Graded column removal for homomorphisms between Specht modules

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Joint work with Matthew Fayers.

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For $\lambda \vdash n$ and any $c \geqslant 0$, define $\lambda_L(c)$ and $\lambda_R(c)$ to be the partitions on the left and right of a vertical cut after column c of $[\lambda]$.

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Theorem (Fayers–Lyle '03, Lyle–Mathas '05)

Suppose $e \neq 2$. Then $\mathrm{Hom}_{R_n}(S_\lambda, S_\mu) \cong \mathrm{Hom}_{R_L}(S_{\lambda_L}, S_{\mu_L}) \otimes \mathrm{Hom}_{R_R}(S_{\lambda_R}, S_{\mu_R})$ *as $\mathbb{F}$-vector spaces, where R_n, R_L, R_R are Hecke algebras of type A of the appropriate degrees.*

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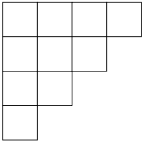
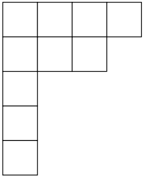
$[\lambda] =$ 

$[\mu] =$ 

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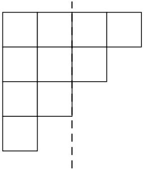
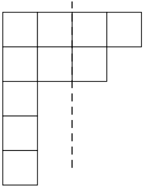
$[\lambda] =$  $[\mu] =$ 

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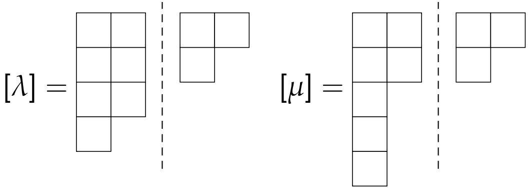

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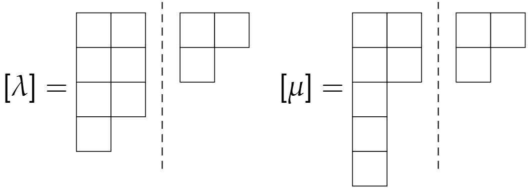

$$\mathrm{Hom}_{R_{10}}(S_\lambda, S_\mu) = \mathrm{Hom}_{R_7}(S_{(2^3,1)}, S_{(2^2,1^3)}) \otimes \mathrm{Hom}_{R_3}(S_{(2,1)}, S_{(2,1)}).$$

The KLR algebra

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$\mathbb{F}$ a field of characteristic p, $e \in \{0, 2, 3, 4, \dots\}$ and define $I = \mathbb{Z}/e\mathbb{Z}$. The *affine Khovanov–Lauda–Rouquier algebra* or *quiver Hecke algebra* R_n is the unital associative $\mathbb{F}$-algebra with generating set

$$\left\{ e(i) \;\middle|\; i \in I^n \right\} \cup \{y_1, \dots, y_n\} \cup \{\psi_1, \dots, \psi_{n-1}\}$$

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$$\begin{aligned} e(i)e(j) &= \delta_{i,j} e(i); \\ \sum_{i \in I^n} e(i) &= 1; \quad * \\ y_r e(i) &= e(i) y_r; \\ \psi_r e(i) &= e(s_r i) \psi_r; \end{aligned}$$

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y_r y_s &= y_s y_r; \\
\psi_r y_s &= y_s \psi_r && \text{if } s \neq r, r+1; \\
\psi_r \psi_s &= \psi_s \psi_r && \text{if } |r - s| > 1;
\end{aligned}$$

$$\begin{aligned} y_r \psi_r e(i) &= (\psi_r y_{r+1} - \delta_{i_r, i_{r+1}}) e(i); \\ y_{r+1} \psi_r e(i) &= (\psi_r y_r + \delta_{i_r, i_{r+1}}) e(i); \end{aligned}$$

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$$\psi_r^2 e(i) = \begin{cases} 0 & \text{if } i_r = i_{r+1}, \\ e(i) & \text{if } i_{r+1} \neq i_r, i_r \pm 1, \\ (y_{r+1} - y_r)e(i) & \text{if } i_r \rightarrow i_{r+1}, \\ (y_r - y_{r+1})e(i) & \text{if } i_r \leftarrow i_{r+1}, \\ (y_{r+1} - y_r)(y_r - y_{r+1})e(i) & \text{if } i_r \rightleftarrows i_{r+1}; \end{cases}$$

$$\psi_r\psi_{r+1}\psi_r = \begin{cases} (\psi_{r+1}\psi_r\psi_{r+1} + 1)e(i) & \text{if } i_{r+2} = i_r \rightarrow i_{r+1}, \\ (\psi_{r+1}\psi_r\psi_{r+1} - 1)e(i) & \text{if } i_{r+2} = i_r \leftarrow i_{r+1}, \\ (\psi_{r+1}\psi_r\psi_{r+1} + y_r - 2y_{r+1} + y_{r+2})e(i) & \text{if } i_{r+2} = i_r \rightleftarrows i_{r+1}, \\ (\psi_{r+1}\psi_r\psi_{r+1})e(i) & \text{otherwise}; \end{cases}$$

for all admissible r, s, i, j.

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Corollary

Cyclotomic Hecke algebras can be non-trivially $\mathbb{Z}$-graded.

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Example: Take $\lambda = (3,1)$. S_λ has a basis indexed by the three standard λ-tableaux $\begin{array}{|c|c|c|}\hline 1 & 3 & 4 \\ \hline 2 \\ \cline{1-1}\end{array}$, $\begin{array}{|c|c|c|}\hline 1 & 2 & 4 \\ \hline 3 \\ \cline{1-1}\end{array}$ and $\begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 \\ \cline{1-1}\end{array}$.

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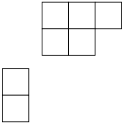
Example: $\lambda = ((3,2),(1,1))$.Then $[\lambda] =$  .

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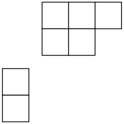
Example: $\lambda = ((3,2),(1,1))$.Then $[\lambda] =$  .

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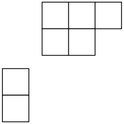
Example: $\lambda = ((3,2),(1,1))$.Then $[\lambda] =$ .

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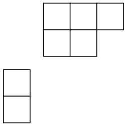
Example: $\lambda = ((3,2),(1,1))$.Then $[\lambda] =$ .

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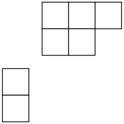

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3	5	7
4	6	

1
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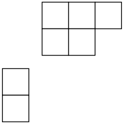
Example: $\lambda = ((3,2),(1,1))$.Then $[\lambda] =$  .

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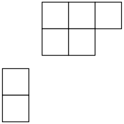
Example: $\lambda = ((3,2),(1,1))$.Then $[\lambda] =$ .

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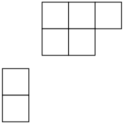
Example: $\lambda = ((3,2),(1,1))$.Then $[\lambda] =$ .

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$$\mathtt{T}_\lambda = \begin{array}{l} \qquad \begin{array}{|c|c|c|}\hline 3 & 5 & 7 \\ \hline 4 & 6 \\ \cline{1-2} \end{array} \\ \begin{array}{|c|}\hline 1 \\ \hline 2 \\ \hline \end{array} \end{array}. \text{ Let } \mathtt{T} = \begin{array}{l} \qquad \begin{array}{|c|c|}\hline 3 & 5 \\ \hline 6 & 7 \\ \hline \end{array} \\ \begin{array}{|c|c|}\hline 1 & 4 \\ \hline 2 \\ \cline{1-1} \end{array} \end{array} \text{ and } \mathtt{S} = \begin{array}{l} \qquad \begin{array}{|c|c|}\hline 3 & 4 \\ \hline 5 & 6 \\ \hline \end{array} \\ \begin{array}{|c|c|}\hline 1 & 7 \\ \hline 2 \\ \cline{1-1} \end{array} \end{array}.$$

Then $\mathtt{T}$ is λ-dominated, but $\mathtt{S}$ is not.

Dominated homomorphisms

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If $e \neq 2$ and $\kappa_i \neq \kappa_j$ whenever $i \neq j$, then $\mathrm{DHom}_{R_n}(\mathrm{S}_\lambda, \mathrm{S}_\mu) = \mathrm{Hom}_{R_n}(\mathrm{S}_\lambda, \mathrm{S}_\mu)$.

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Definition

For any l-multipartition λ of n and any $1 \leqslant m \leqslant l$, $c \geqslant 0$, define $\lambda_L(c, m)$ and $\lambda_R(c, m)$ to be the multipartitions on the left and right of a vertical cut after column c of component m of $[\lambda]$.

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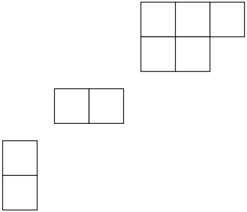

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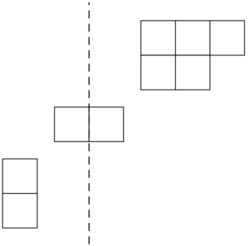
$[\lambda] =$ 

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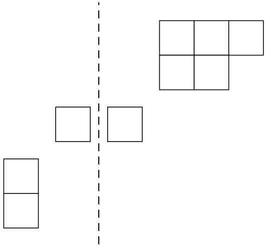
$[\lambda] =$ 

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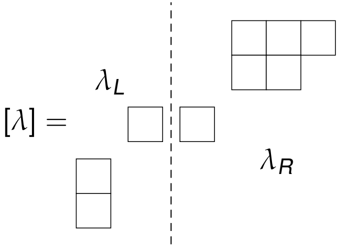

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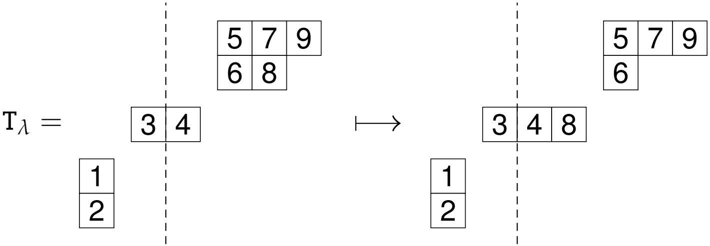

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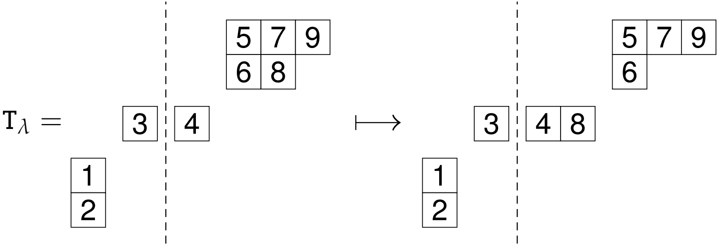

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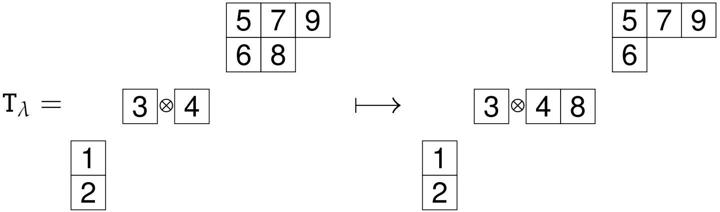

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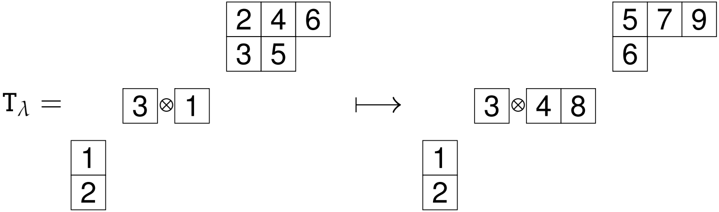

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Conjecture

If $e = 2$, $\kappa_i \neq \kappa_j$ whenever $i \neq j$, and λ is *conjugate-Kleshchev*, then $\mathrm{DHom}_{R_n}(\mathrm{S}_\lambda, \mathrm{S}_\mu) = \mathrm{Hom}_{R_n}(\mathrm{S}_\lambda, \mathrm{S}_\mu)$.