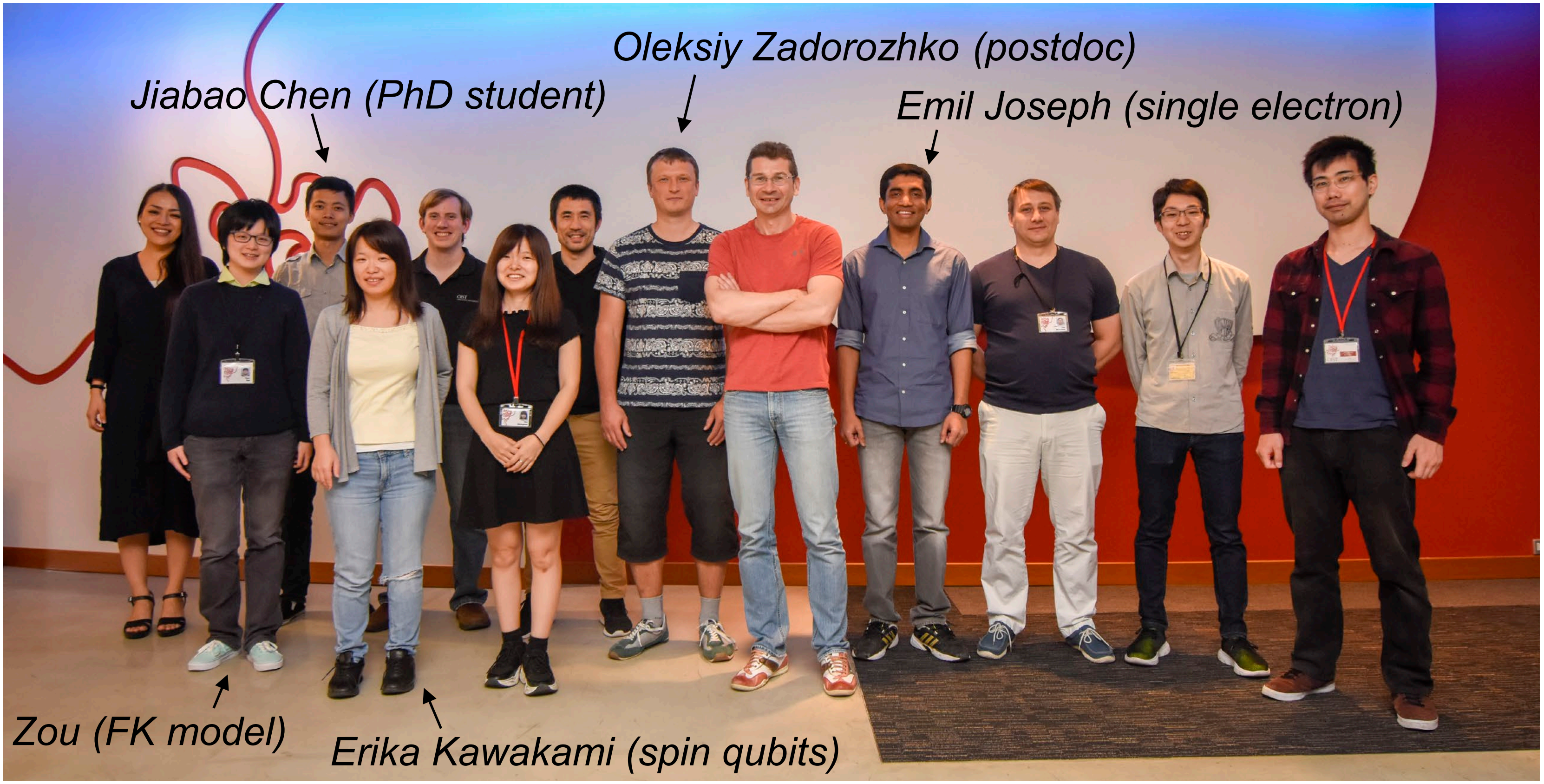
Energy eigenstates of electrons on liquid helium in tilted magnetic fields

Quantum Dynamics Unit
Okinawa Institute of Science and Technology (OIST)

QFS2019, Edmonton

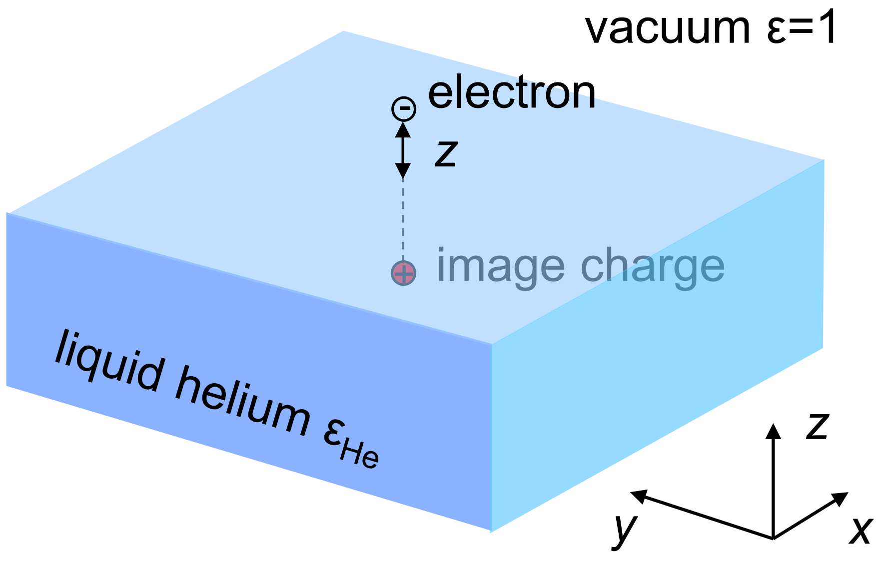
Quantum Dynamics Unit (QDU)



Alexei Chepelianskii
(University Paris-Sud)

Funded by
Okinawa Institute of Science and Technology (OIST)

2D Electrons on Helium



Surface bound states

$$H=\underbrace{\frac{p_x^2}{2m}+\frac{p_y^2}{2m}}_{\text{in-plane motion}}+\underbrace{\frac{p_z^2}{2m}-\frac{(\varepsilon_{He}-1)}{4(\varepsilon_{He}+1)}\frac{e^2}{z}+V_{rep}\Theta(-z)}_{\text{perpendicular motion}}$$

Complement to 2DEG in semiconductors!

- Wigner solid: Coulomb energy >> $k_B T$

C.C. Grimes and G. Adams (1979)

- Collective excitations

D.C. Glattli et al (1985), A. Dahm et al (1985)

- Possibility for quantum melting

P. Leiderer et al. Surface Science (1996)

- Many-electron transport

M. Lea and M. Dykman (1997)

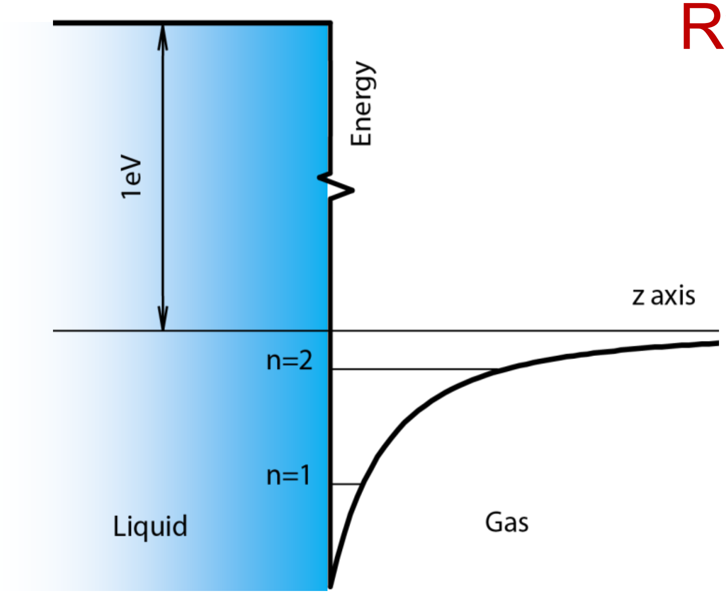
K. Kono and K. Shirahama (1996)

- Quantum computing

M. Dykman and P. Platzman (1998)

S. Lyon (2006), D. Shuster et al (2010)

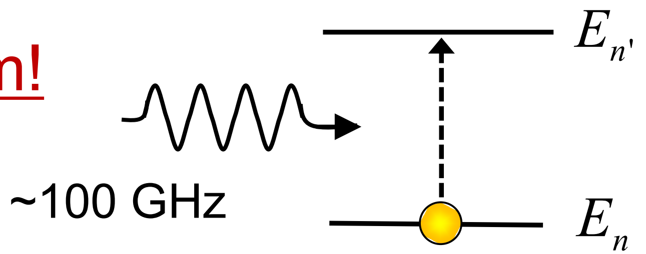
Rydberg States of Electrons on Helium



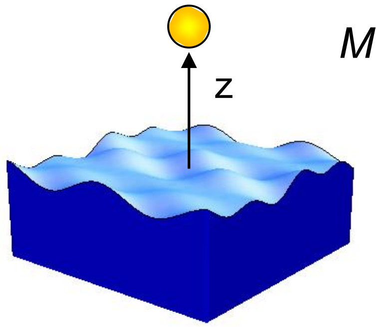
Rydberg spectrum

$$H = \underbrace{\frac{p_x^2}{2m} + \frac{p_y^2}{2m}}_{\text{in-plane motion}} + \underbrace{\sum_n E_n |n\rangle\langle n|}_{\text{perpendicular motion}}, \qquad \boxed{E_n = -\frac{R_e}{n^2},\ \ n = 1,2,..}$$

Effectively a 1D Rydberg atom!



- Ripplonic Lamb shift



M. Dykman, K. Kono, DK, M. Lea (2017)

- Coulomb shift = Rydberg blockade

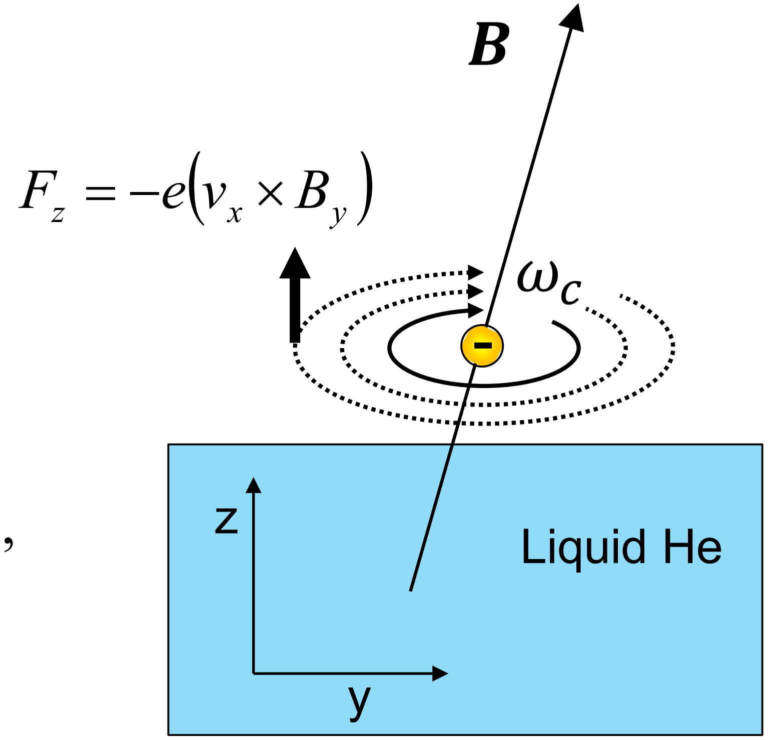
DK, M. Dykman, M. Lea, Yu. Monarkha, K. Kono (1998)

Electrons in Tilted *B*-field

Electron in B_z-field

$$\hat{H} = \underbrace{\sum_n E_n |n\rangle\langle n|}_{\text{perpendicular motion}} + \underbrace{\hbar\omega_c\left(\hat{a}^+\hat{a}+\frac{1}{2}\right)}_{\text{in-plane motion}}$$

1D Rydberg atom + 2D oscillator



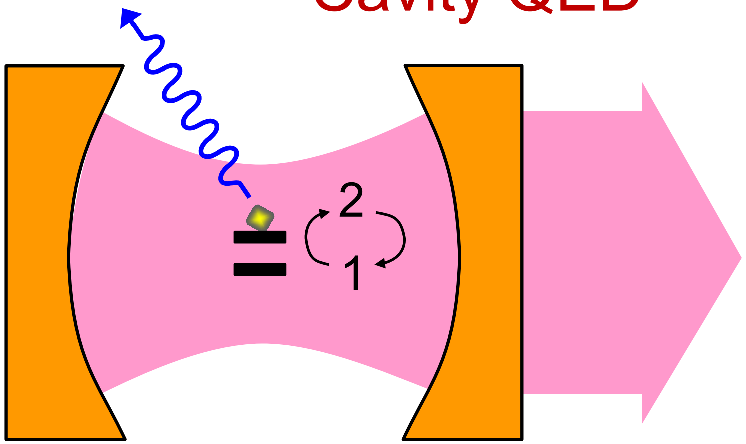
Electron in tilted *B*-field (B_y≠0)

$$\hat{H} = \underbrace{\sum_n E_n |n\rangle\langle n|}_{\text{perpendicular motion}} + \underbrace{\hbar\omega_c\left(\hat{a}^+\hat{a}+\frac{1}{2}\right)}_{\text{in-plane motion}} + \underbrace{\frac{\hbar\omega_y}{\sqrt{2}\,l_B}\left(\hat{a}^+ + \hat{a}\right)\hat{z}}_{\text{COUPLING}},$$

K. Yunusova, DK, H. Bouchiat, A. Chepelianskii, PRL (2019)

Effective Jaynes-Cummings Model (JCM)

Cavity QED



Jaynes-Cummings Hamiltonian

$$\boxed{\hat{H} = (E_2 - E_1)\hat{s}_z + \hbar\omega_c\left(\hat{a}^+\hat{a} + \frac{1}{2}\right) + \hbar g(\hat{a}^+ + a)\hat{s}_x}$$

$$\hat{s}_z = \frac{1}{2}(|2\rangle\langle 2| - |1\rangle\langle 1|), \quad \hat{s}_x = \frac{1}{2}(|1\rangle\langle 2| + |2\rangle\langle 1|).$$

Electron in tilted *B*-field ($B_y \neq 0$)

$$\hat{H} = \underbrace{\sum_n E_n |n\rangle\langle n|}_{\text{perpendicular motion}} + \underbrace{\hbar\omega_c\left(\hat{a}^+\hat{a} + \frac{1}{2}\right)}_{\text{in-plane motion}} + \underbrace{\frac{\hbar\omega_y}{\sqrt{2}l_B}(\hat{a}^+ + \hat{a})\sum_{nn'} z_{nn'}|n\rangle\langle n'|}_{\text{COUPLING}}$$

$$\boxed{g_{nn'} = \sqrt{\frac{\hbar m\omega_z\omega_y^2}{2}}\, z_{nn'}}$$

coupling strength

K. Yunusova, DK, H. Bouchiat, A. Chepelianskii, PRL (2019)

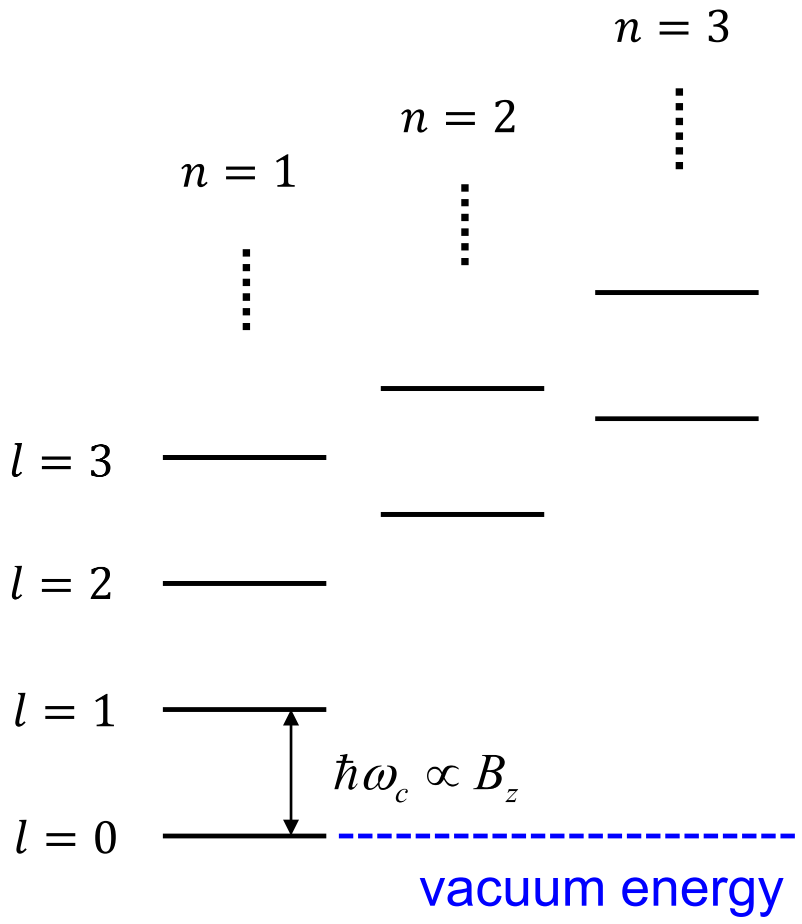
Energy eigenvalues of electrons in tilted B-field

Can be described by two numbers

- n=1,2,3, … for the bound states in z
- l=0,1,2,… for cyclotron motion in xy

For uncoupled motion (B_y=0)

$$E_{n,l} = E_n + \hbar\omega_c l + \underbrace{\frac{\hbar\omega_c}{2}}_{\text{vacuum energy}}$$



Energy eigenvalues of electrons in tilted B-field

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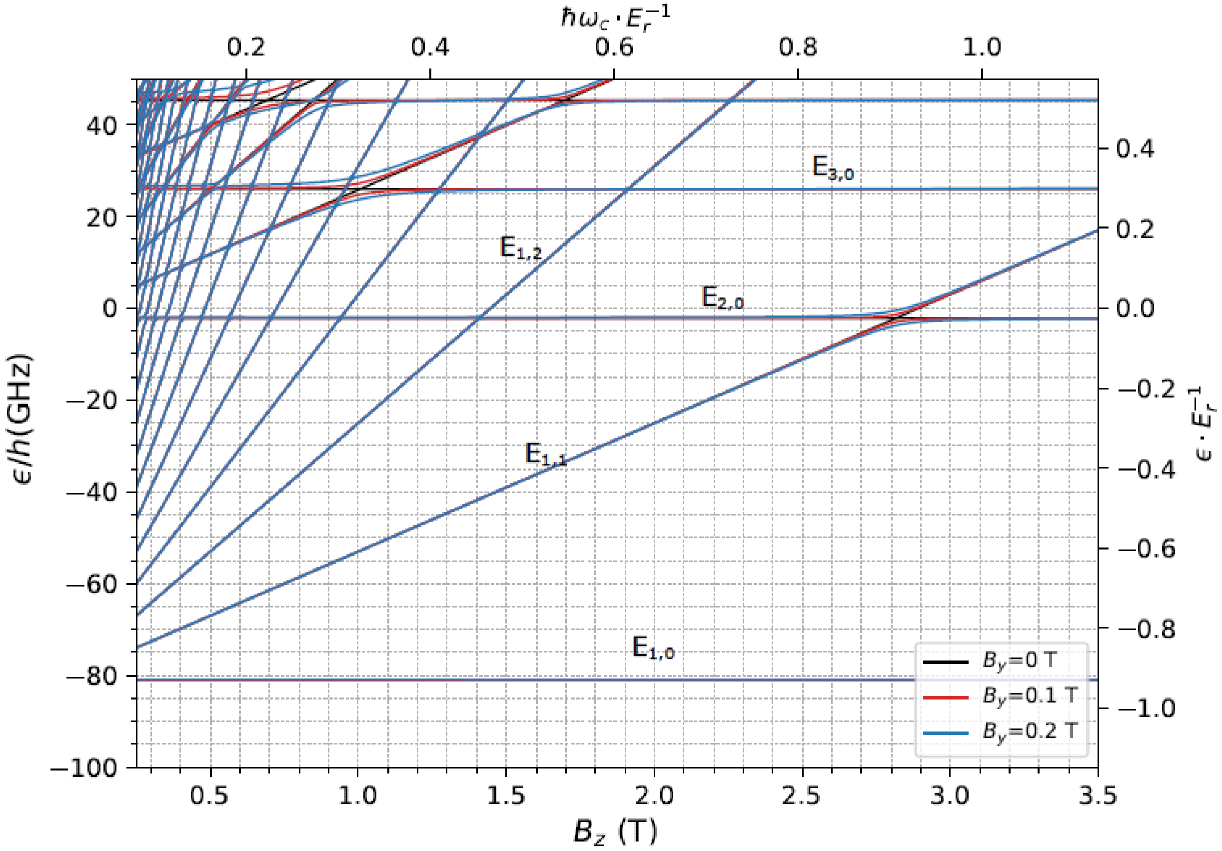
Jiabao Chen (PhD thesis)



For uncoupled motion (B_y=0)

$$E_{n,l} = E_n + \hbar\omega_c l + \underbrace{\frac{\hbar\omega_c}{2}}_{\text{vacuum energy}}$$

For coupled motion (B_y≠0)

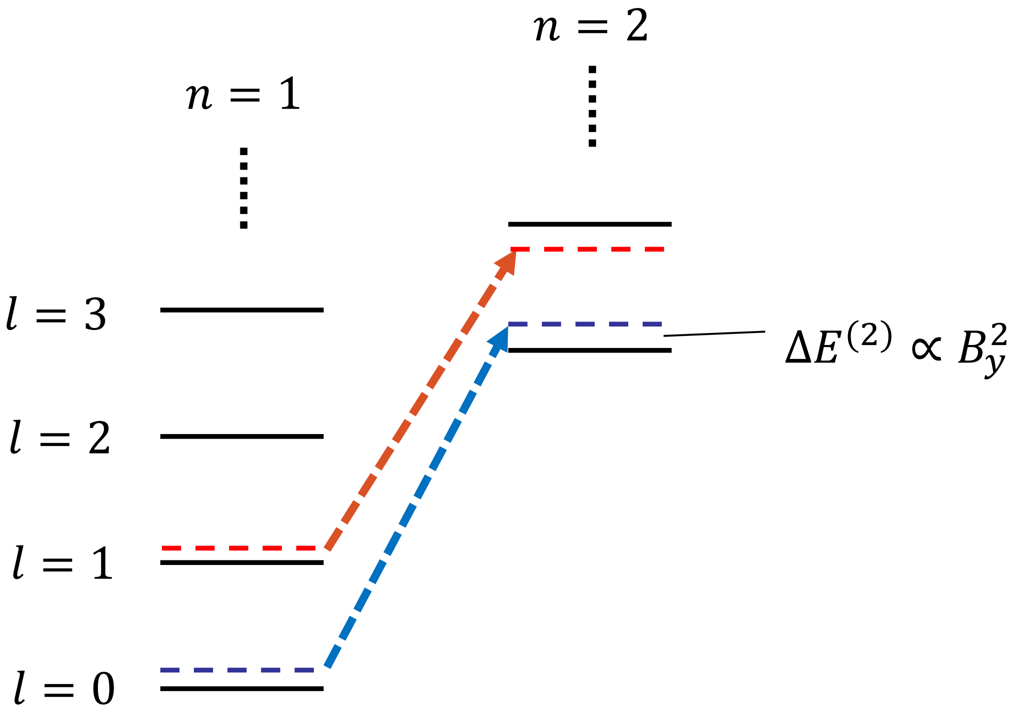
numerical diagonalization

- 1 ≤ n ≤ 10 Rydberg levels
- 0 ≤ l ≤ 50 Landau levels

Shifts of energy levels

Second-order perturbation theory (light shift or ac Stark shift in JCM)

$$\boxed{\Delta E^{(2)}_{n,l} = \frac{\hbar^2\omega_y^2}{2l_B^2}\sum_{(n',l')\neq(n,l)} \frac{|z_{nn'}|^2 \left|\langle l'|\hat{a}^+ + \hat{a}|l\rangle\right|^2}{E_n - E_{n'} + \hbar\omega_c(l-l')},}$$



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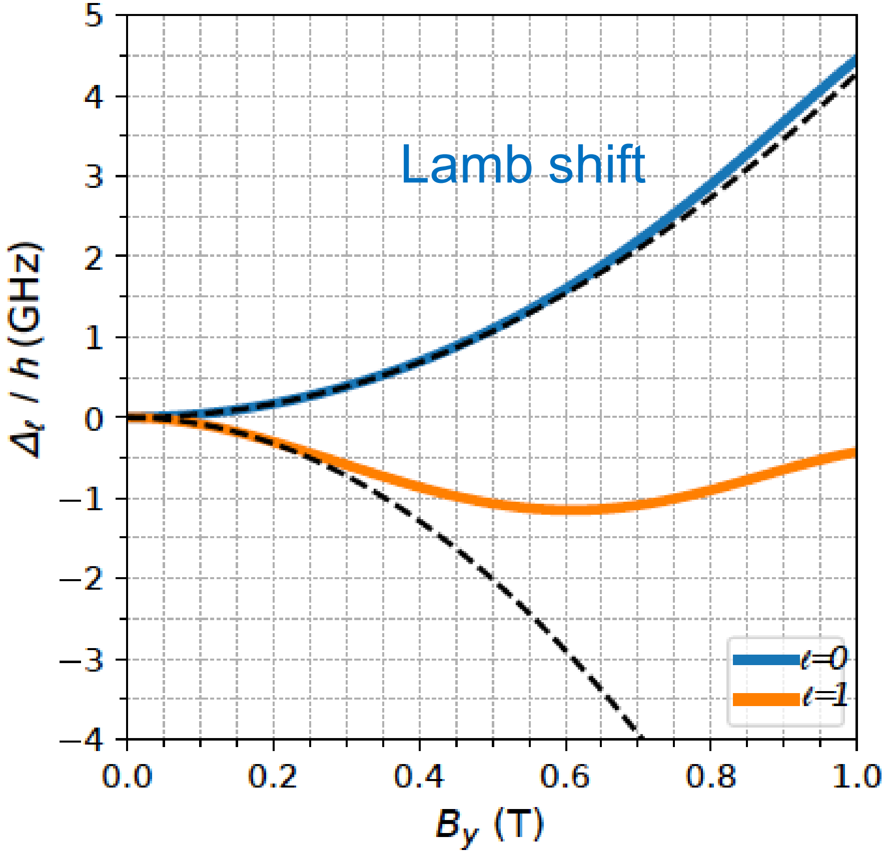


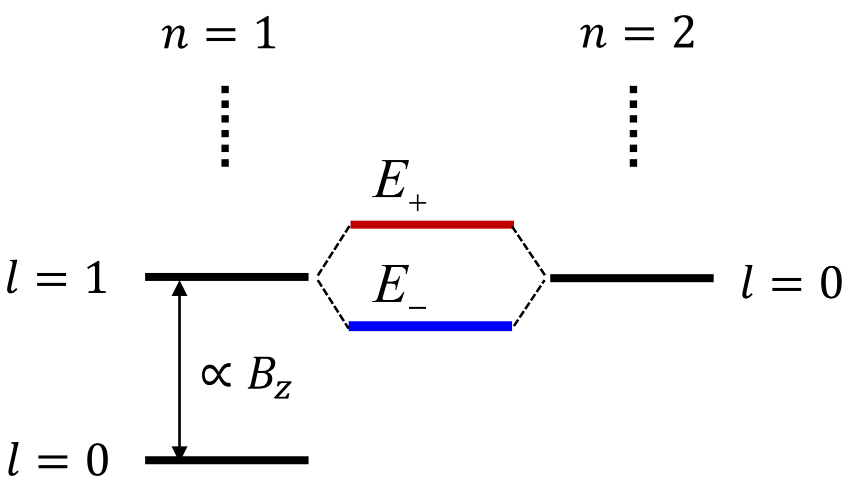
Anti-crossings of energy levels

Consider subspace n=1,2 and l=0,1:

$$\boxed{\hat{H} = \sum_n E_n |n\rangle\langle n| + \hbar\omega_c\left(\hat{a}^+\hat{a} + \frac{1}{2}\right) + \frac{\hbar\omega_y}{\sqrt{2}l_B}\left(\hat{a}^+ + \hat{a}\right)\hat{z}}$$

$$\hat{H} = \begin{pmatrix} E_{1,1} & g_{12} \\ g_{21} & E_{2,0} \end{pmatrix} \rightarrow \begin{pmatrix} E_+ & 0 \\ 0 & E_- \end{pmatrix}$$

$$E_\pm = \frac{1}{2}\left(E_{1,1} + E_{2,0} \pm \sqrt{\left(E_{1,1} - E_{2,0}\right) + 4g_{12}^2}\right)$$



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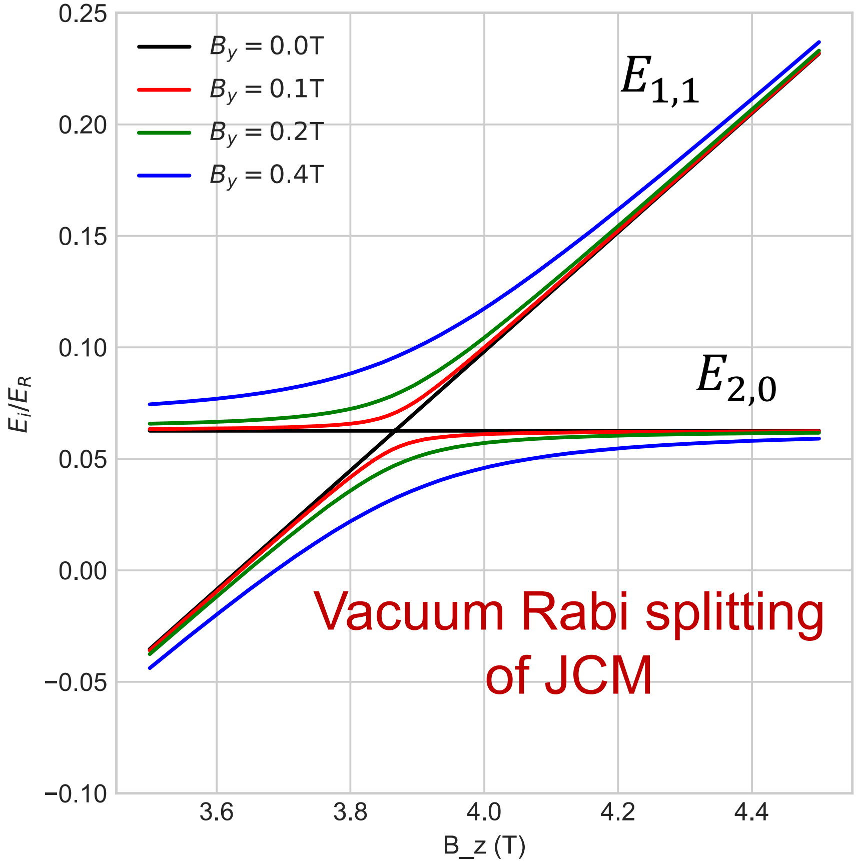
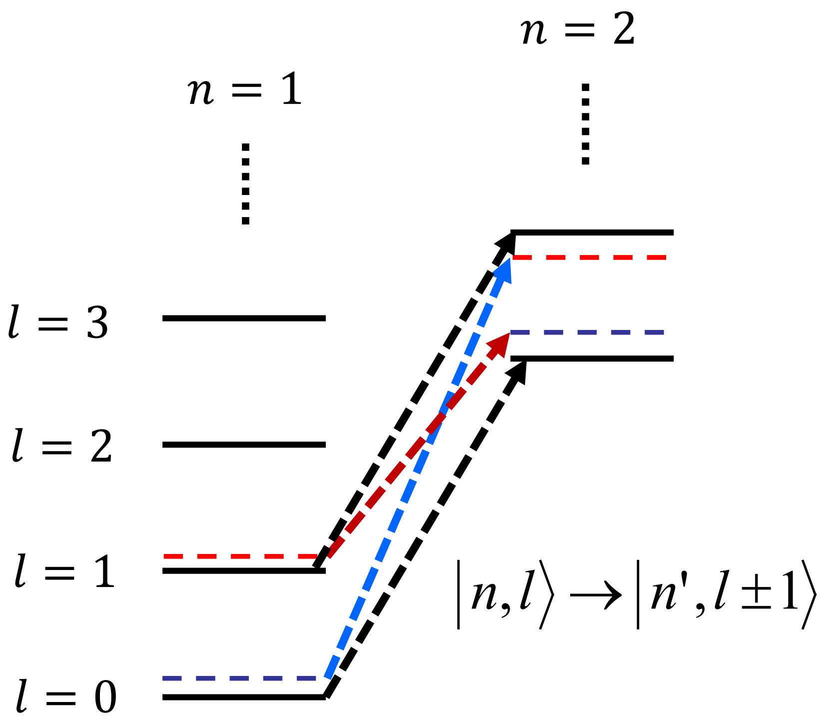


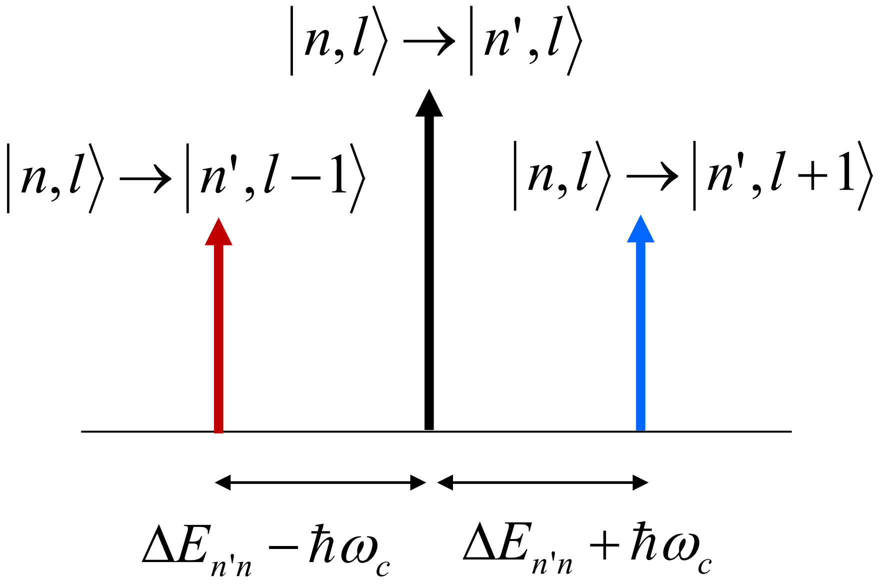
Vacuum Rabi splitting of JCM

Sideband transitions between Rydberg levels

First-order perturbation theory (mixing of states)

$$|\varphi\rangle = |n,l\rangle + \underbrace{\frac{m\omega_y^2}{2}\sum_{n'\neq n}\frac{\left(z^2\right)_{n'n}}{E_n - E_{n'}}|n',l\rangle + \frac{\hbar\omega_y}{\sqrt{2}l_B}\sum_{n'}\frac{\left(z^2\right)_{n'n}\sqrt{l+\frac{1}{2}(l\pm1)}}{E_n - E_{n'} \mp \hbar\omega_c}|n',l\pm1\rangle}_{\text{1}^{\text{st}}\text{ order correction to } |n,l\rangle}$$

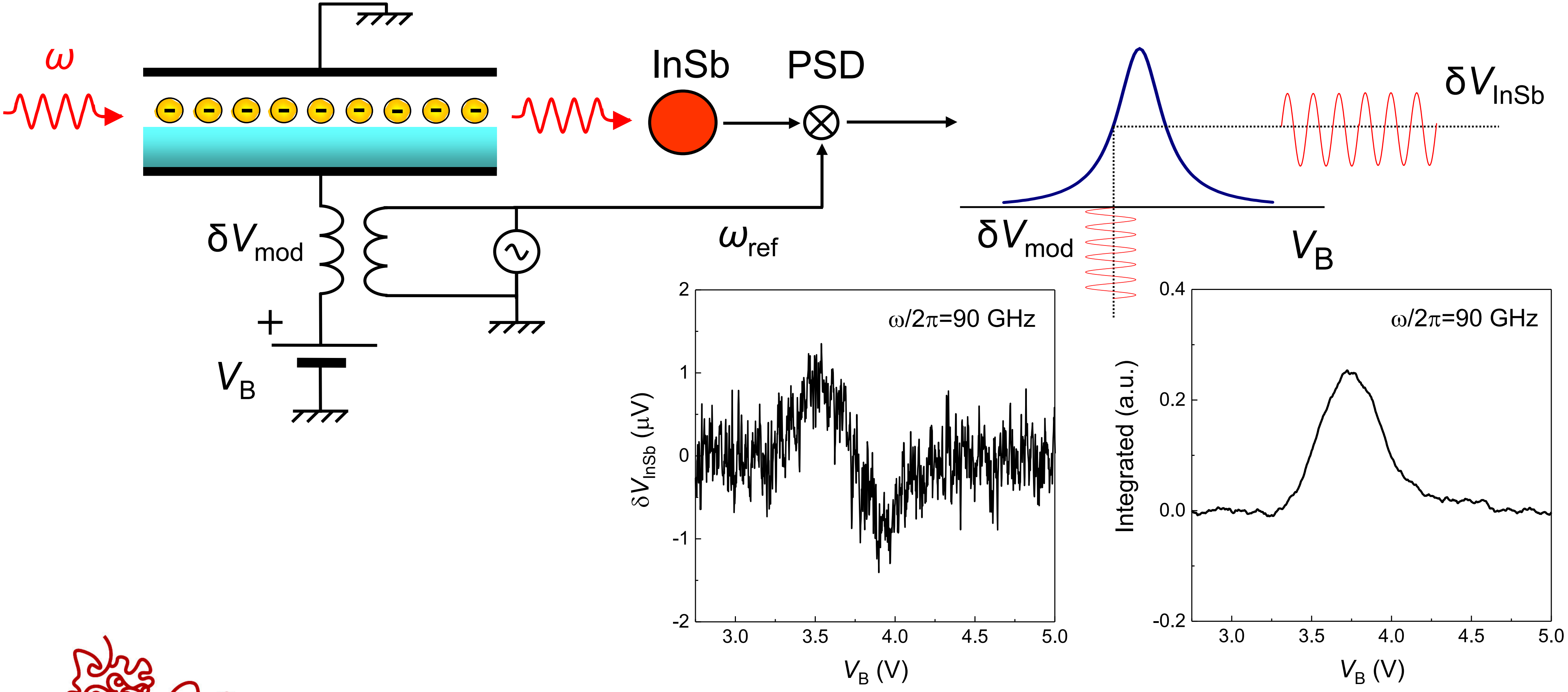




Experiment (Stark spectroscopy method)

Linear DC Stark shift

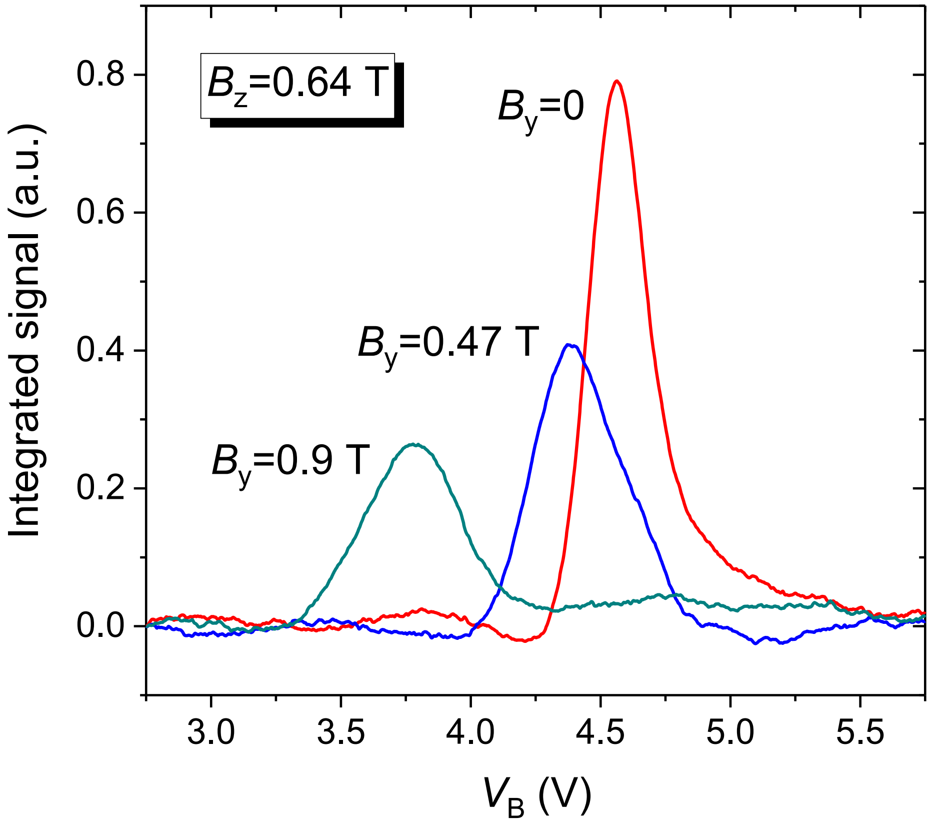
$$E_n = -\frac{R_e}{n^2} + \underbrace{eE_z\langle n|\hat{z}|n\rangle}_{\text{Stark shift in DC electric field } E_z}, \quad n = 1,2,..$$



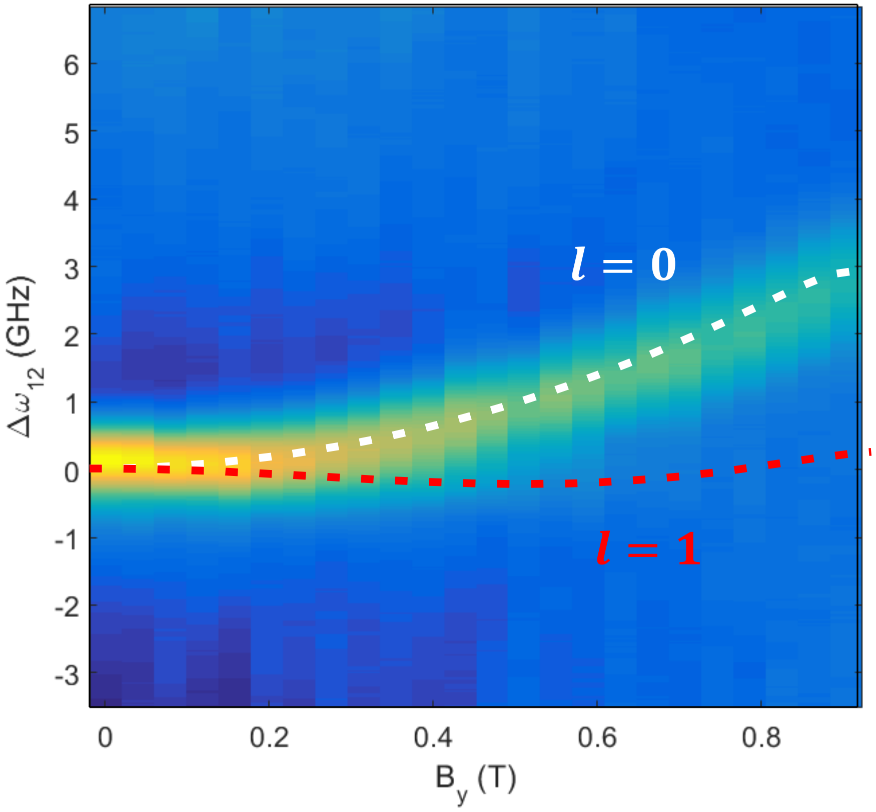
Lamb shift (l=0 vacuum state)

Measure n=1-2 transition line in tilted *B*-field

Oleksiy Zadorozhko (experiment)



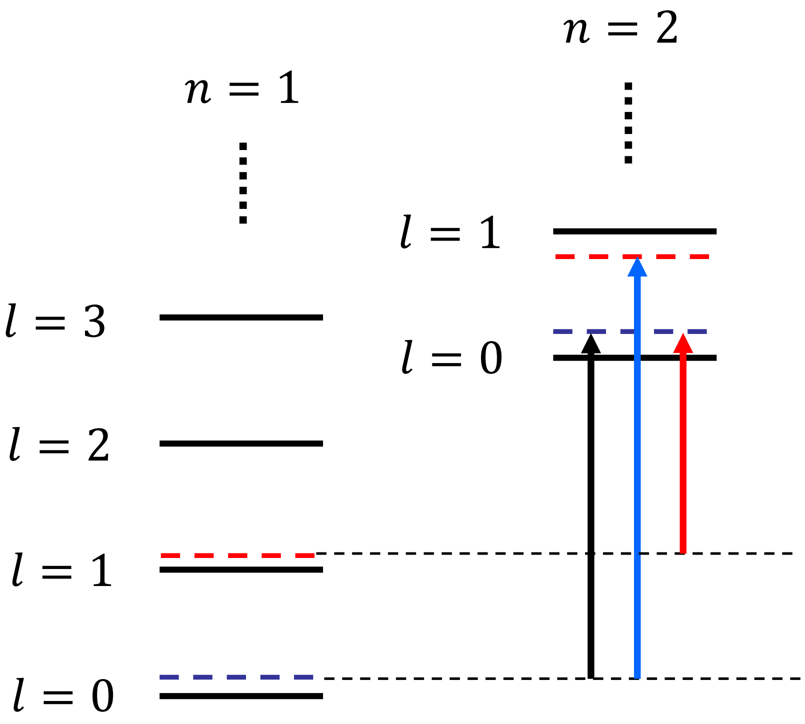
Jiabao Chen (PhD thesis)



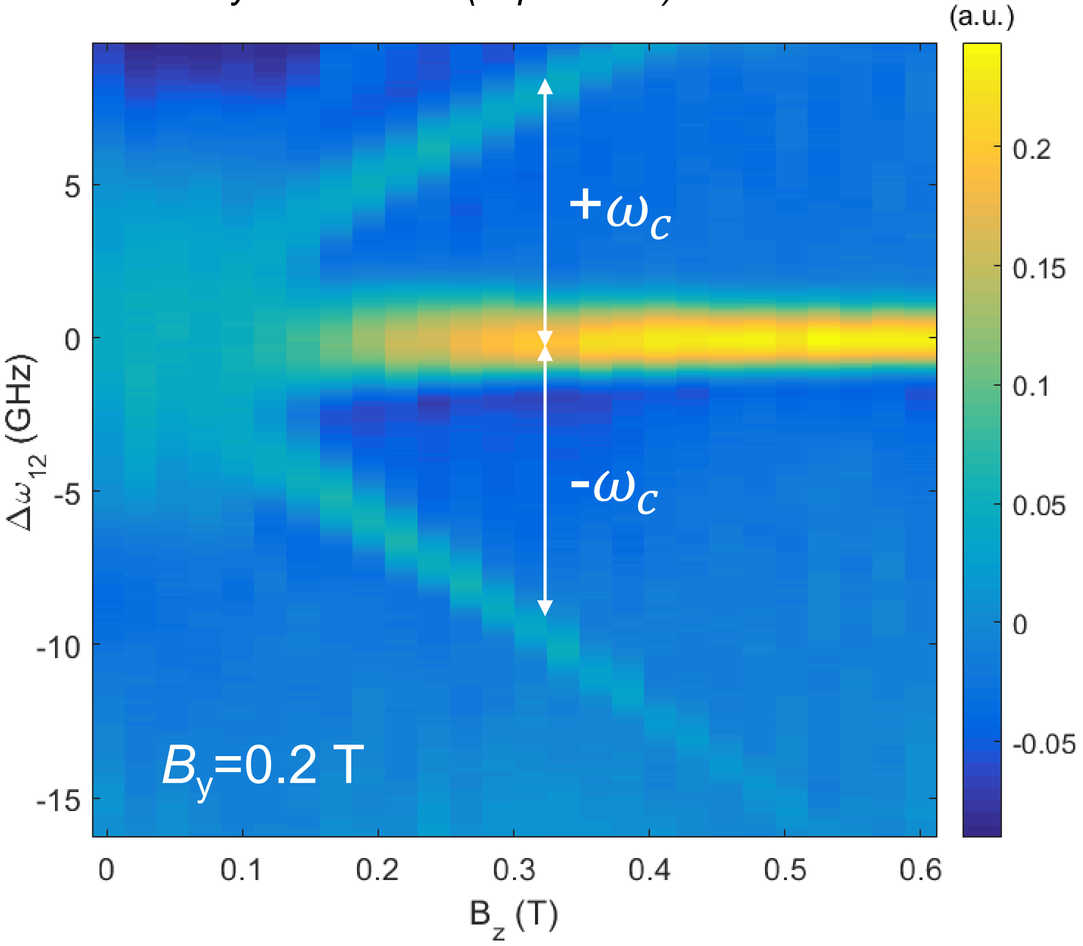
No adjustable parameters!

Sideband transitions

Mixing of energy eigenstates leads to sideband transitions!

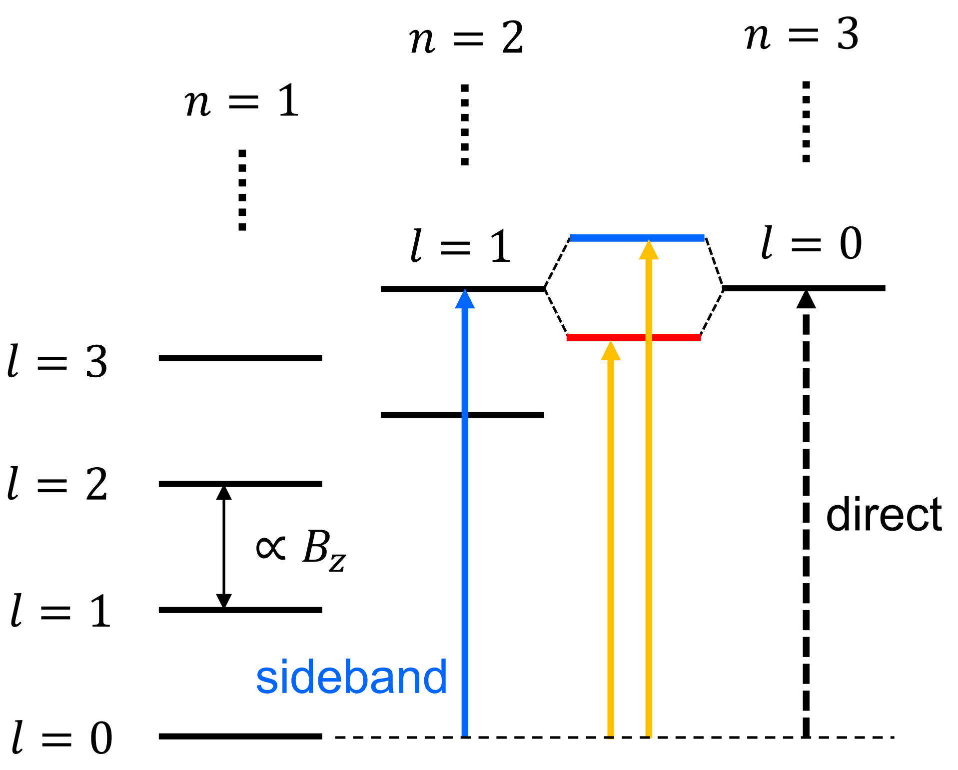


Oleksiy Zadorozhko (experiment)

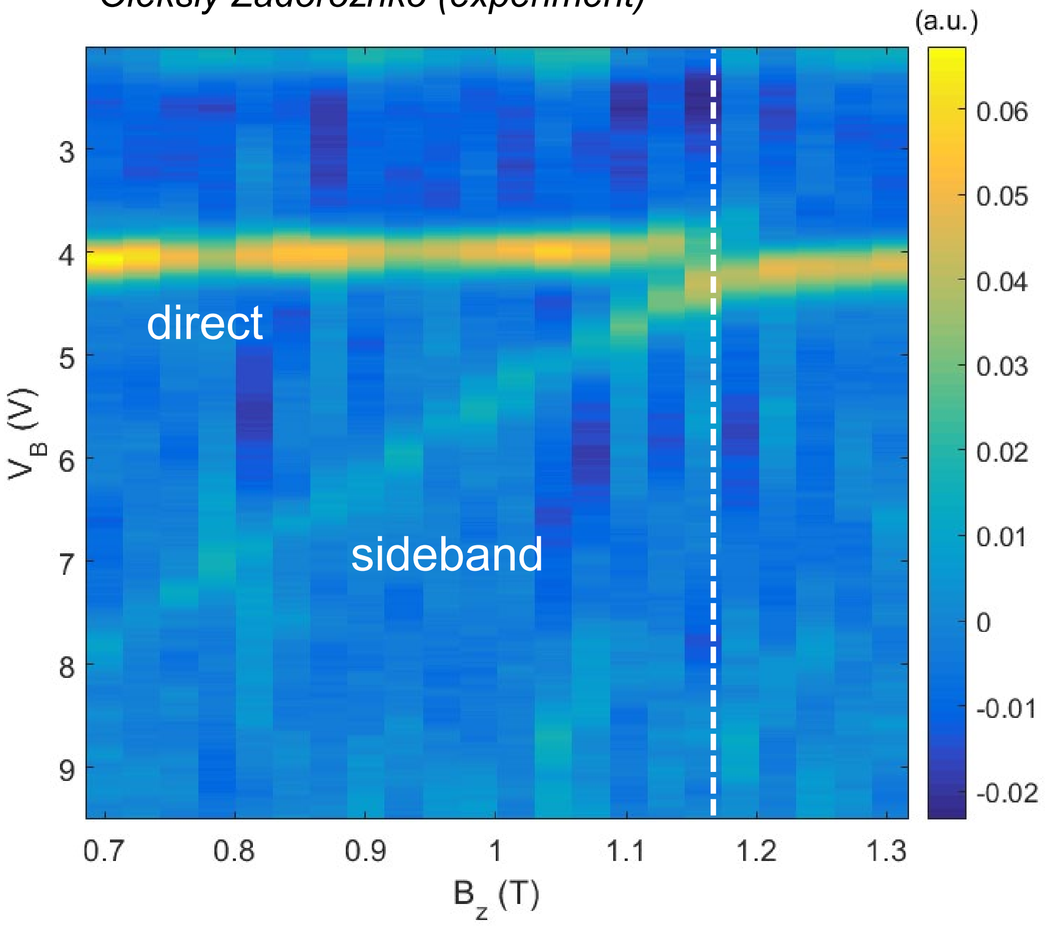


Level anti-crossing

Consider energy crossing between $|n = 2, l = 1\rangle$ and $|n = 3, l = 0\rangle$



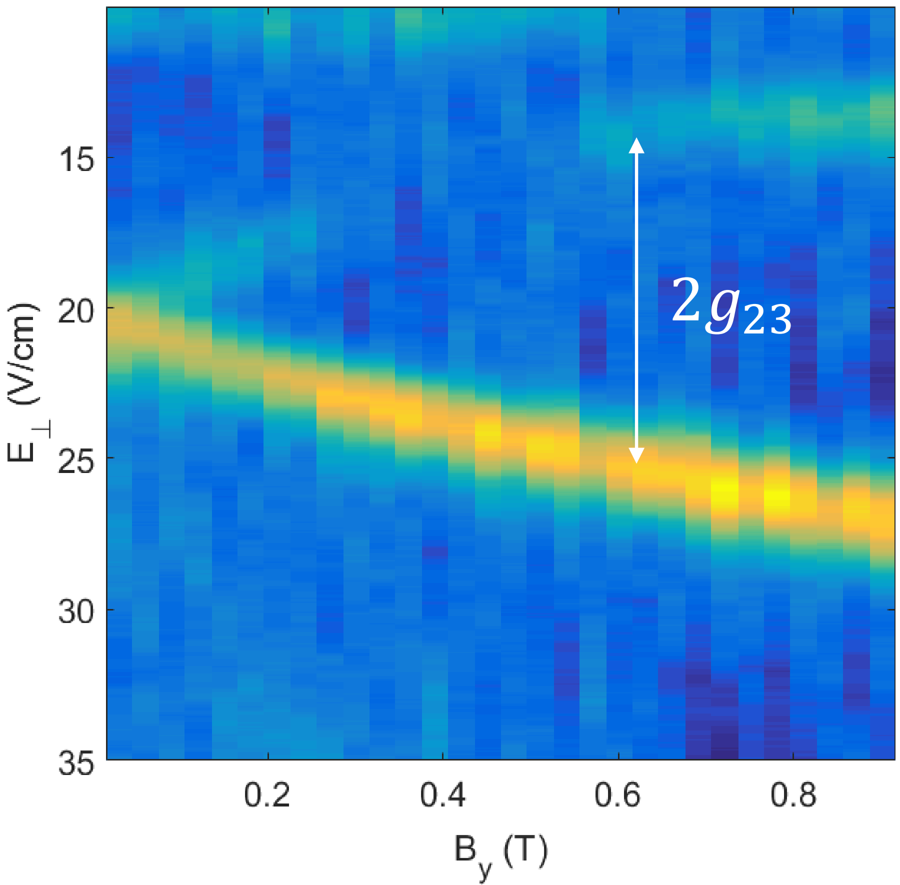
Oleksiy Zadorozhko (experiment)



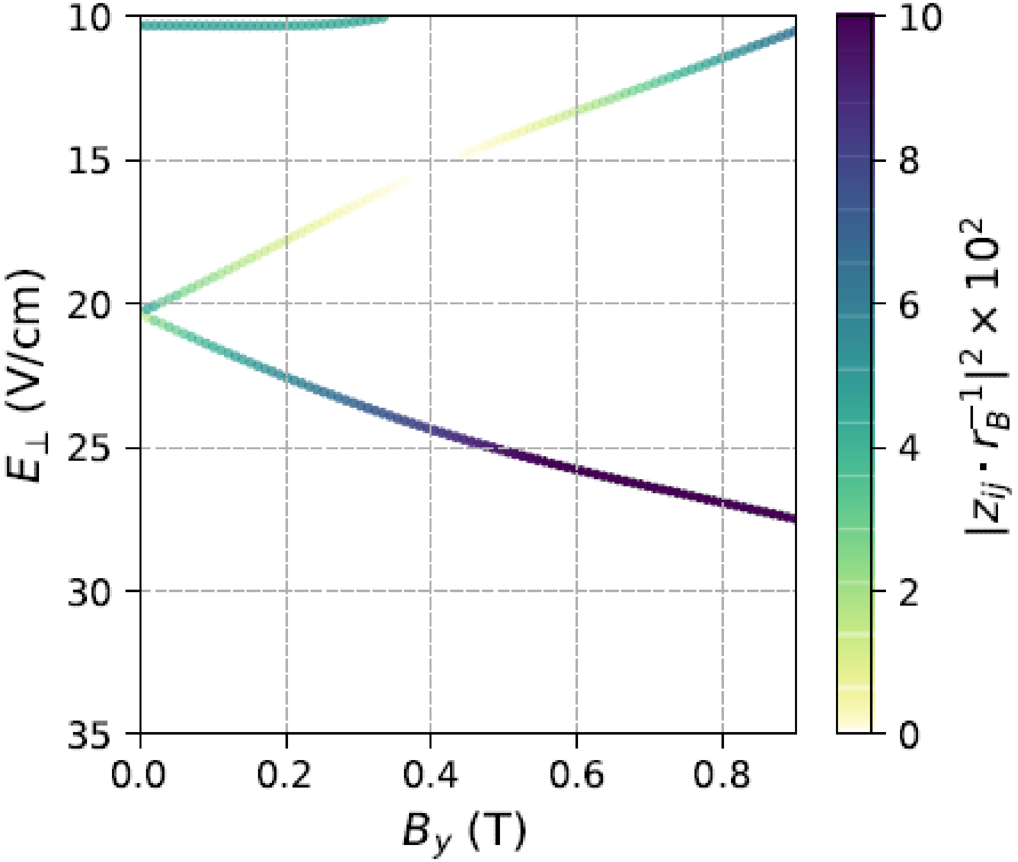
Vacuum Rabi splitting

Vacuum Rabi splitting - $2g_{23} = \sqrt{2\hbar m \omega_z \omega_y^2} z_{nn'} \propto B_y$

Oleksiy Zadorozhko (experiment)

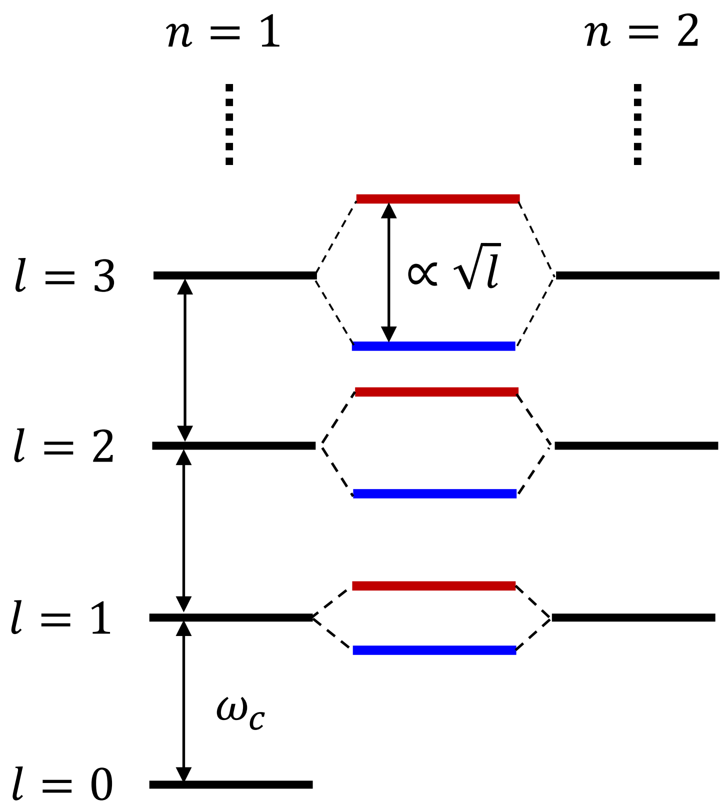


Jiabao Chen (PhD thesis)

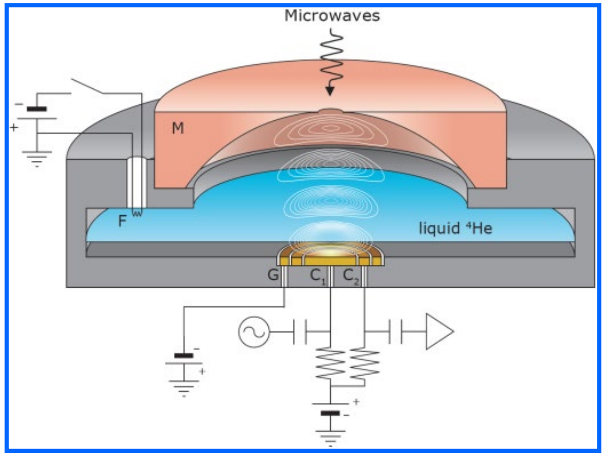


Electrons in cavity

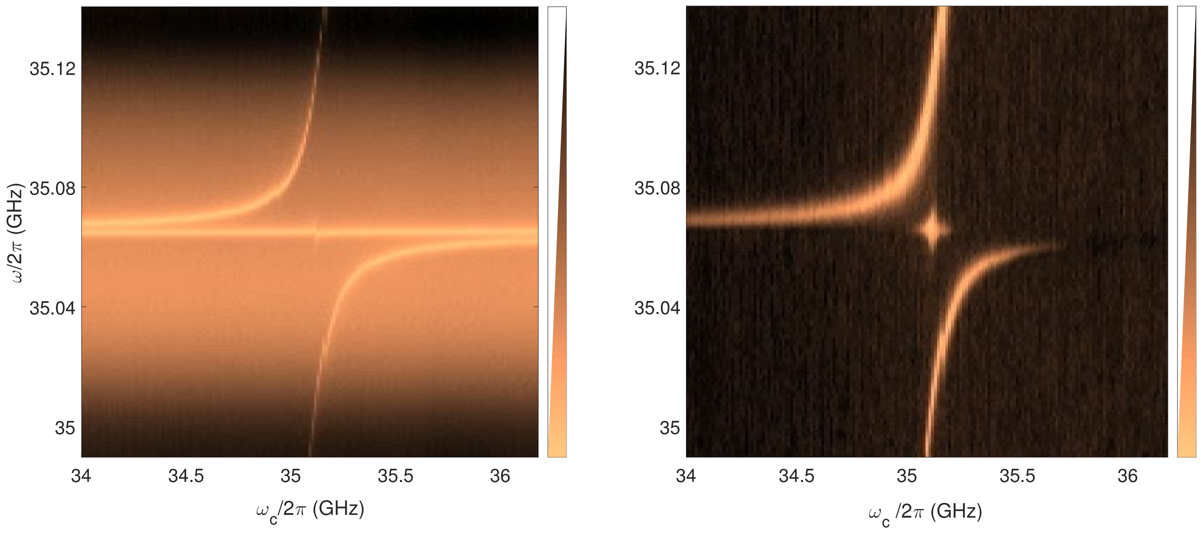
Strong non-linearity of cyclotron spectrum due to coupling!



Cavity QED -type experiments with electrons on helium!



L. Abdurakhimov, R. Yamashiro, A. Badrutdinov, DK, PRL (2016)



J. Chen, O. Zadorozhko, DK, PRB (2019)

Summary

Electrons on helium

Unique compliment to
2DEG in semiconductors

- Quantum transport
- Many-electron effects
- Topological effects

Model quantum systems
(1D Rydberg atom)

- Coherent control of states
- Quantum engineering
- Cavity QED

Quantum computing