the surface of liquid helium to a microwave cavity

Denis Konstantinov, Quantum Dynamics Unit
OIST Graduate University

OIST

OKINAWA INSTITUTE OF SCIENCE AND TECHNOLOGY GRADUATE UNIVERSITY

Okinawa Institute of Science and Technology (OIST) Graduate University

https://www.oist.jp

Mission:

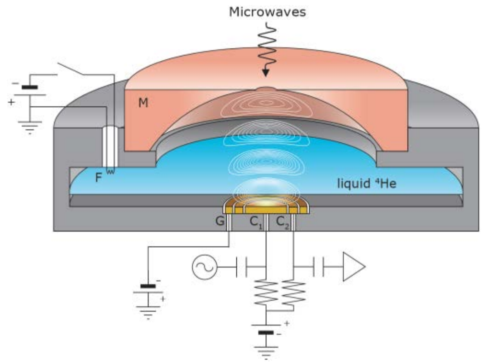
- sustainable development of Okinawa
- advancement of science and technology in Japan

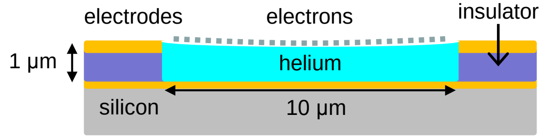
People: 56 faculty members / 440 researchers / 134 graduate students

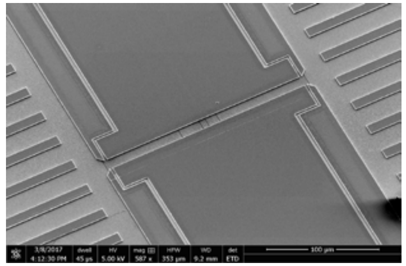
Expect 100 faculty members in 2024 (towards 300 faculty members)

Quantum Dynamics Unit (QDU)

2D electrons on helium





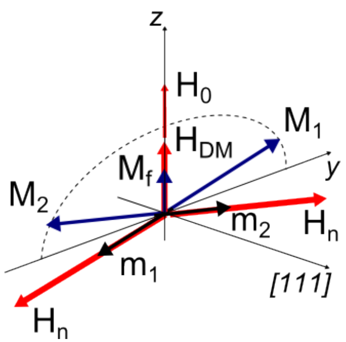


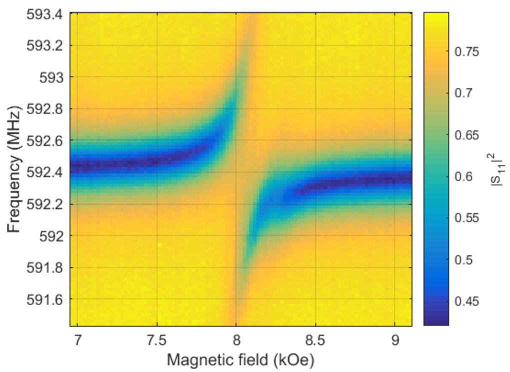
PRL115, 256802 (2015)
PRL117, 056803 (2016)

PRB 94, 195311 (2016)

Nuclear spins ensembles in $MnCO_3$

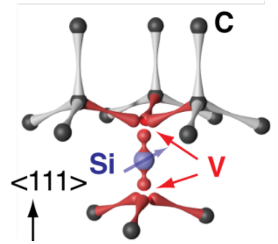
PRL114, 226402 (2015)



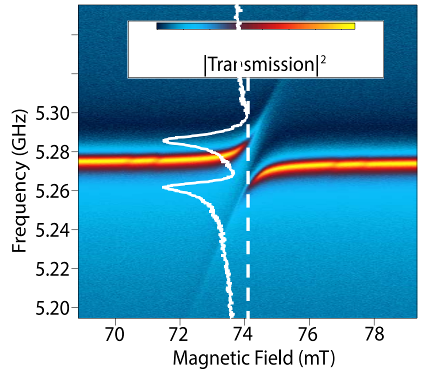


Impurity spins in diamond

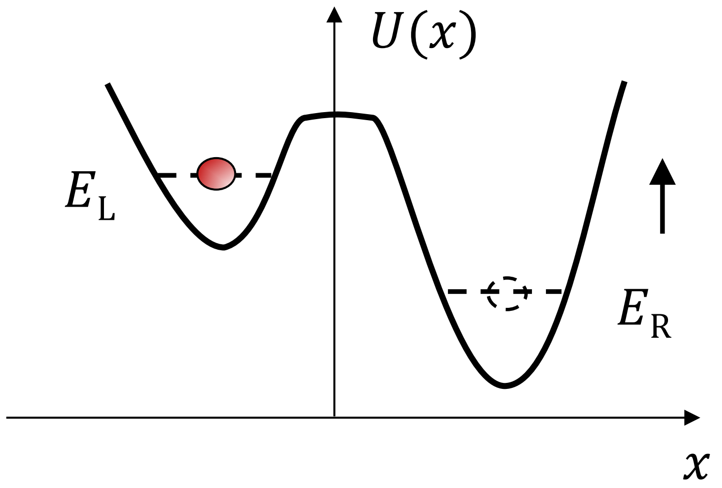
Current experiments

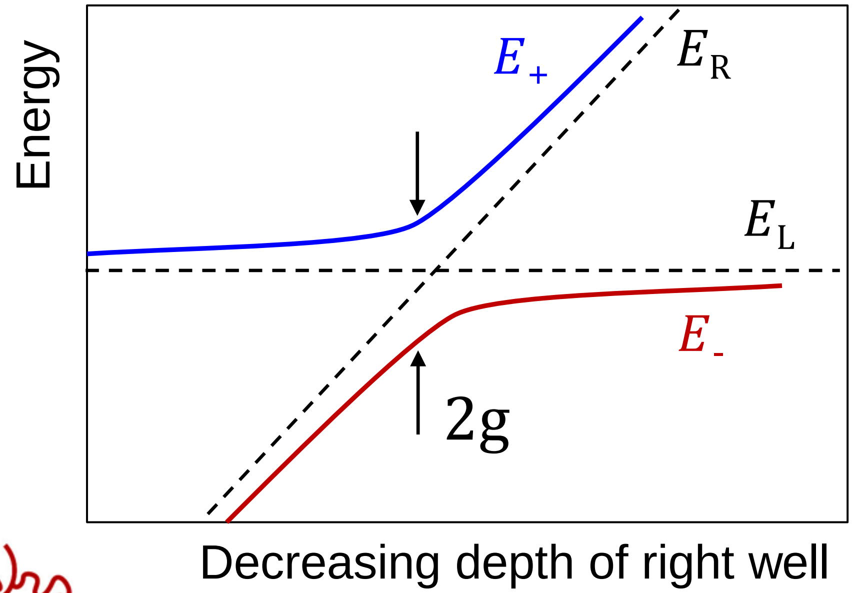


NV and SiV centers



Avoided crossing: coupled potential wells





Particle in two potential wells $U(x)$

$$H = \begin{pmatrix} E_L & 0 \\ 0 & E_R \end{pmatrix}$$

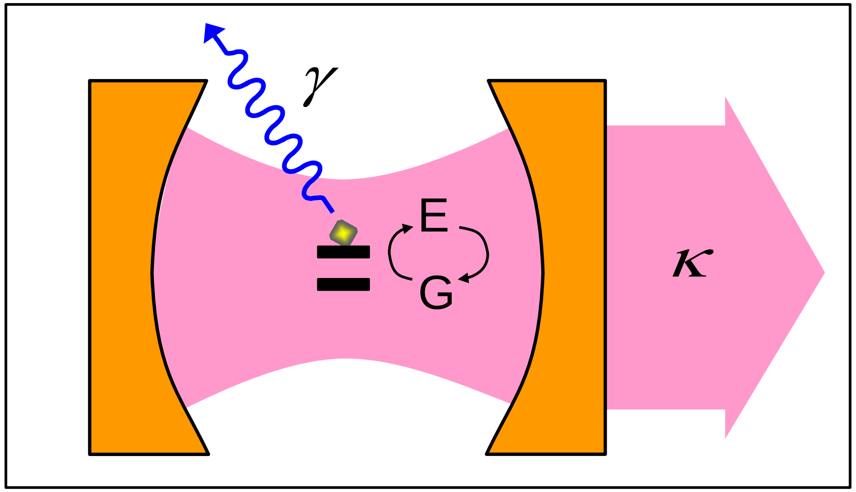
Add tunneling between two wells

$$H = \begin{pmatrix} E_L & g \\ g & E_R \end{pmatrix}$$

$$E_{\pm} = \frac{E_L + E_R}{2} \pm \sqrt{\frac{(E_L - E_R)^2}{4} + g^2}$$

[Landau and Lifshitz, Quantum Mechanics, Vol 2]

Avoided crossing: cavity QED



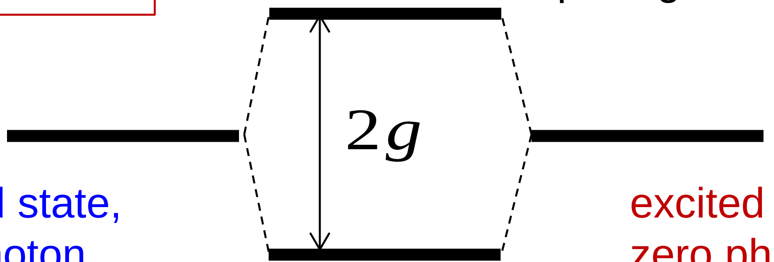
$$H/\hbar = \omega_r a^+ a + \omega_s s_z + g\left(a s^+ + a^+ s^-\right)$$

Jaynes-Cummings Hamiltonian

$$H/\hbar = \begin{pmatrix} -\frac{\omega_s}{2} + \omega_r & g \\ g & \frac{\omega_s}{2} \end{pmatrix}$$

Strong coupling regime: $g >> \gamma, \kappa$

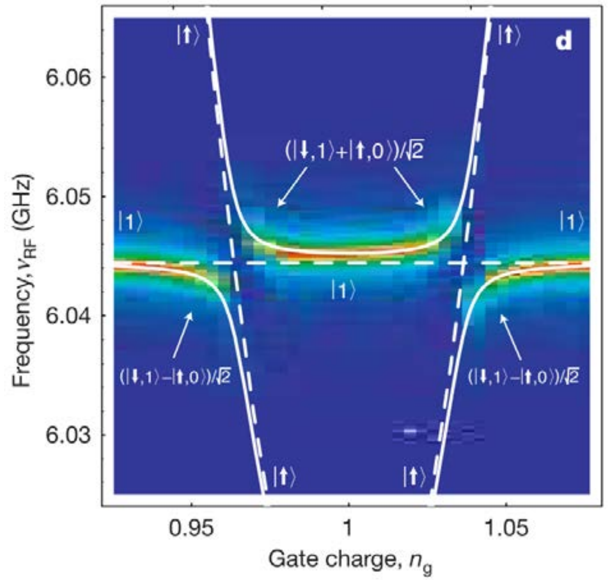
$$\omega_{21} = \omega_c$$



vacuum Rabi splitting

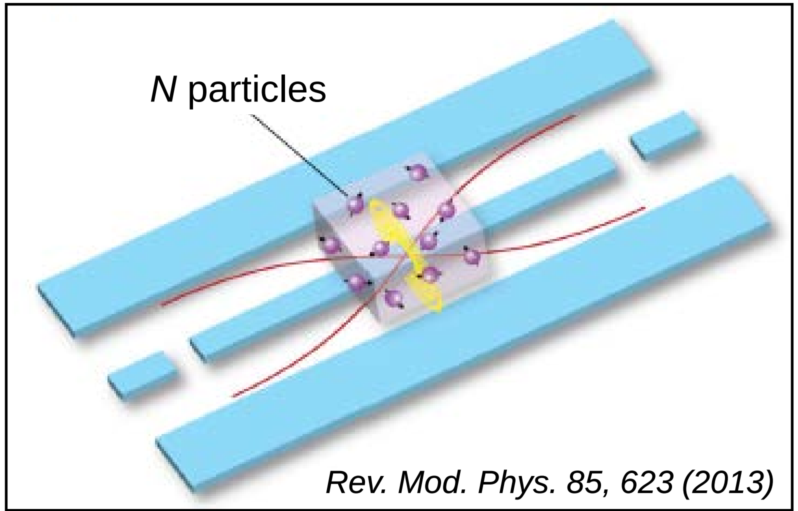
ground state, one photon

excited state, zero photons



[A. Wallraff et al., Nature **431** 162 (2004)]

Avoided crossing: coupling to an ensemble of particles



Rev. Mod. Phys. 85, 623 (2013)

coupling to a single particle

$$H/\hbar = \omega_r a^+ a + \omega_s \sum_{j=1}^{N} s_j^z + g \sum_{j=1}^{N} \left(a s_j^+ + a^+ s_j^- \right)$$

$$\omega_s S_z \qquad g\left(a S^+ + a^+ S^- \right)$$

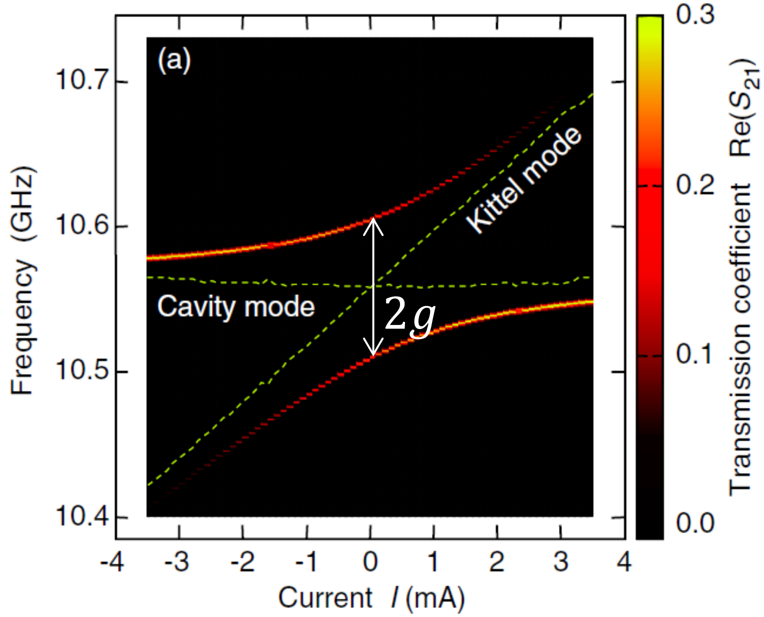
Enhancement of coupling by $\sqrt{N}$!!!

$$|G\rangle = |\uparrow\uparrow\uparrow \ldots\rangle$$

$$|E\rangle = \frac{1}{\sqrt{N}} \left(|\downarrow\uparrow\uparrow \ldots\rangle + |\uparrow\downarrow\uparrow \ldots\rangle + |\uparrow\uparrow\downarrow \ldots\rangle + \ldots \right)$$

$$g_N = g \langle G | \hat{a}^+ \sum_{j=1}^{N} \hat{\sigma}_j^- | E \rangle = g\sqrt{N}$$

Dicke model, 1954



[Y. Tabuchi et al., PRL **113**, 083603 (2014)]

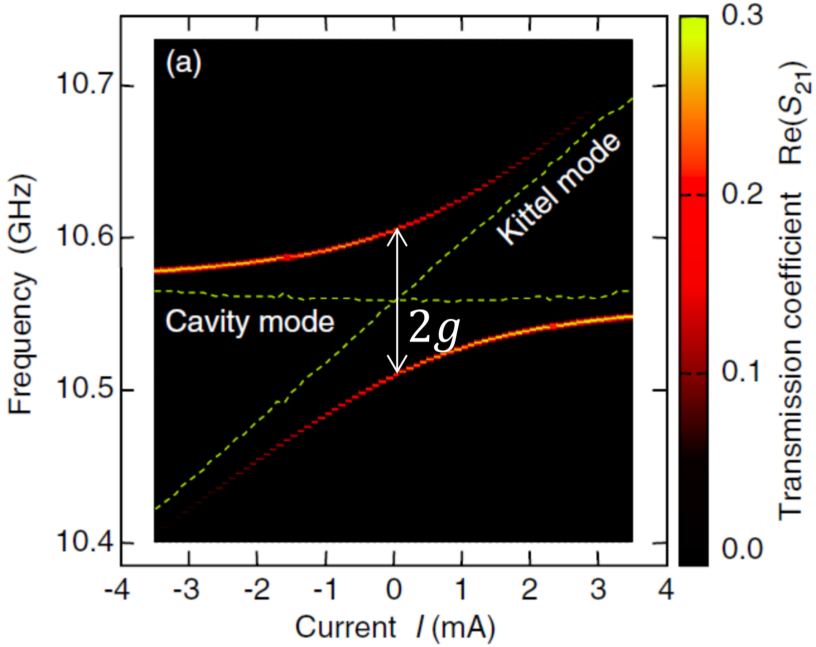
Applications: hybrid quantum computer

"memory" ⟷ photons ⟷ "processor"

spin system:
- difficult to manipulate
- coherence time is long

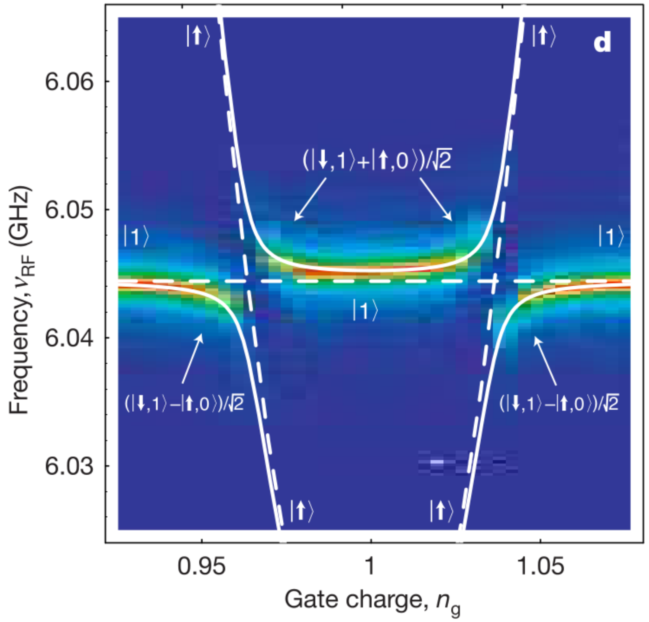
superconducting qubits:
- easy to manipulate
- coherence time is short

Ensemble-photon coupling



[Y. Tabuchi et al., PRL **113**, 083603 (2014)]

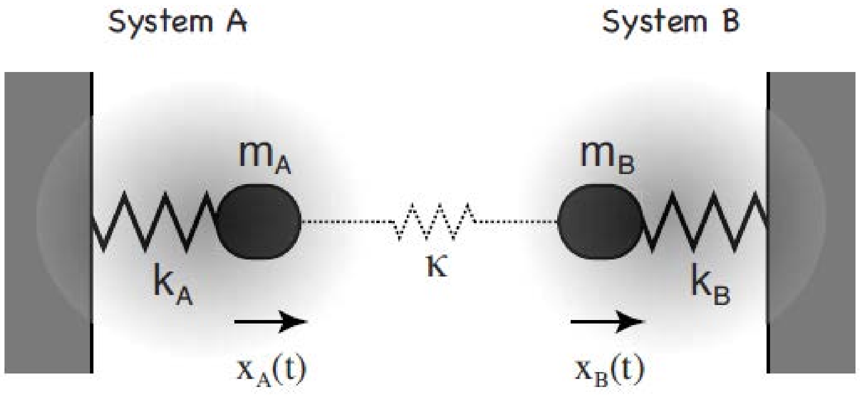
SC-qubit-photon coupling

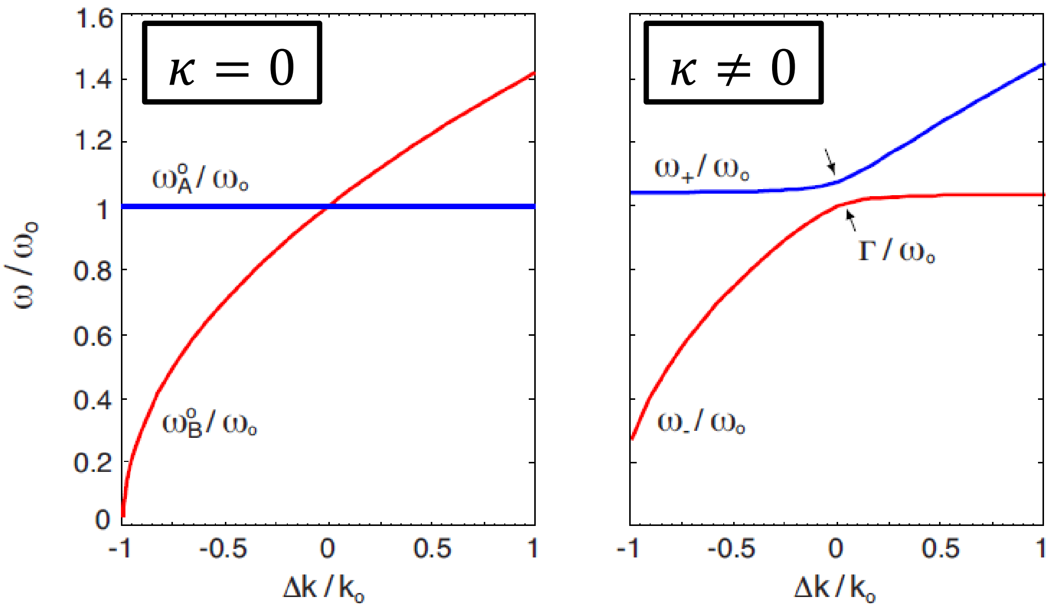


[A. Wallraff et al., Nature **431** 162 (2004)]

Coupling to ensembles: classical of quantum?

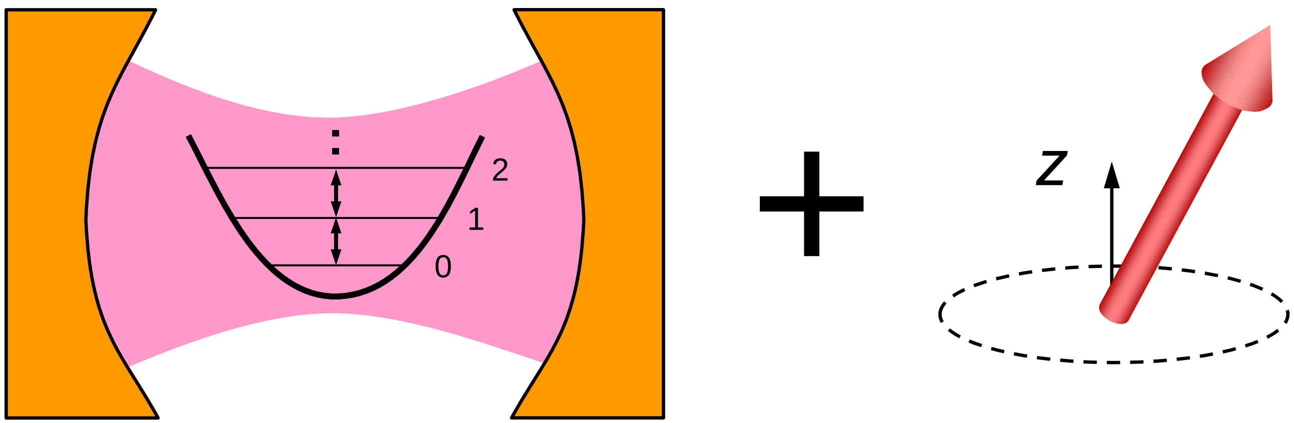
Coupled spring-mass oscillators





Coupled quantum oscillators

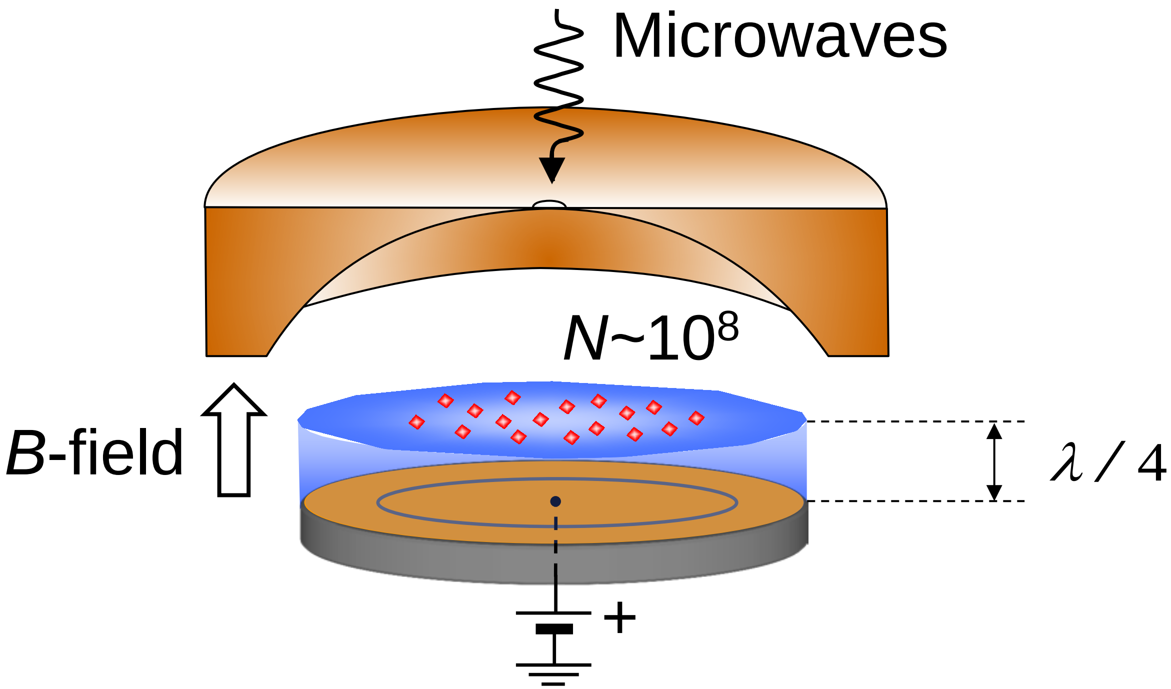
[L. Novotny, Am. J. Phys. **78**, 1199 (2010)]



Macroscopic spin with

$$\langle S \rangle = \frac{\hbar N}{2} >> \hbar$$

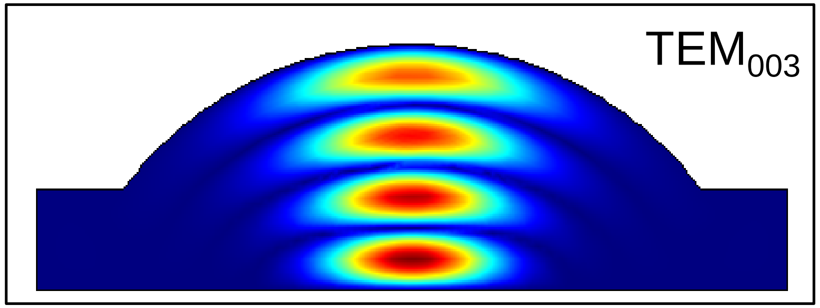
Ensemble of electrons on liquid helium



Resonant mode: TEM_{00q}

Frequency : 35-140 GHz

Quality factor: 1,000-10,000



Second Newton law:

$$m_e \frac{d\vec{v}}{dt} = -e\vec{E}_{MW} - \frac{e}{c}\vec{v}\times\vec{B} - \nu m_e \vec{v}$$

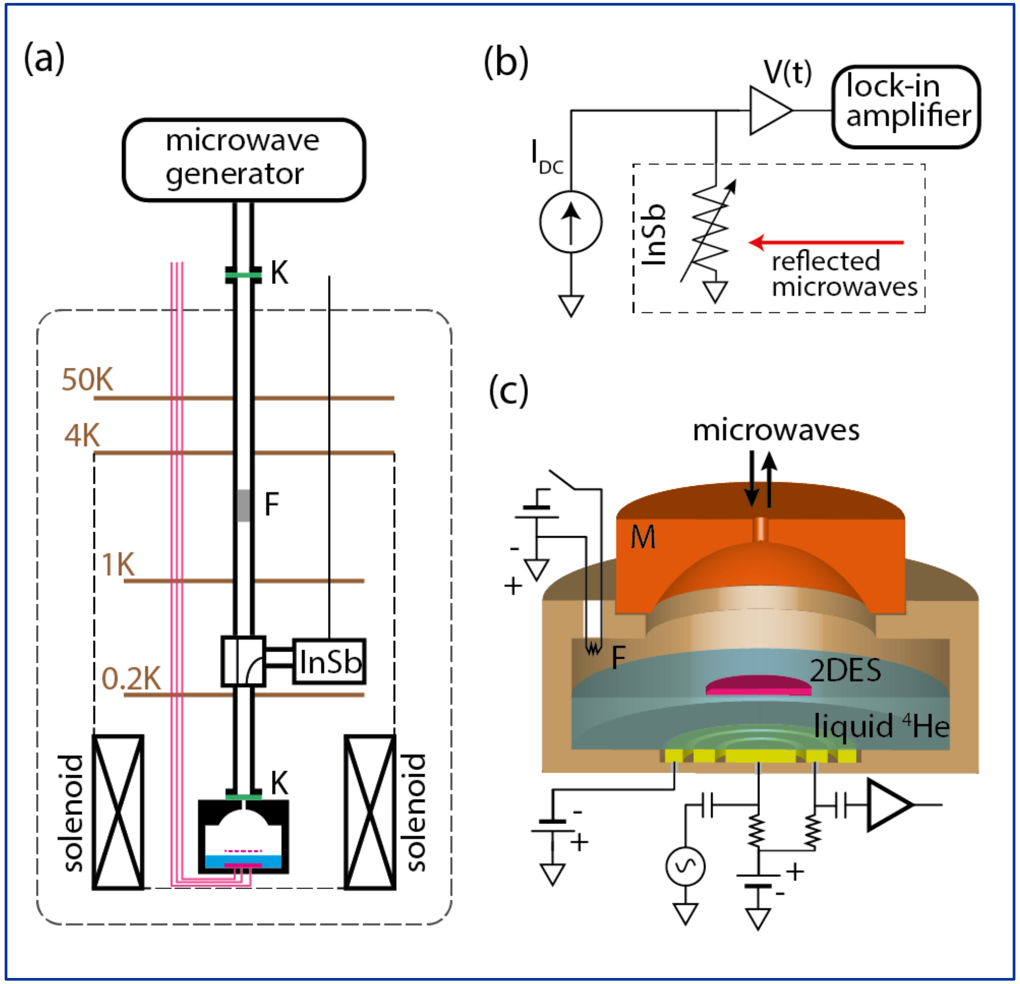
$$j_{\pm} = \frac{n_s e^2}{m_e}\frac{E_{\pm}}{\nu - i(\omega \pm \omega_c)}$$

Quantum treatment:

$$\psi(\vec{r}) = Ce^{ik_y y} H_n\left(\frac{x-X}{l_B}\right)$$

$$\sigma_{\pm} = \frac{n_s e^2}{m_e}\frac{1}{M(\omega) - i(\omega \pm \omega_c)}$$

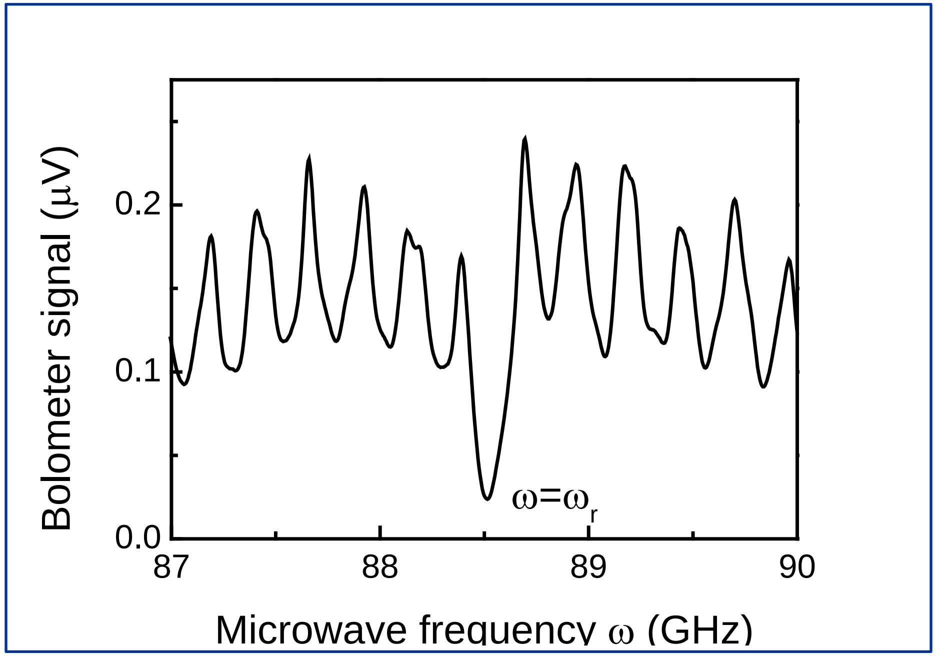
Experimental setup



Semi-confocal FP resonator

TEM_{003} mode: 88.5 GHz

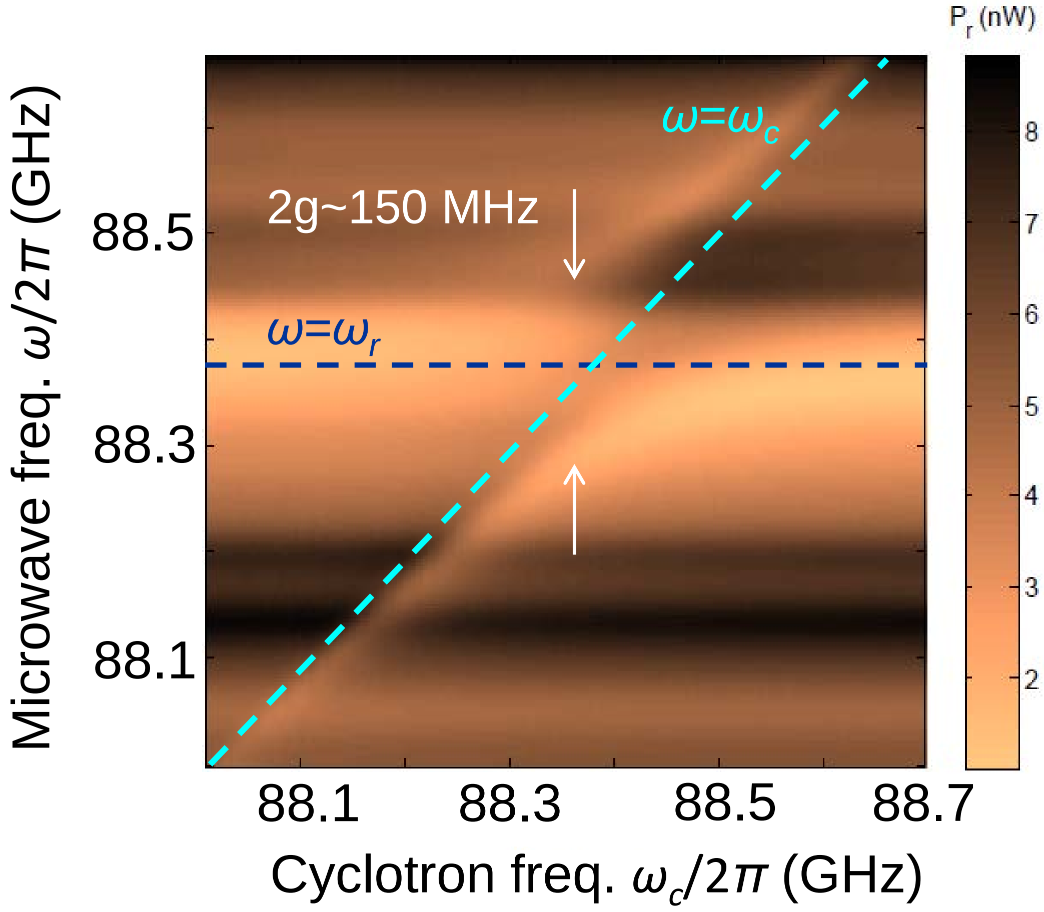
Quality factor Q=900



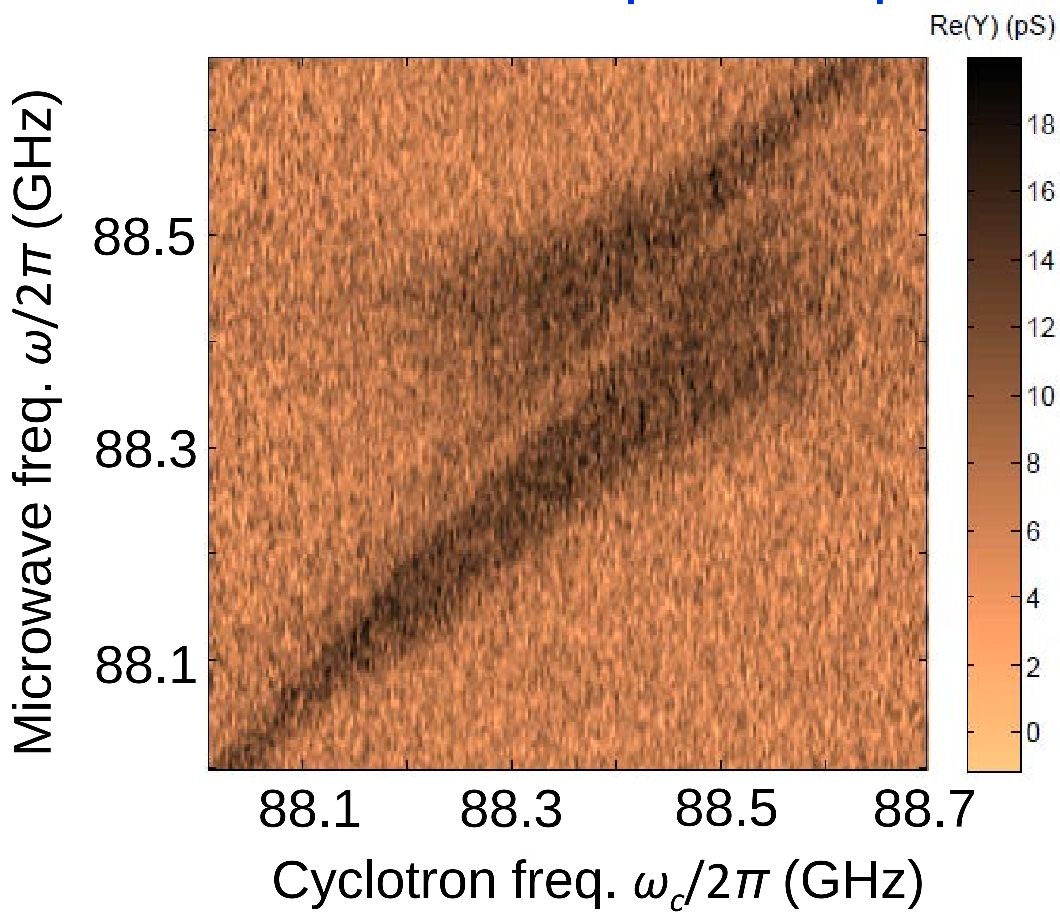
Abdurakhimov et al., PRL117, 056803 (2016)

Avoided crossing: cavity spectrum and electron response

Reflected MW power

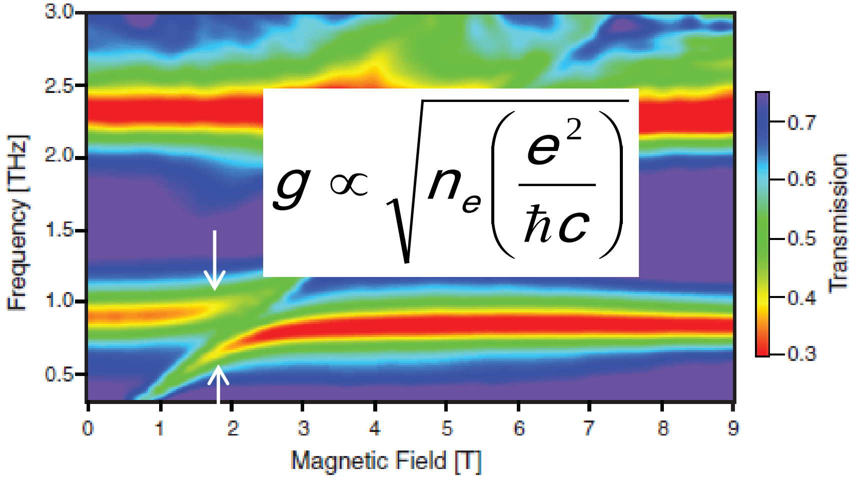


Electron's DC photoresponce

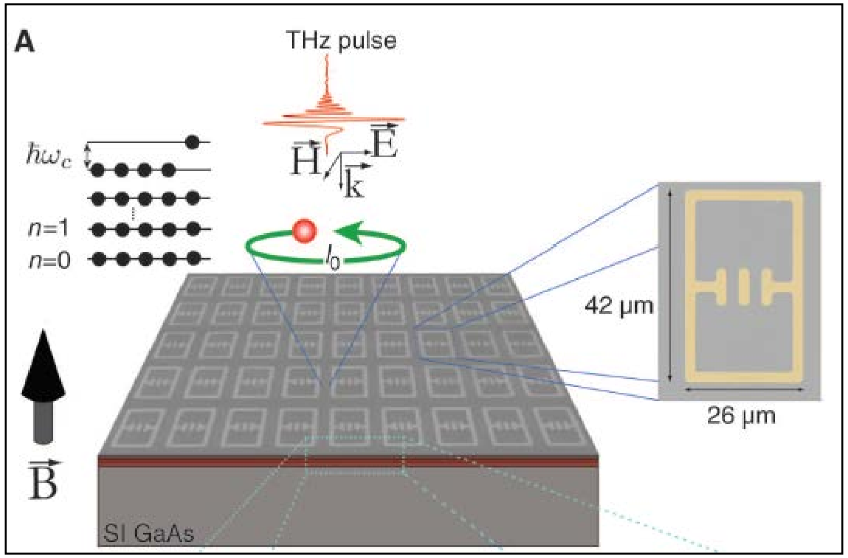


Quantum or classical??

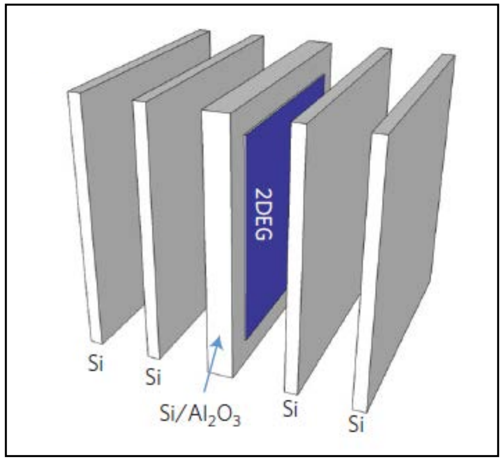
2DEG in GaAs heterostructures:



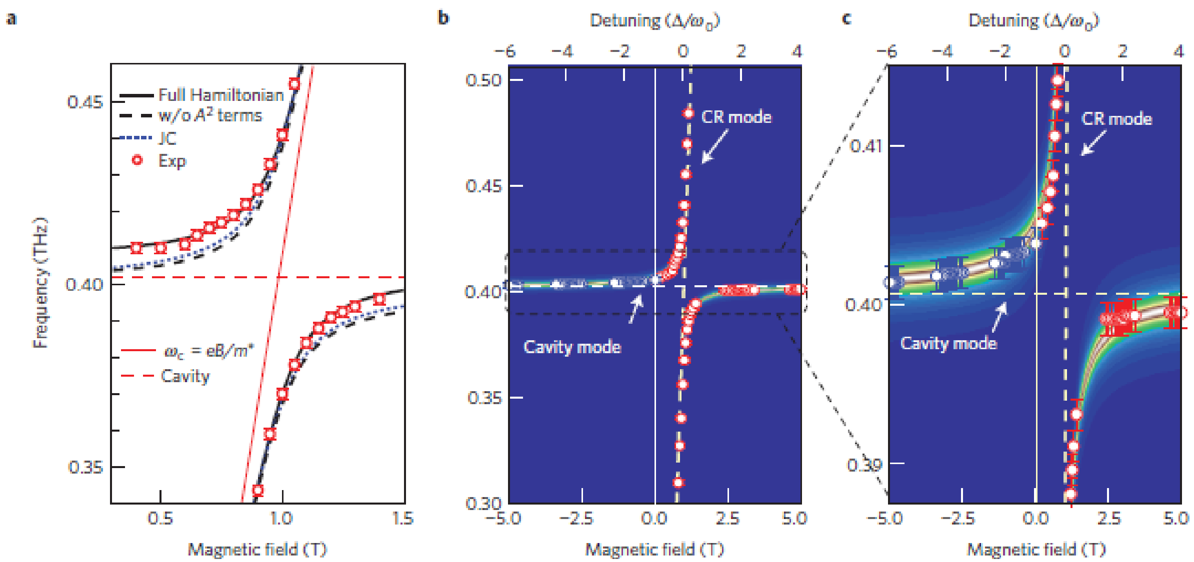
$$g \propto \sqrt{n_e \left(\frac{e^2}{\hbar c}\right)}$$



G. Scalari et al. Science 2012



Q. Zhang et al. Nat. Phys. 2016



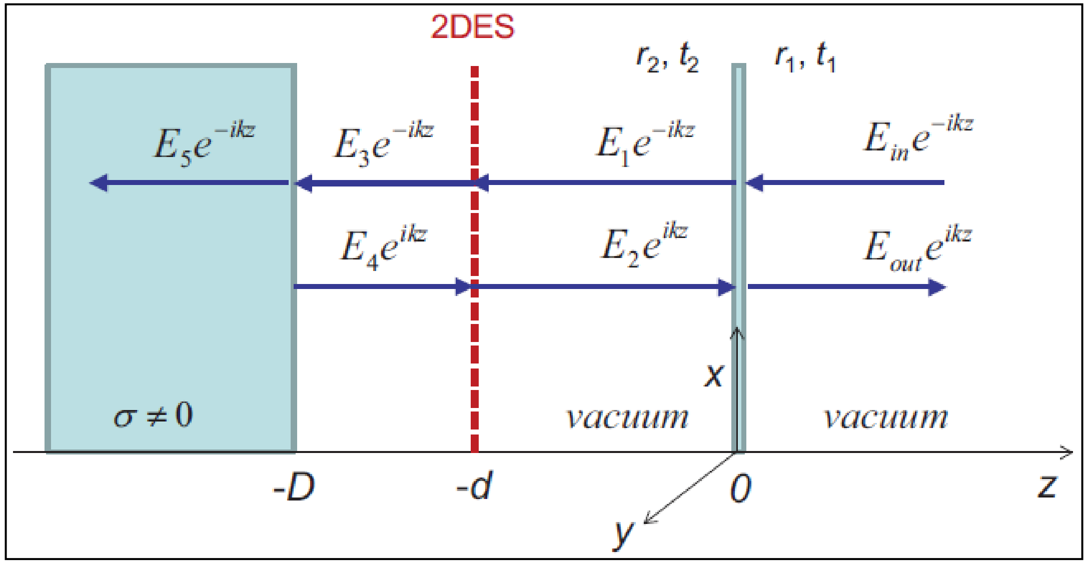
Our classical model

Coupled equations for two oscillators

Current of 2D electrons: $j_\pm = \sigma_\pm E^\pm_{z=-d}$

$$\sigma_\pm = \frac{n_s e^2}{m_e} \frac{E_\pm}{\nu - i(\omega - \omega_c)}$$

B.C. for magnetic field : $H_{z=-d_+} - H_{z=-d_-} = j_e$



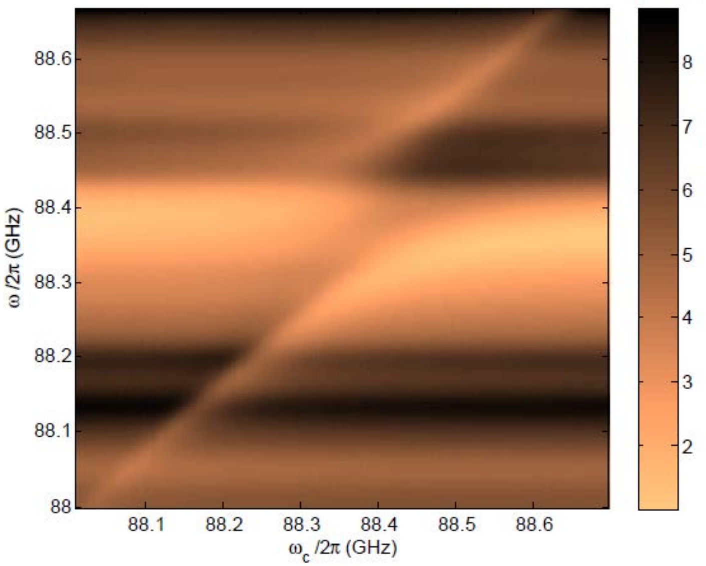
$$\begin{pmatrix} \frac{D}{2c}(\omega - \omega_r + i\gamma) & i\eta_0 / 2 \\ \sigma_{xx} & 1 \end{pmatrix} \begin{pmatrix} E_{z=-d} \\ j_s \end{pmatrix} = \begin{pmatrix} E_{in} \\ 0 \end{pmatrix}$$

$$\mathrm{Det} \begin{pmatrix} \frac{D}{2c}(\omega - \omega_r + i\gamma) & i\eta_0 / 2 \\ \sigma_{xx} & 1 \end{pmatrix} = 0$$

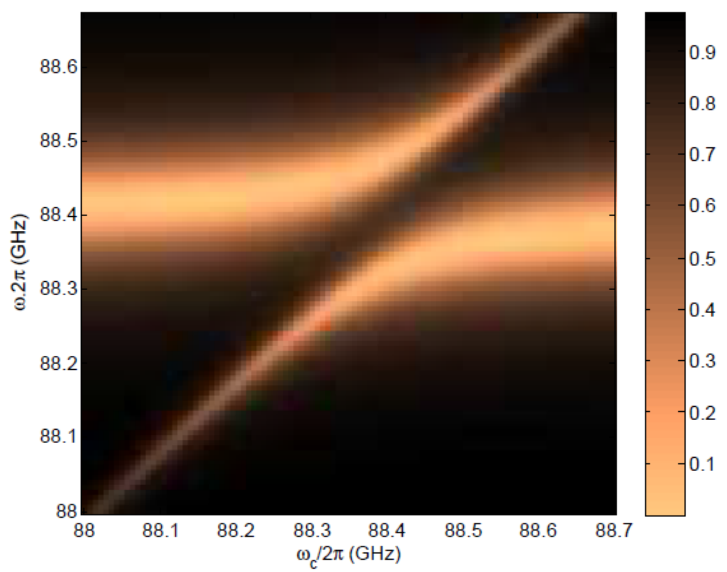
$$g_N = \sqrt{\frac{n_s e^2}{2 m_e \varepsilon_0 D}} = \frac{e}{\hbar} \sqrt{\frac{\hbar}{m_e \omega_c}} \sqrt{\frac{\hbar \omega_c}{2\varepsilon_0 V}} \sqrt{n_s A} = \underbrace{\frac{e l_B E_{rms}}{\hbar} \sqrt{N}}_{\text{"quantum" result}} \propto \sqrt{n_e \frac{e^2}{\hbar c}}$$

Comparison with experiment

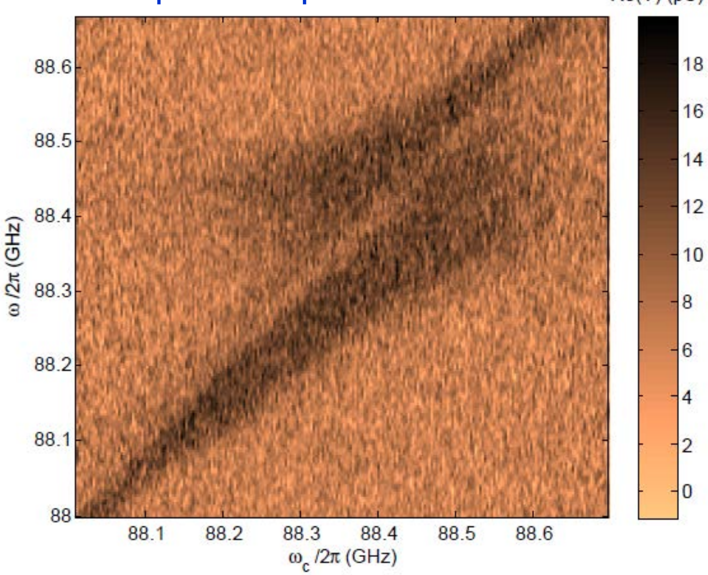
Reflected power



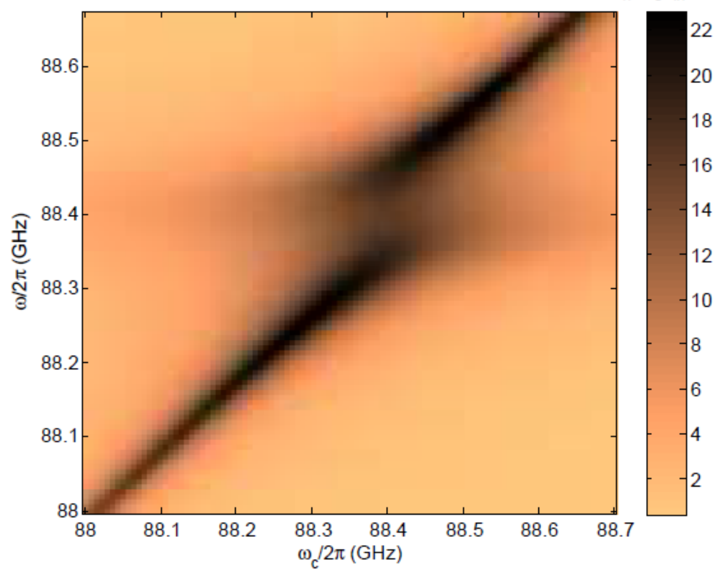
$|S_{11}|=|E_{out}/E_{in}|^2$



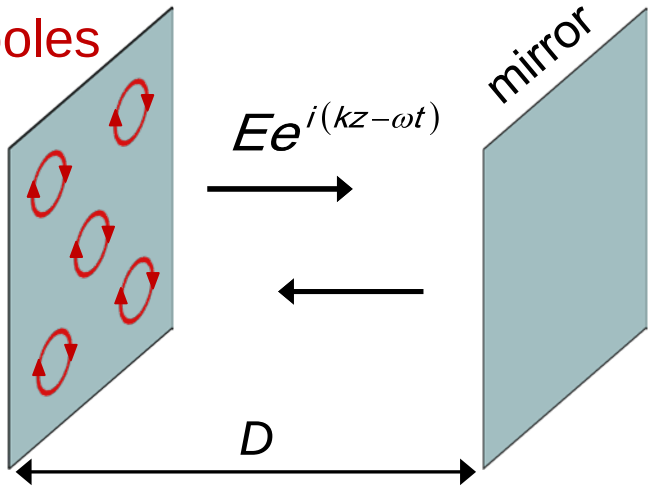
DC photoresponce



AC current j_e



Superradians and Rabi oscillations



$$\frac{dj_e}{dt} + (i\omega_c + \nu)j_e = \frac{e^2 n_s}{m_e}E$$

$$E = -\eta_0 j_e$$

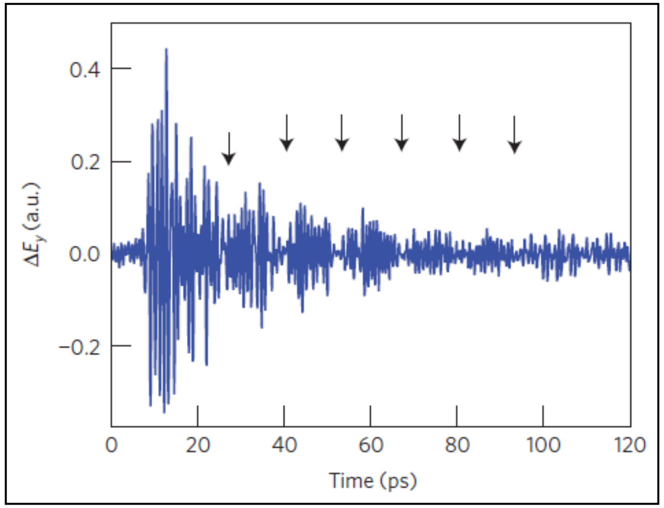
$$\frac{dj_e}{dt} + (i\omega_c + \Gamma)j_e = 0$$

$$\Gamma = \nu + \Gamma_s = \nu + \frac{\eta_0 e^2 n_s}{m_e}$$

"Superradiant" decay

Qi Zhang et al. PRL, 047601 (2014)

Vacuum Rabi oscillations?



Q. Zhang et al. Nat. Phys. 2016

Frequency of (Rabi-like) oscillations:

$$g = \sqrt{\frac{\Gamma_s c}{2D}}$$

Most recent experiment

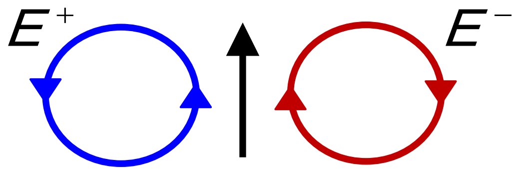
Semi-confocal FP resonator

TEM_{003} mode: 35 GHz

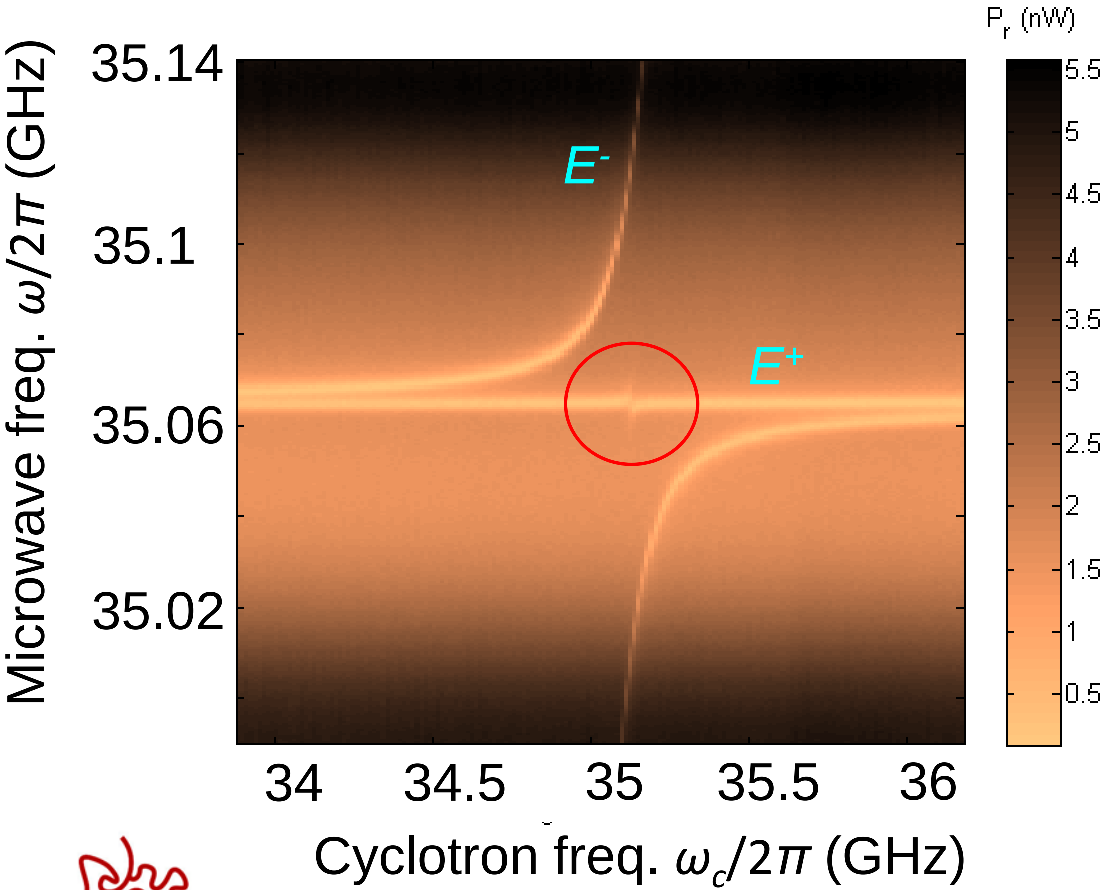
Quality factor Q=7,000

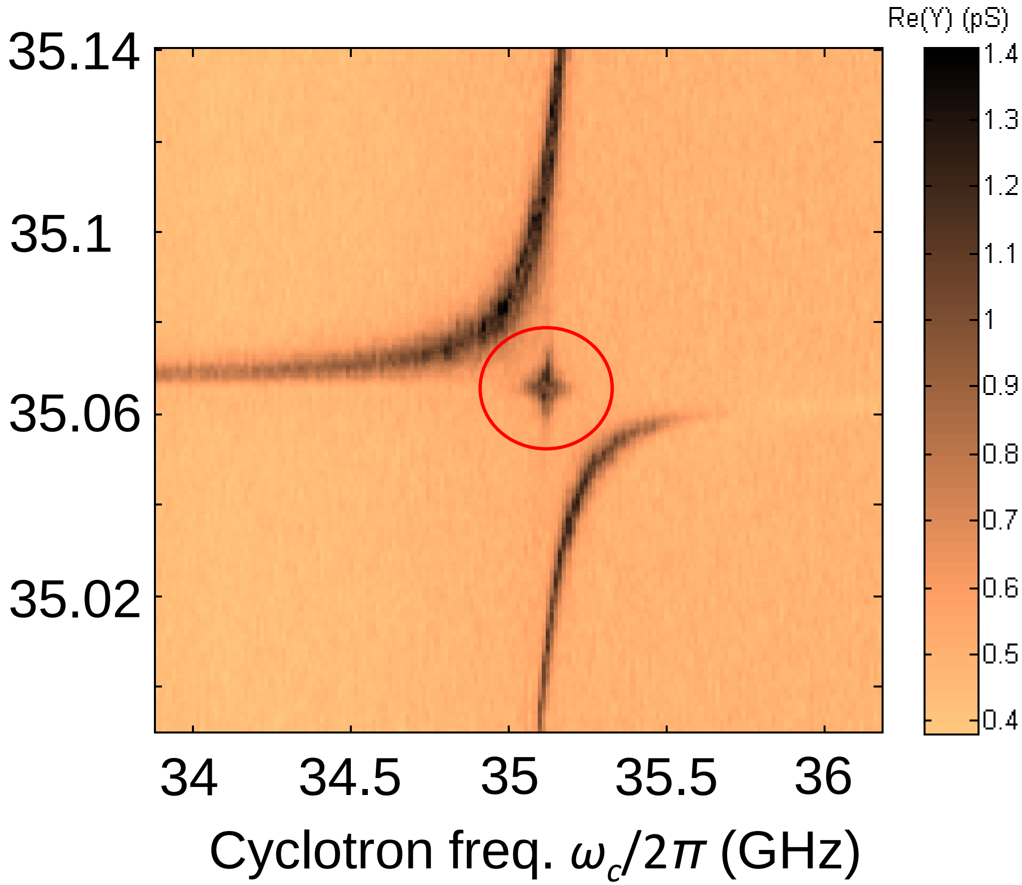
$$E = E^+ + E^-$$

$\pm$ - right/left circular polarized

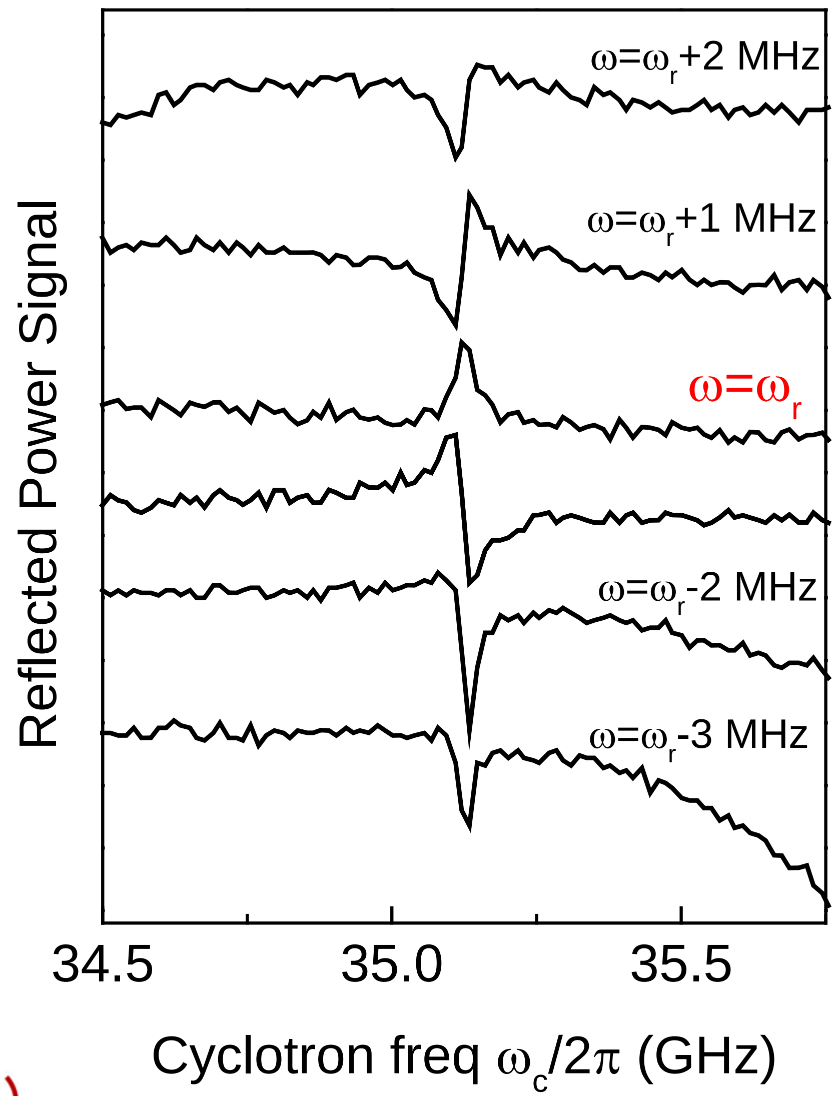


$$j_e^{\pm} = \frac{n_s e^2 \tau}{m_e} \frac{E^{\pm}}{1 - i\tau(\omega \pm \omega_c)}$$

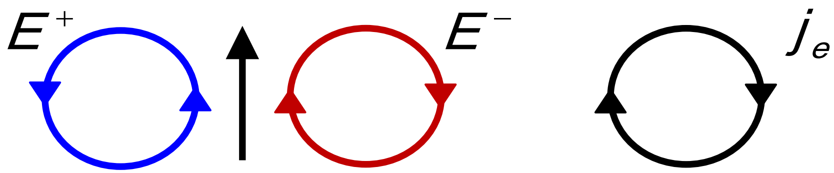




Most recent experiment

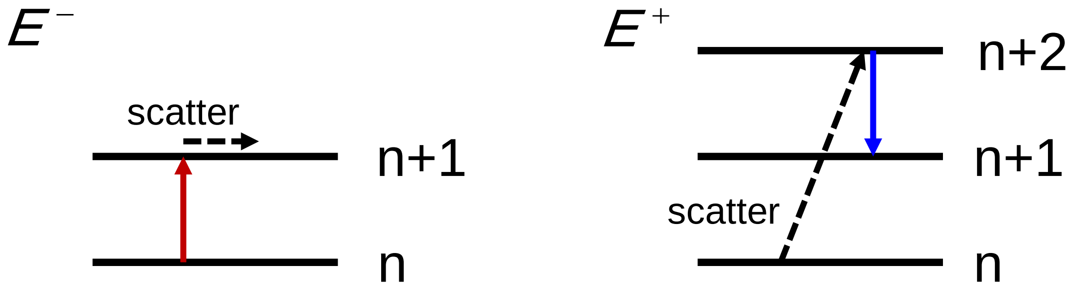


Looks like resonance induced by <u>E^+ mode</u>

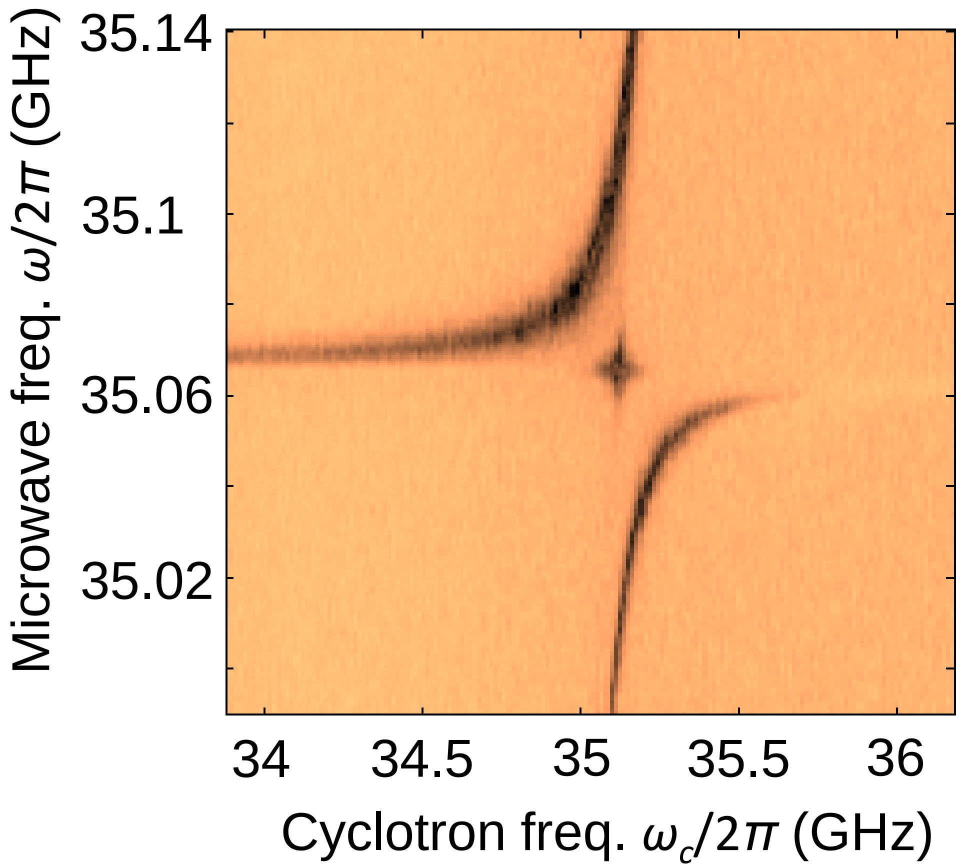


$$w_{i\to f} = \frac{2\pi}{\hbar}\left|\langle f|H_{int}|i\rangle + \sum_m \frac{\langle f|H_{int}|m\rangle\langle m|H_{int}|i\rangle}{E_i - E_f}\right|^2 \delta(E_n - E_i - \hbar\omega)$$

Ripplon-assisted (Raman) transitions



Conclusions



Strong coupling between electron cyclotron motion and a microwave cavity mode

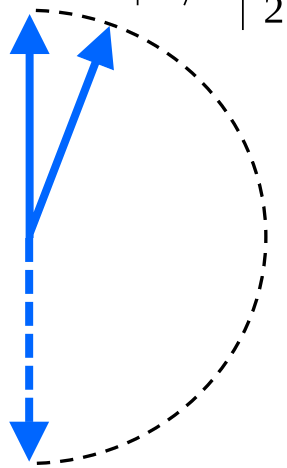
Can be fully described on a completely classical ground

Two coupled oscillators = two coupled linear systems

Cavity QED experiments with electrons on helium? Need a nonlinearity!

Holstein-Primakov bosons

$|G\rangle = \left|\frac{N}{2}, \frac{N}{2}\right\rangle$ $|E\rangle = \left|\frac{N}{2}, \frac{N}{2} - 1\right\rangle$



Dicke model, 1954

Consider spin $S = \frac{N}{2}$ which has states $|S, S_z\rangle$

Apply (Holstein-Primakoff) transformation:

$$\hat{S}_z = S - \hat{b}^+\hat{b}, \ \hat{S}^+ = \sqrt{2S - \hat{b}^+\hat{b}}\ \hat{b}, \ \hat{S}^- = \hat{b}^+\sqrt{2S - \hat{b}^+\hat{b}}$$

Bosonic operator $\hat{b}^+(\hat{b})$ creates (annihilates)

one spin excitation in N-spin system

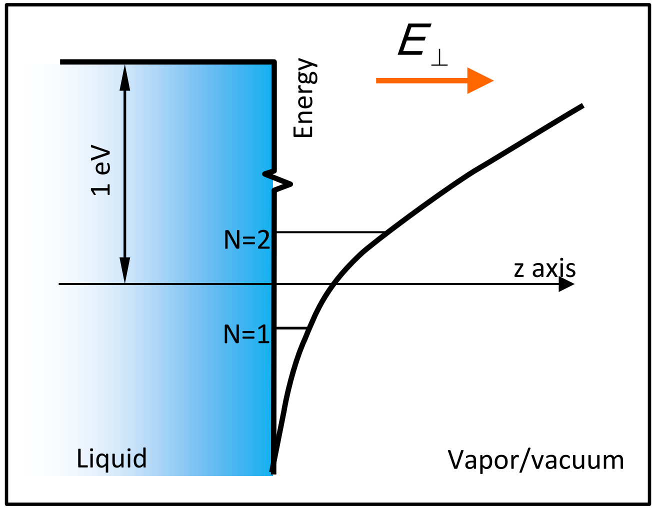
In the low-excitation limit ($S - S_z \ll S$):

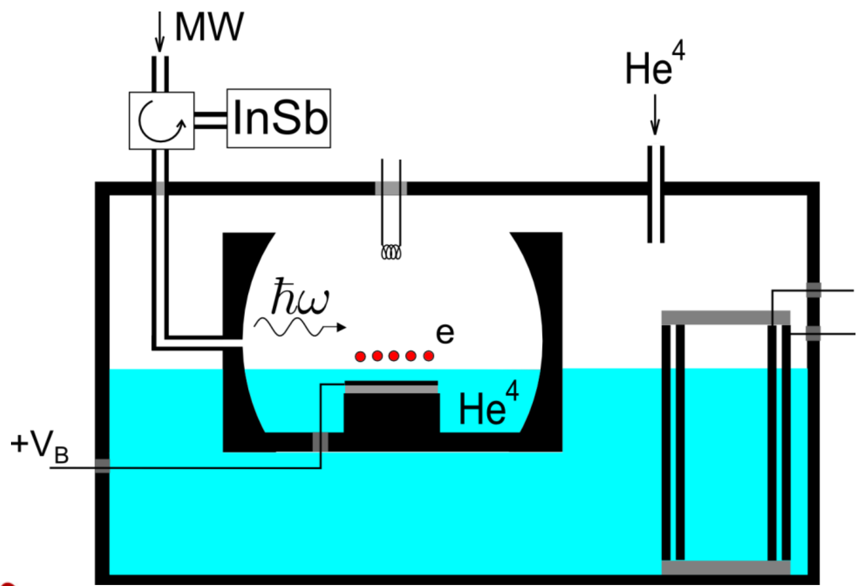
$$\hat{S}_z = S - \hat{b}^+\hat{b}, \ \hat{S}^+ = \sqrt{2S}\ \hat{b}, \ \hat{S}^- = \hat{b}^+\sqrt{2S}$$

Full Hamiltonian becomes:

$$\hat{H}/\hbar = \omega_r \hat{a}^+\hat{a} - \omega_s \hat{b}^+\hat{b} + g\sqrt{N}\left(\hat{a}\hat{b}^+ + \hat{a}^+\hat{b}\right)$$

Ensemble of two-level systems on helium surface





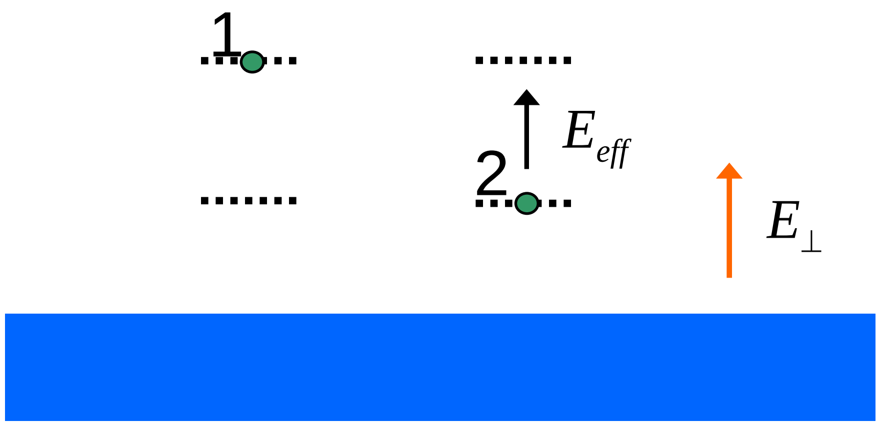
Use inter-subband resonance rather then CR

Dipole matrix element $\sim a_B \approx 10$ nm

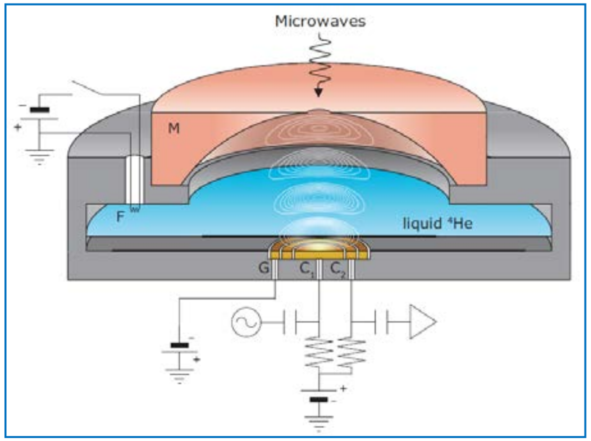
Coupling to single electron:

$$g = ea_B E_{rms} = ea_B \sqrt{\frac{\hbar\omega_{21}}{2\varepsilon_0 V}} \approx 100\ \text{kHz}$$

Nonlinearity: Coulomb shift of energy levels



Cyclotron resonance harmonics



Yamashiro et al., PRL 115, 256802 (2015)