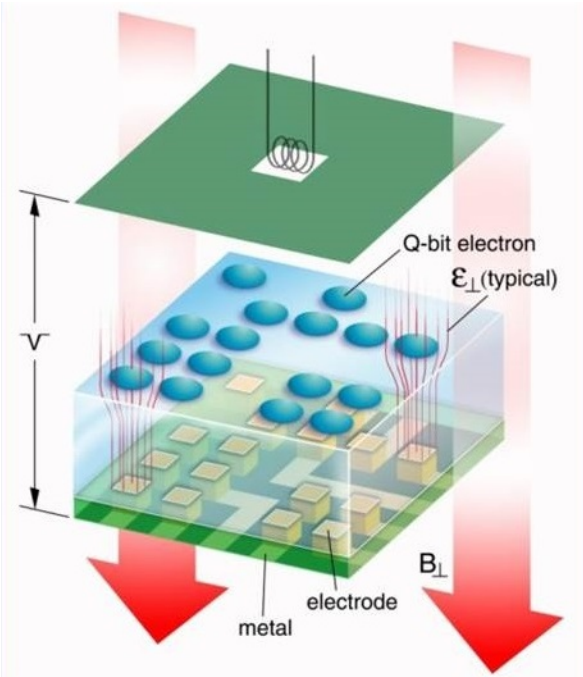
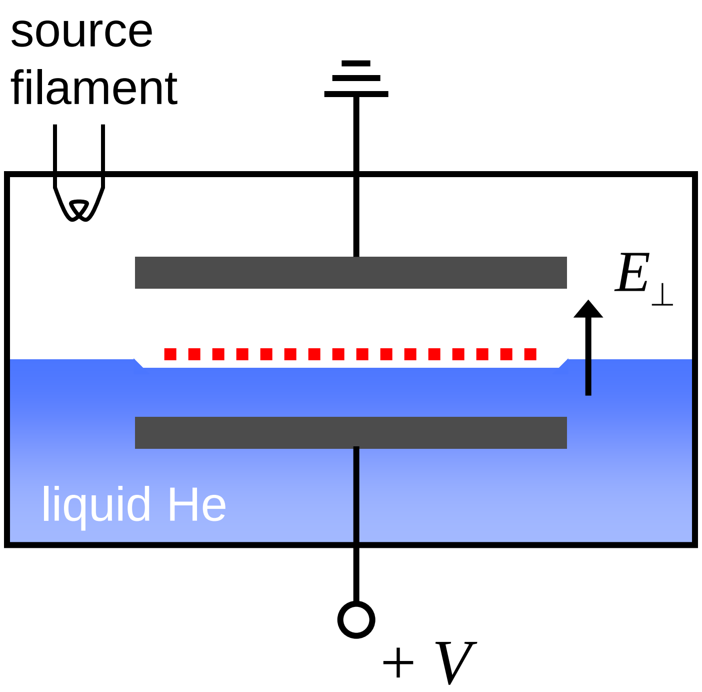


Electrons on Helium for Quantum Computing

Denis Konstantinov
Quantum Dynamics Unit, OIST

CQD 2014, OIST Sept. 22

Electrons on helium



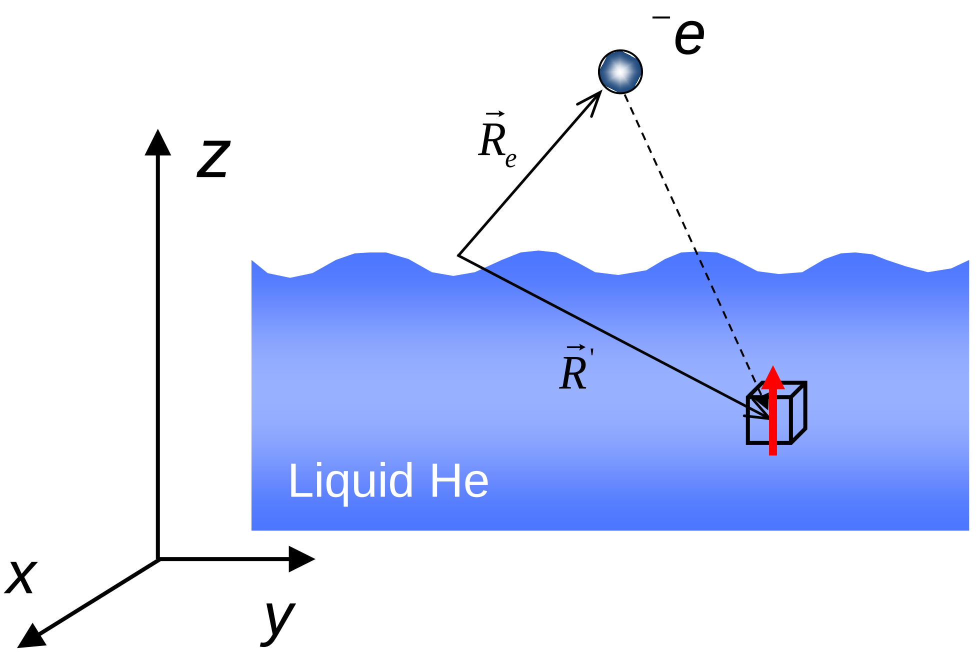
Cryogenic non-polar substrates:

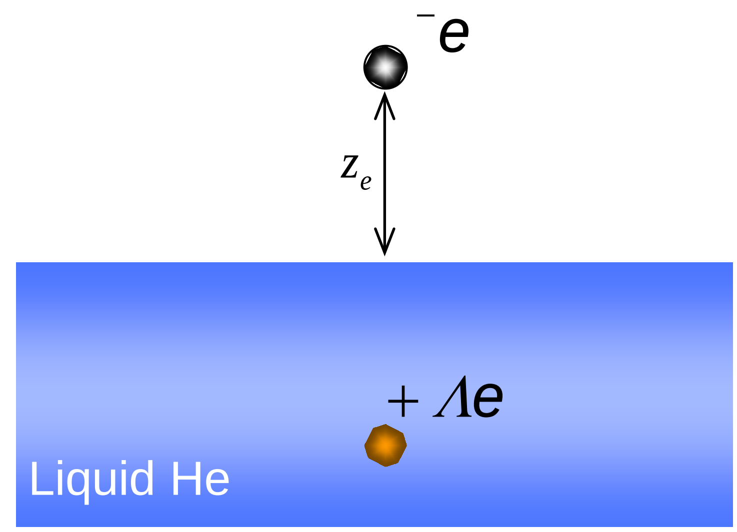
- solid hydrogen
- liquid neon
- liquid helium

Why liquid helium?

- remains liquid down to T=0
- no impurities
- the smoothed surface

Polarization potential



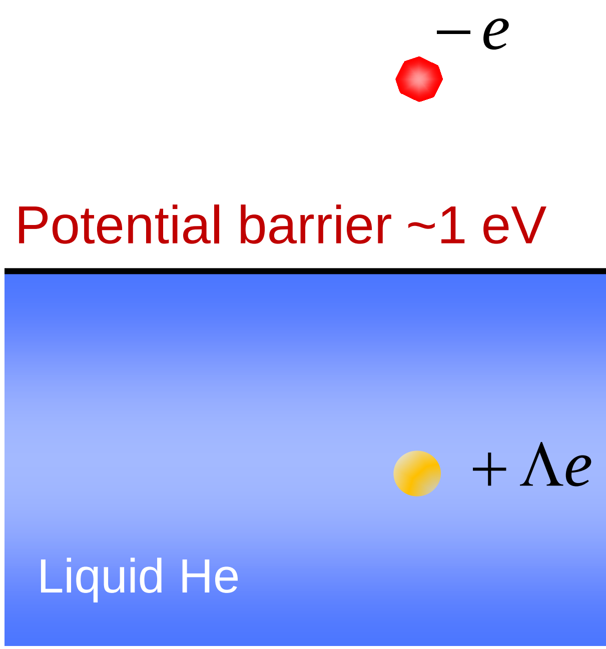


$$U_{pol}(\vec{R}_e) = -\frac{(\varepsilon - 1)e^2}{4(\varepsilon + 1)} \int d^3\vec{R}' \frac{1}{\left|\vec{R}' - \vec{R}_e\right|^4}$$

$$U_{pol}(z_e) = -\frac{(\varepsilon - 1)e^2}{4(\varepsilon + 1)z_e}$$

Surface barrier

Sommer, 1964



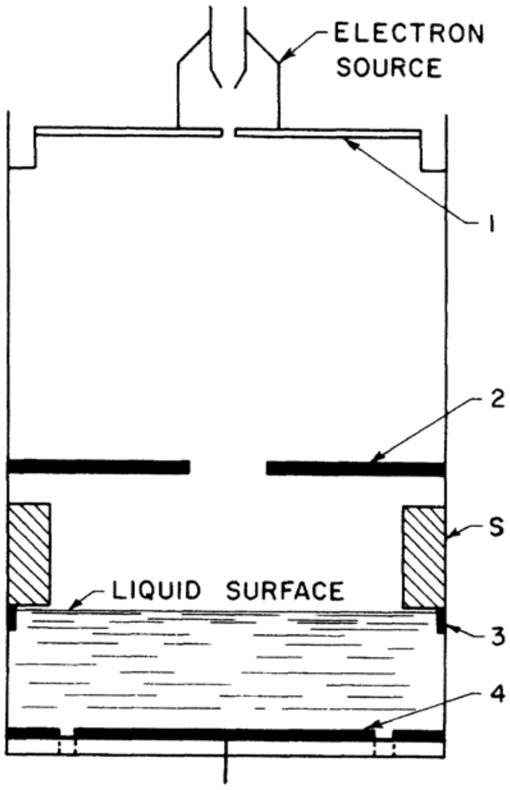
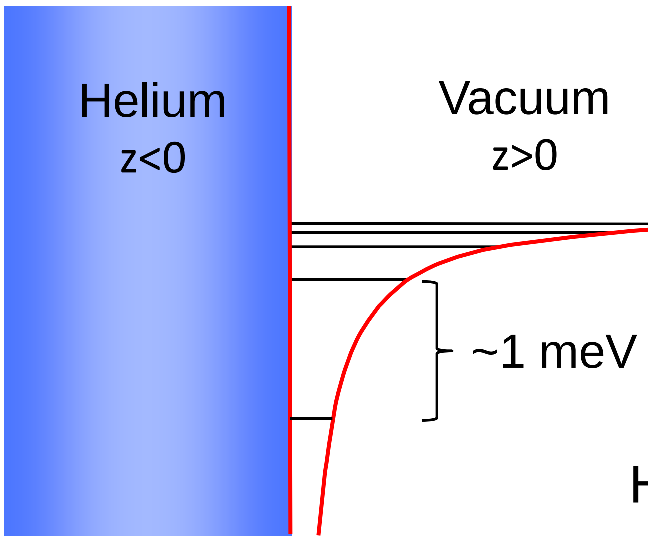


FIG. 1. The experimental chamber.

The Pauli exclusion principle -

electron avoids He atoms

Surface states



Equation of motion in *z*-direction:

$$\psi_n''(z)+\left(\frac{2mE_n}{\hbar^2}+\frac{\alpha}{z}\right)\psi_n(z)=0$$

Hydrogen-like spectrum:

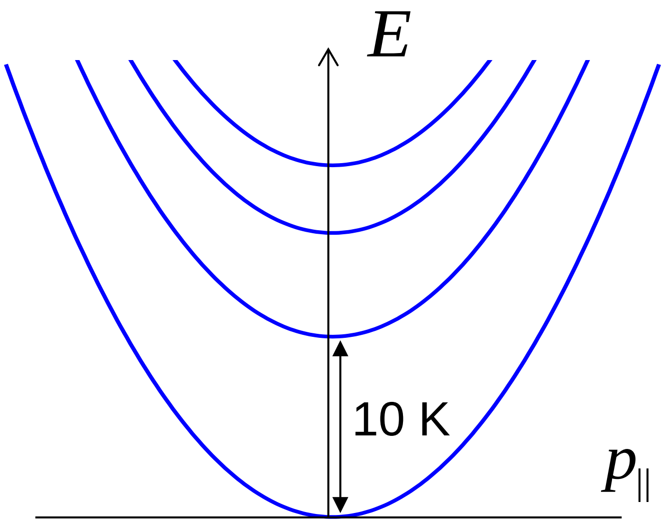
$$E_n=-\frac{R}{n^2}$$

$^4\mathrm{He}:\quad \varepsilon=1.057$

$$R=\frac{(\varepsilon-1)^2 m}{32(\varepsilon+1)^2\hbar}\approx 8\,\mathrm{K}\quad (120\,\mathrm{GHz})$$

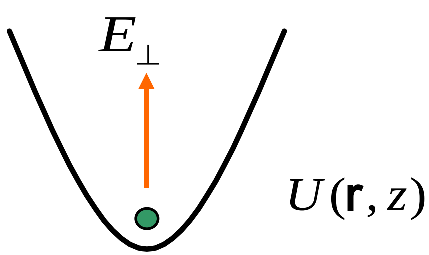
$$a_B=\frac{\hbar}{\sqrt{2m_e R}}\approx 7.6\,\mathrm{nm}$$

2D electron system



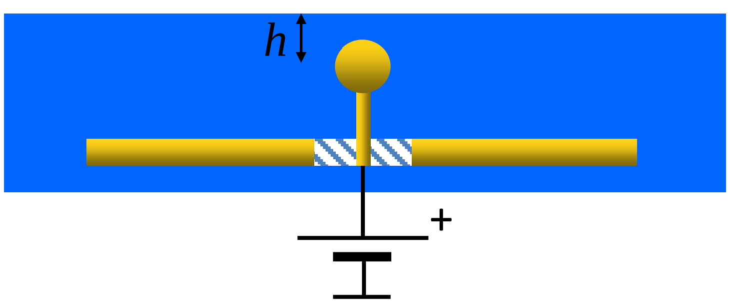
Total energy:

$$E = E_n + \underbrace{\frac{p_{\parallel}^2}{2m}}_{\text{2D motion}} \pm \overbrace{\mu_B B_{\parallel}}^{\text{due to spin}}$$



$$U(\mathbf{r}, z) = -\frac{\Lambda e^2}{z} + eE_{\perp}e + \frac{1}{2} m\omega_{\parallel}^2 r^2$$

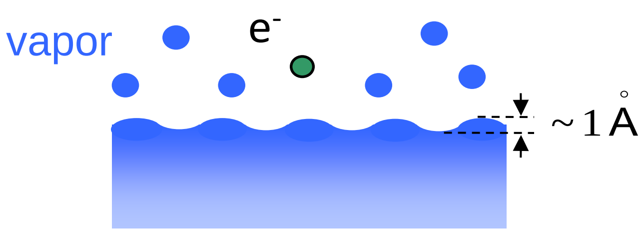
* quantization of in-plane motion

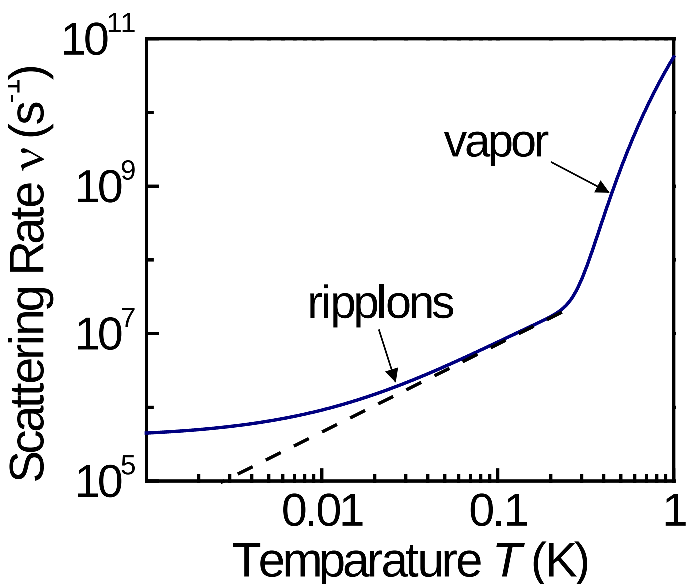


States in parabolic potential $|n, \underbrace{\nu, m}_{\nu+1 \text{ degenerate}}\rangle$

Scattering of electrons

T-dependent scattering





Vapor-atoms (short range):

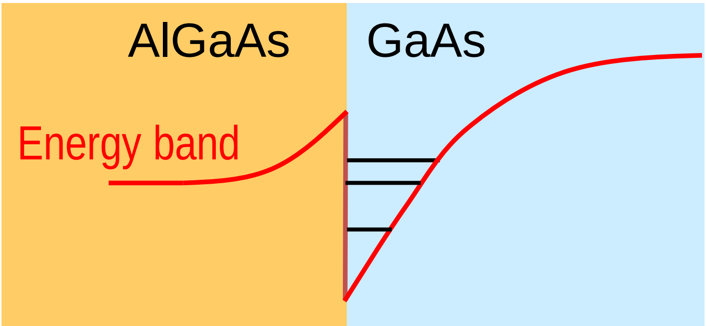
$$N_{vapor} \propto T^{3/2} \exp(-Q/kT)$$

Vibration of surface:

$$n_q = \frac{1}{\exp(\hbar\omega_q/kT) - 1} \propto T$$

(Presumably) negligible interaction with bulk excitations

Basic properties

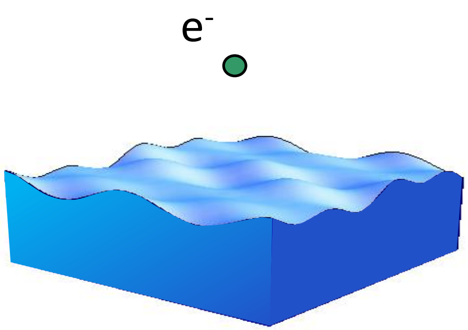


Complement to degenerate 2DEG in

Si MOSFETs and

GaAs/AlGaAs heterostructures

BUT…

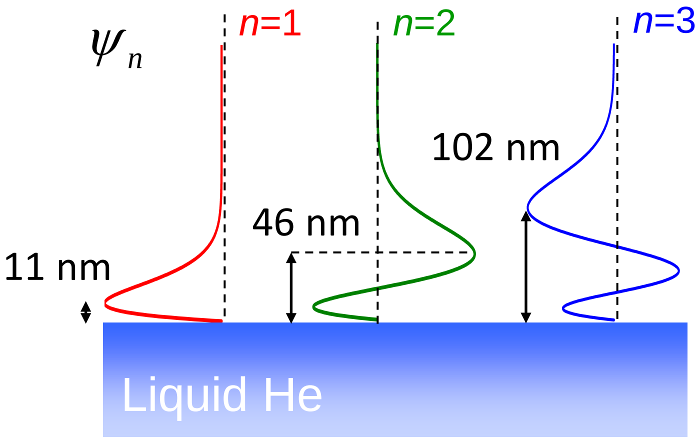


- Classical non-degenerate electron system
- No impurities, scattering from ripplons
- Electron mobility exceeding 10^8 $cm^2/V{\cdot}s$ - highest known in nature!
- Unscreened Coulomb interaction – plasmon excitations, Wigner solid
- Magneto-transport under excitation – zero-resistance states etc.

DK and Kono, PRL (2010)

DK, Monarkha and Kono, PRL (2013)

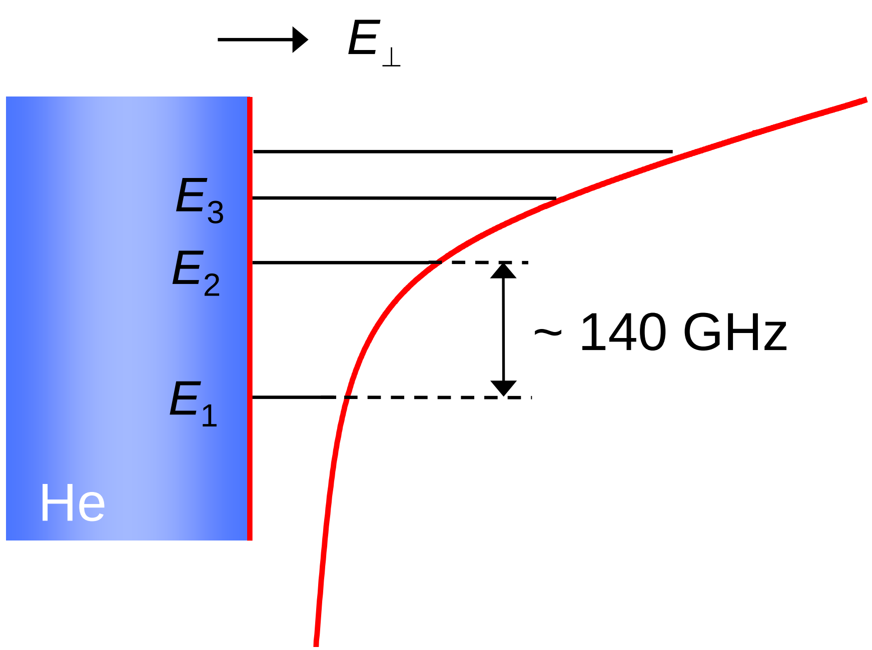
Rydberg states



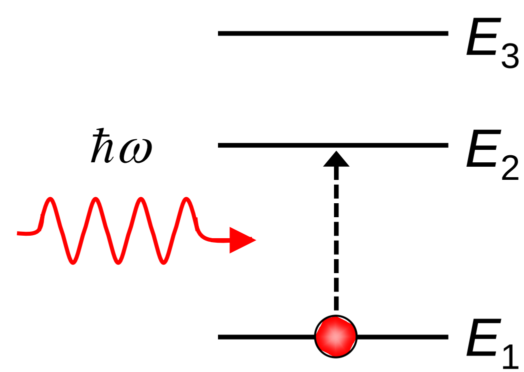
Parity symmetry-breaking of states ψ_n :

$$\langle n | z | n \rangle \neq 0$$

linear Stark effect

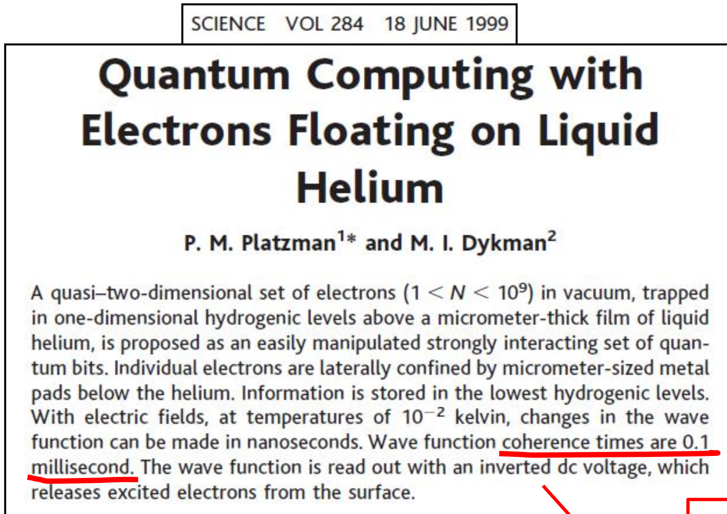


MW resonance: $\hbar\omega = E_2 - E_1$



2-level system

Proposal for qubits



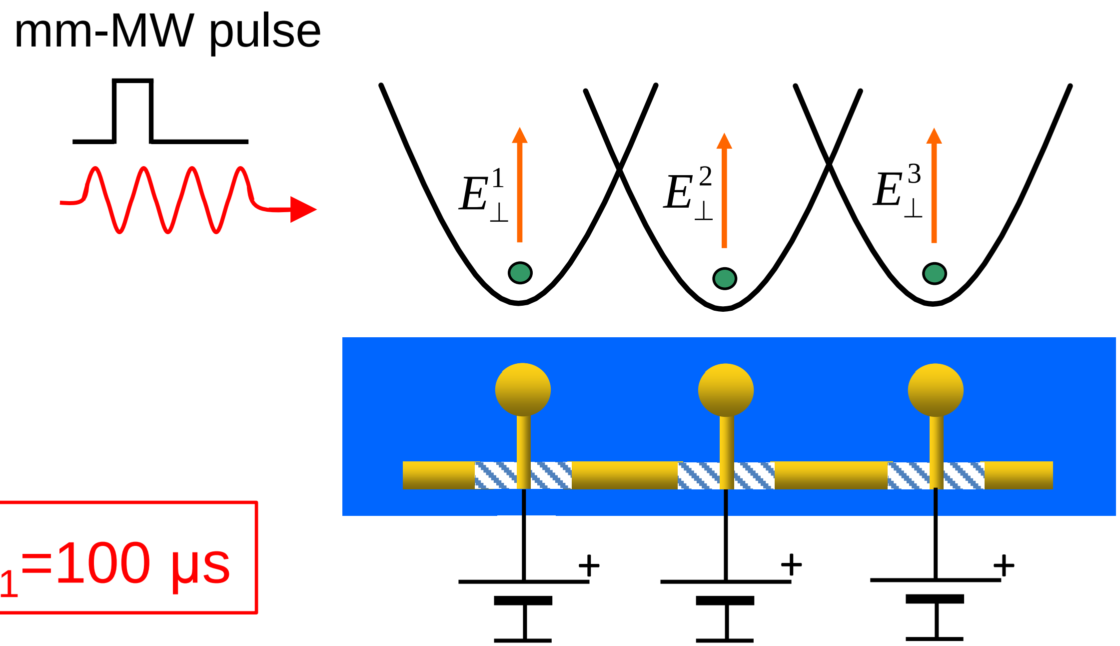
SCIENCE VOL 284 18 JUNE 1999

Quantum Computing with Electrons Floating on Liquid Helium

P. M. Platzman[1]* and M. I. Dykman[2]

A quasi–two-dimensional set of electrons ($1 < N < 10^9$) in vacuum, trapped in one-dimensional hydrogenic levels above a micrometer-thick film of liquid helium, is proposed as an easily manipulated strongly interacting set of quantum bits. Individual electrons are laterally confined by micrometer-sized metal pads below the helium. Information is stored in the lowest hydrogenic levels. With electric fields, at temperatures of 10^{-2} kelvin, changes in the wave function can be made in nanoseconds. Wave function coherence times are 0.1 millisecond. The wave function is read out with an inverted dc voltage, which releases excited electrons from the surface.

T_1=100 μs



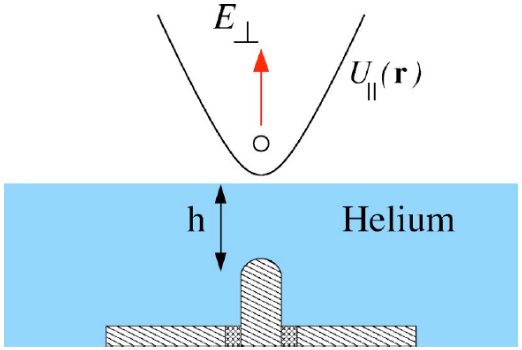
- Identification of well-defined qubits:
  I0> and I1> states of individual surface electrons
- Reliable state preparation:
  Below 1 K almost all qubits will be in the quantum ground state I0>
- Low decoherence (?)
- Scalability

- Decoherence
- Qubit coupling
- Read out

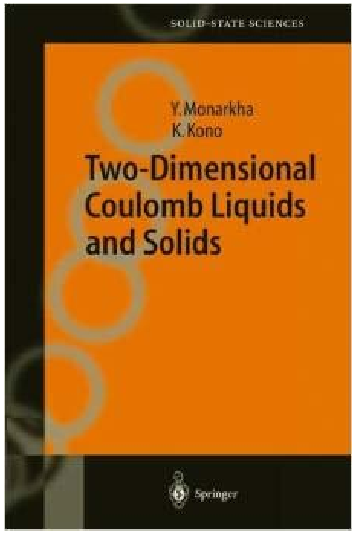
PHYSICAL REVIEW B **67**, 155402 (2003)

Qubits with electrons on liquid helium

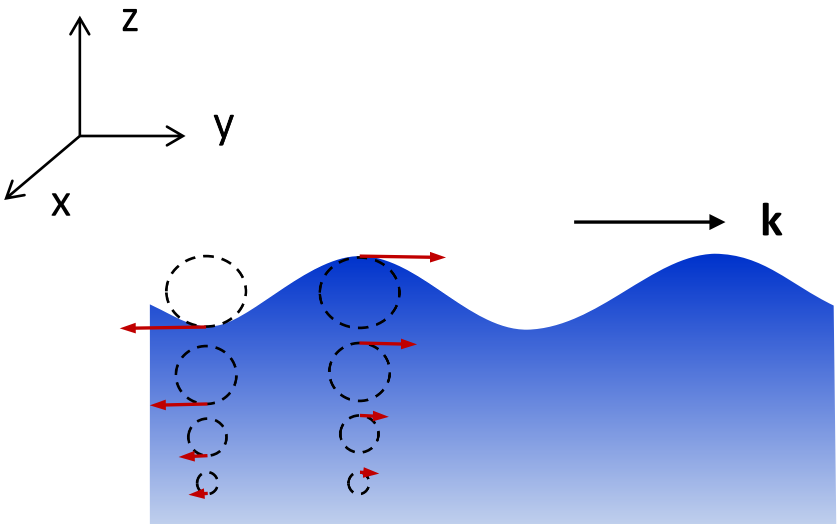
M. I. Dykman,[1,*] P. M. Platzman,[2] and P. Seddighrad[1]



Y. Monarkha, K. Kono
Two-dimensional Coulomb Liquids and Solids



Capillary-gravity waves



Ideal incompressible fluid

$$\nabla^2 \varphi = 0$$

$$\varphi = \Phi_0 e^{-kz} e^{i(\mathbf{kr} - \omega t)}$$

Euler equation

$$\nabla\left(-\rho \frac{\partial \phi}{\partial t} + P + \rho g z\right) = 0$$

Dispersion relation

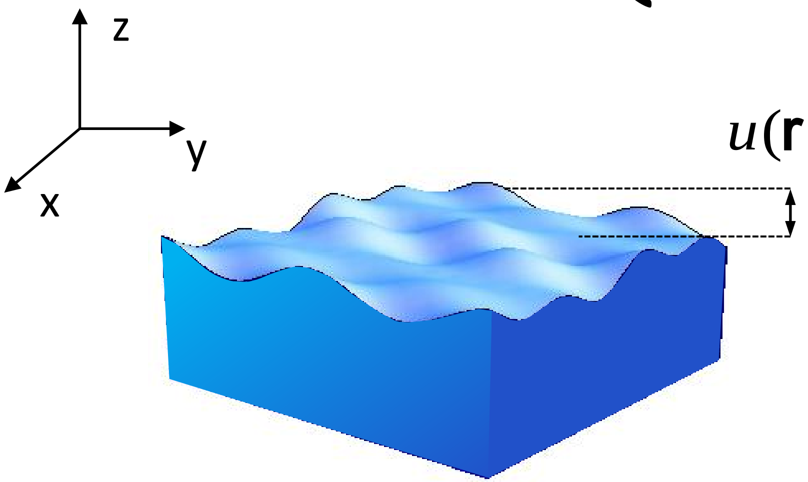
$$\omega^2 = \cancel{gk} + \frac{\sigma k^3}{\rho}$$

Typically interested with

$$k^{-1} << \kappa = \sqrt{\frac{\sigma}{g\rho}} \approx 1\,\text{mm}$$

capillary length

Quantization of ripples



$u(\mathbf{r})$ - displacement of surface

Periodic boundary conditions

$$u(\mathbf{r}) = A^{-1/2} \sum_{\mathbf{k}} Q_{\mathbf{k}} e^{i\mathbf{kr}}$$

$$\dot{u}(\mathbf{r}) = A^{-1/2} \sum_{\mathbf{k}} \dot{Q}_{\mathbf{k}} e^{i\mathbf{kr}}$$

Hamilton function

$$H = \sum_{\mathbf{k}} \Pi_{\mathbf{k}} \Pi_{\mathbf{-k}} \frac{k}{2\rho} + \sum_{\mathbf{k}} Q_{\mathbf{k}} Q_{\mathbf{-k}} \frac{\rho g + \sigma k^3}{2} = \sum_{\mathbf{k}} \hbar \omega_{\mathbf{k}} \left(a_{\mathbf{k}}^{+} a_{\mathbf{-k}} + \frac{1}{2} \right)$$

Ripplons with $\omega_{\mathbf{k}}^2 \approx \dfrac{\sigma k^3}{\rho}$

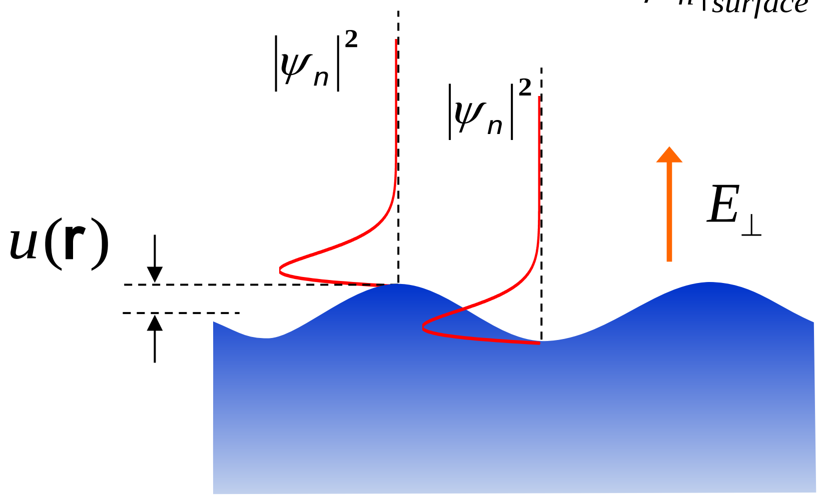
Surface displacement

$$u(\mathbf{r}) = \sqrt{\frac{\hbar k}{2\rho A \omega_k}} \sum_{\mathbf{k}} \left(a_{\mathbf{-k}}^{+} - a_{\mathbf{k}} \right) e^{i\mathbf{kr}}$$

$$\sqrt{\langle u^2(\mathbf{r}) \rangle} \approx 1\,\mathbf{A}$$ at T=0

Electron-ripplon interaction

$\psi_n\big|_{surface} \to 0$ - adiabatic adjustment to surface motion



Electron Hamiltonian

$$H_e = \underbrace{\frac{P^2}{2m_e}}_{K} + \underbrace{U_{pol} + eE_\perp z}_{\text{potential energy}}$$

Electron wave function over deformed surface

$$\tilde{\Psi}(\mathbf{R}) = \langle \mathbf{R} | \tilde{\Psi} \rangle = \langle \tilde{\mathbf{R}} | \Psi \rangle = \langle \mathbf{R} | e^{-i\frac{\hat{p}_z u(\mathbf{r})}{\hbar}} | \Psi \rangle$$

$$\langle \tilde{\Psi} | H_e | \tilde{\Psi} \rangle = \langle \Psi | \underbrace{e^{i\frac{\hat{p}_z u(\mathbf{r})}{\hbar}} H_e e^{-i\frac{\hat{p}_z u(\mathbf{r})}{\hbar}}} | \Psi \rangle \approx \langle \Psi | H_e + \underbrace{\delta H_e}_{\text{perturbation}} | \Psi \rangle$$

Perturbation to kinetic energy

So $$\delta H_e = e^{i\frac{\hat{p}_z u(\mathbf{r})}{\hbar}} H_e e^{-i\frac{\hat{p}_z u(\mathbf{r})}{\hbar}} - H_e$$, expansion parameter $$\frac{u}{a_B} << 1$$

Largest contribution from kinetic energy $$K = \frac{p_{\mathbf{r}}^2}{2m_e} + \frac{p_z^2}{2m_e}$$

$$\delta K = \frac{p_z}{2m_e}\left(\nabla_{\mathbf{r}} u(\mathbf{r}) \cdot p_{\mathbf{r}} + p_{\mathbf{r}} \cdot \nabla_{\mathbf{r}} u(\mathbf{r})\right) + \frac{p_z^2}{2m_e}\left(\nabla_{\mathbf{r}} u(\mathbf{r})\right)^2 + ..$$

δK_1 - linear in $u(\mathbf{r})$ δK_2 - quadratic in $u(\mathbf{r})$

$$-\frac{ip_z}{2m_e}\sum_{\mathbf{k}} ...\left(a_{-\mathbf{k}}^{+} - a_{\mathbf{k}}\right)e^{i\mathbf{kr}} \qquad + \frac{p_z^2}{2m_e}\sum_{\mathbf{k},\mathbf{k}'} ...\left(a_{-\mathbf{k}}^{+} a_{-\mathbf{k}'}^{+} - a_{\mathbf{k}} a_{-\mathbf{k}'}^{+} - a_{-\mathbf{k}}^{+} a_{-\mathbf{k}} + a_{-\mathbf{k}} a_{-\mathbf{k}'}\right)e^{i(\mathbf{k}+\mathbf{k}')\mathbf{r}}$$

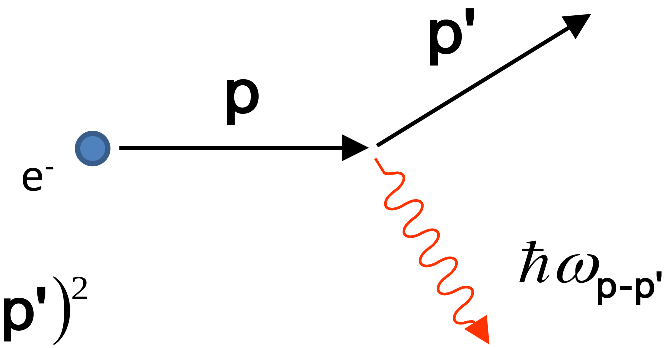
Surface displacement

$$u(\mathbf{r}) = \sqrt{\frac{\hbar k}{2\rho A \omega_k}}\sum_{\mathbf{k}}\left(a_{-\mathbf{k}}^{+} - a_{\mathbf{k}}\right)e^{i\mathbf{kr}}$$

One- and two-ripplon processes

One-ripplon scattering

$$\delta K_1 = -\frac{ip_z}{2m_e}\sum_{\mathbf{k}}\ldots\left(a^+_{-\mathbf{k}} - a_{\mathbf{k}}\right)e^{i\mathbf{kr}}$$

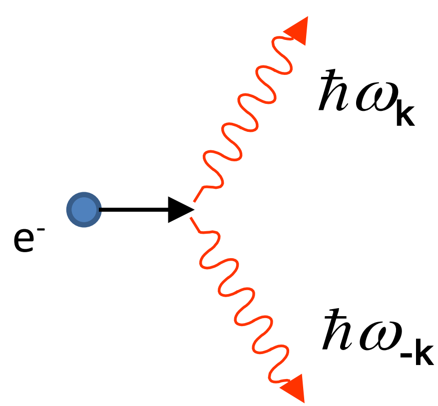


Ripplons modes are soft $\hbar\omega_{\mathbf{p-p'}} << \frac{(\mathbf{p-p'})^2}{2m_e}$

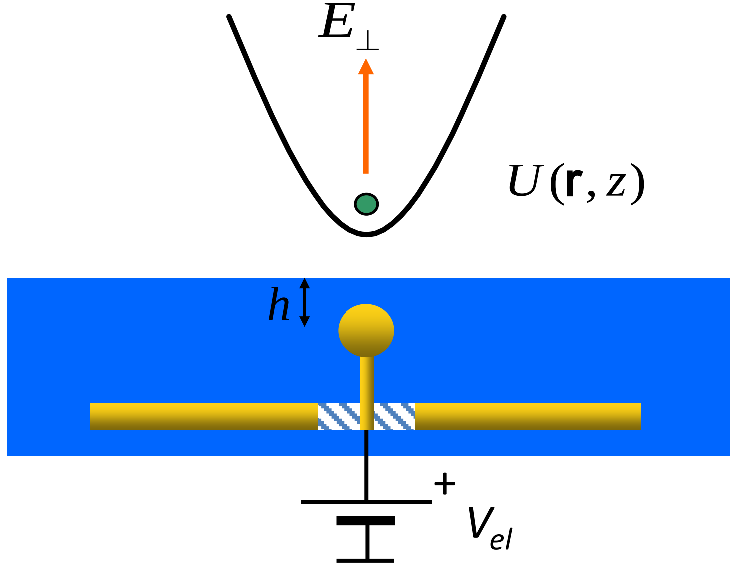
One-ripplon processes are <u>elastic</u>!

Transfer energy by two-ripplon emission!

$$\delta K_2 \approx \frac{p_z^2}{2m_e}\sum_{\mathbf{k},\mathbf{k'}}\ldots\left(a^+_{-\mathbf{k}}a^+_{-\mathbf{k'}}\right)e^{i(\mathbf{k}+\mathbf{k'})\mathbf{r}}$$

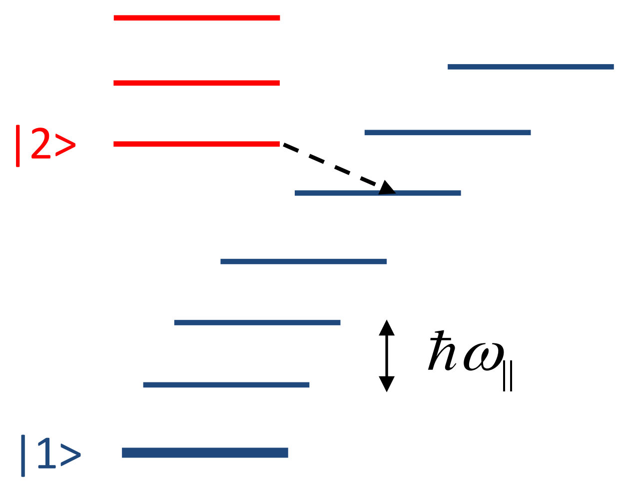


Qubit life time



$$U(\mathbf{r}, z) = -\frac{\Lambda e^2}{z} + eE_\perp e + \frac{1}{2} m\omega_\parallel^2 r^2$$

* quantization of in-plane motion



Can suppress one-ripplon decay!

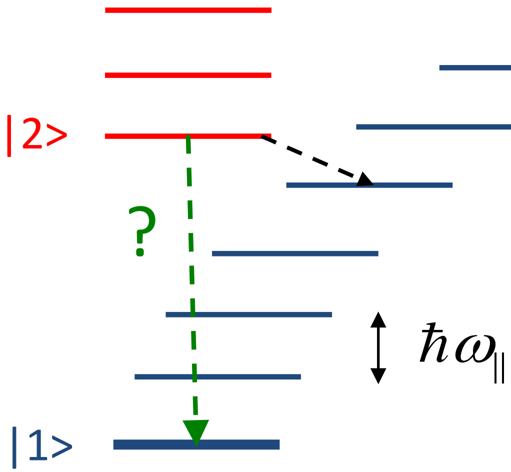
$h \sim 0.5\,\mu\text{m}$

$\omega_\parallel = 20\text{ GHz}$ for $V_{\text{el}} \sim 10\text{ meV}$

Decay due to two-ripplon emission

$$\tau_{decay}^{-1} = \frac{2\pi}{\hbar} \sum_{\nu,m,k} |< 2,0,0 | \delta K_2 | 1,\nu,m >|^2 \delta(E_{2,0} - E_{1,\nu} + 2\hbar\omega_k)$$

*Fermi's golden rule



with $\delta K_2 = \frac{p_z^2}{2m_e}(\nabla_r u(r))^2$

*kinematic 2-ripplon coupling

$$\tau_{decay}^{-1} \propto \left\langle 2 \right| \frac{p_z^2}{2m_e} \left| 1 \right\rangle \times k_0^5$$

were $\Delta E = 2\hbar\omega_{k_0}$

$$\Delta E = \hbar\omega_{||}$$

$$\omega_{||} = 2\pi \cdot 20 \text{ GHz} \qquad k_0 = 1.2\times 10^7 \text{ cm}^{-1}$$

Decay rate

$$\tau_{decay}^{-1} \approx 10^4 \text{ s}^{-1}$$

Decay due to two-ripplon emission

SCIENCE VOL 284 18 JUNE 1999

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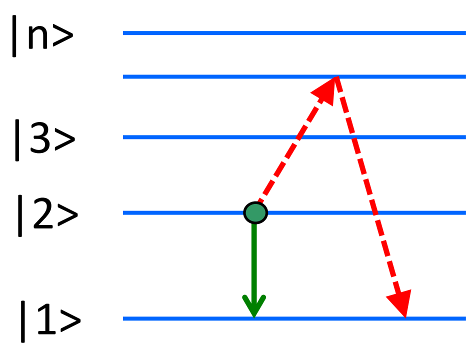
Elephant in the room!

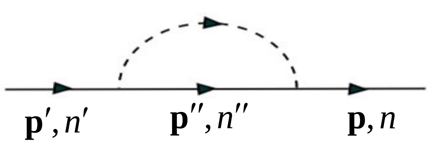
Is there anything we missed?

Second order terms? May be!

Introduced cut-off in *k*

$$k < 10^7 \text{ cm}^{-1}$$



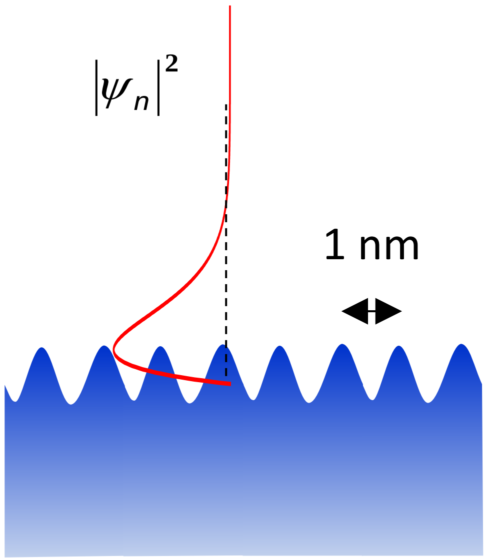


$$\tau_{decay}^{-1} = \frac{2\pi}{\hbar} \sum_{k,\nu,m} \left| <2,0,0|\delta K_2|1,\nu,m> + \sum_{n,\nu',m'} \frac{<2,0,0|\delta K_1|n,\nu',m'><n,\nu',m'|K_1|1,\nu,m>}{E_{n,\nu'} - E_{1,\nu} - \hbar\omega_k} \right|^2 \delta(E_{2,0} - E_{1,\nu} - 2\hbar\omega_k)$$

One-ripplon in second order perturbation compensates two-ripplon in first order

Work in progress!

Breakdown of adiabatic approximation



Finite potential barrier! $V_0 \approx 1\,\text{eV}$

Main interaction term

$$\delta H = V_0 \Theta(u(\mathbf{r}) - z) - V_0 \Theta(-z) \approx$$

$$\approx V_0 \delta(z) u(\mathbf{r}) + \frac{1}{2} V_0 \delta'(-z) u^2(\mathbf{r})$$

Two-ripplon emission decay

$$\tau_{decay}^{-1} = \frac{V_0}{8\pi\rho^2} \left(\psi_2^{'}(0)\psi_1^{'}(0)\right)^2 \int_0^{k_0} \frac{k^3}{\omega_k^2} dk \approx 10^6\ \text{s}^{-1}$$

Need experiments!

Dephasing rate

Due to random fluctuations in energy difference caused by fluctuations in $u(\mathbf{r})$

$$\Delta E_{21}(t) = \langle 200 | \delta K(t) | 200 \rangle - \langle 100 | \delta K(t) | 100 \rangle$$

Elastic scattering is different for electrons in different states!

$$\left\langle [\varphi_{21}(t) - \varphi_{21}(0)]^2 \right\rangle = \frac{1}{\hbar^2} \int_0^t dt' \int_0^t dt'' \left\langle \Delta E_{21}(t') \Delta E_{21}(t'') \right\rangle = D_\varphi t$$

Dephasing rate

$$D_\varphi = \frac{2\pi}{\hbar} \sum_{\mathbf{k},\mathbf{k}'} \left| \langle 200 | \delta K_2 | 200 \rangle - \langle 100 | \delta K_2 | 100 \rangle \right|^2 \delta\left(\hbar\omega_{\mathbf{k}} - \hbar\omega_{\mathbf{k}'}\right)$$

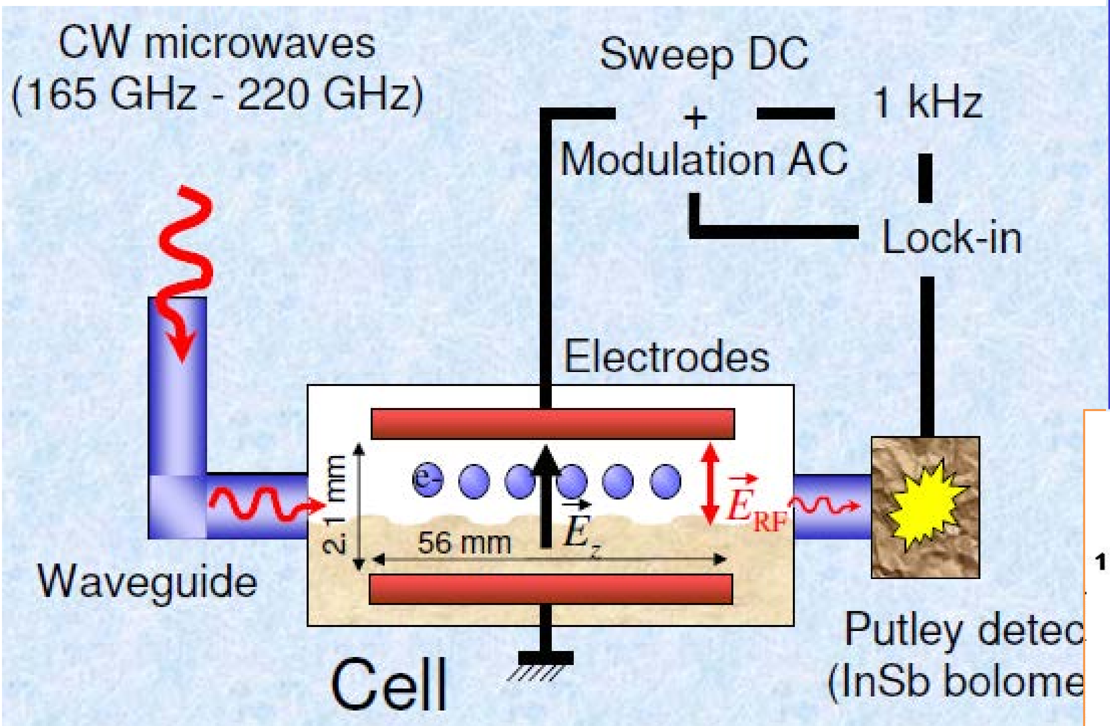
Elastic two-ripplon scattering

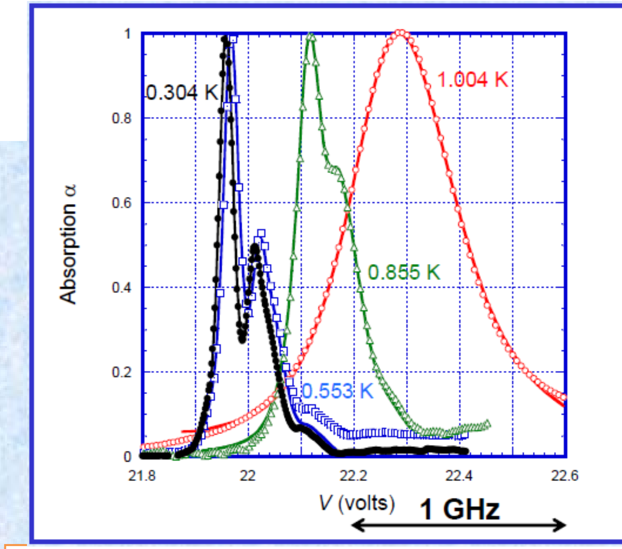
$$D_\varphi = 10^2 \ \mathbf{s}^{-1}$$

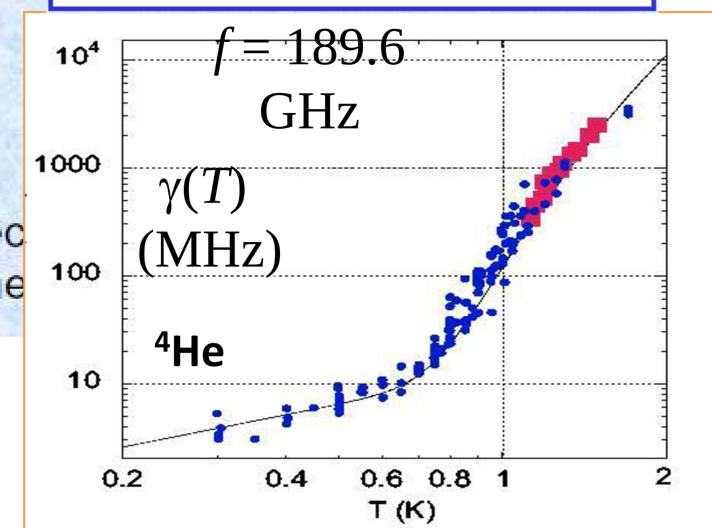
Significantly smaller!

Experiments on microwave absorption

Measure attenuation of MW power passing through cell containing electrons

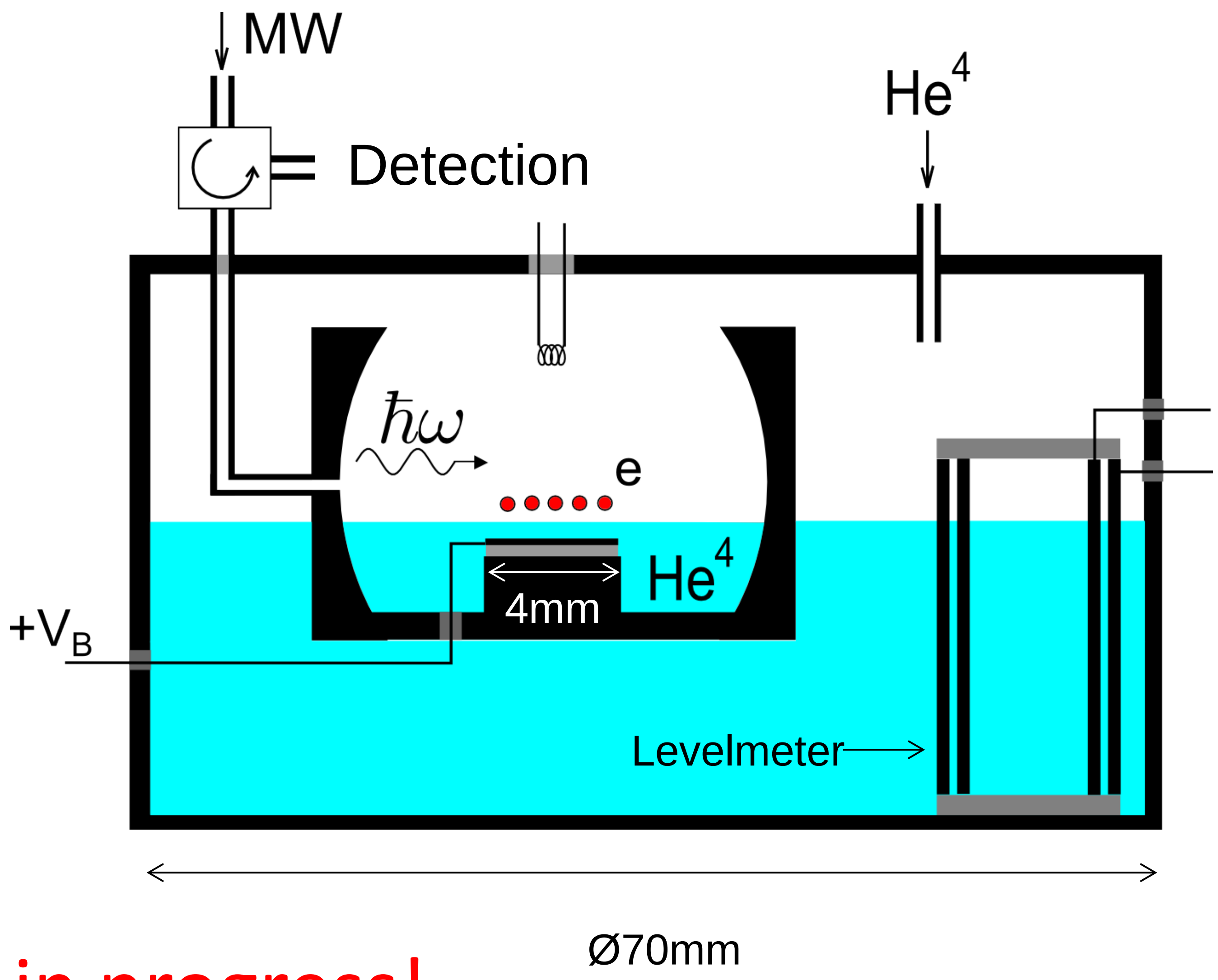






E. Collin et al. PRL 2002

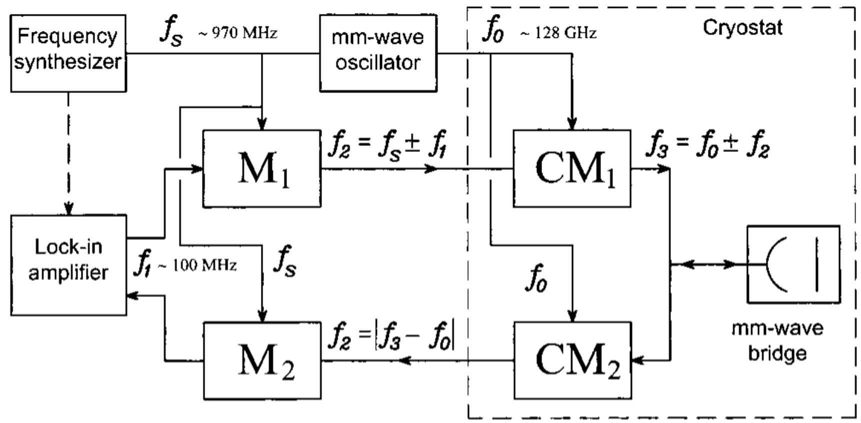
Experiments on microwave absorption in OIST



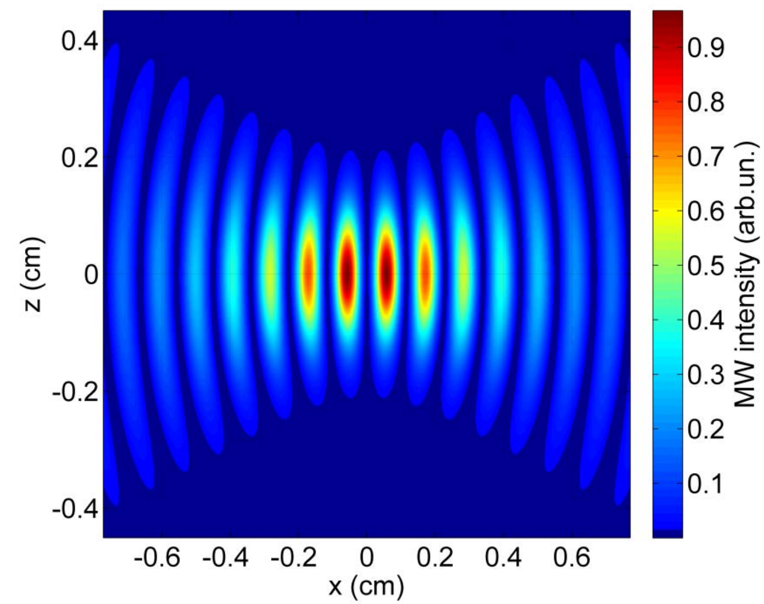
Work in progress!

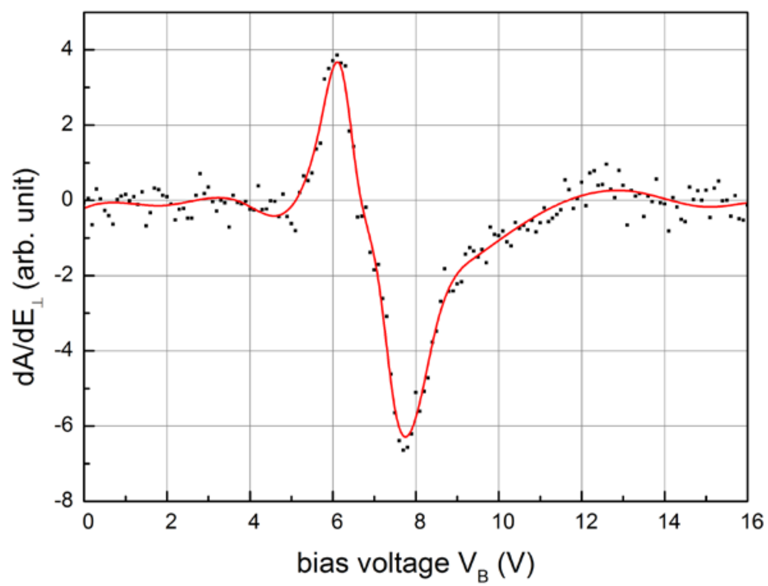
Experiments on microwave absorption in OIST

Heterodyne spectrometer



Fabry-Perot cavity



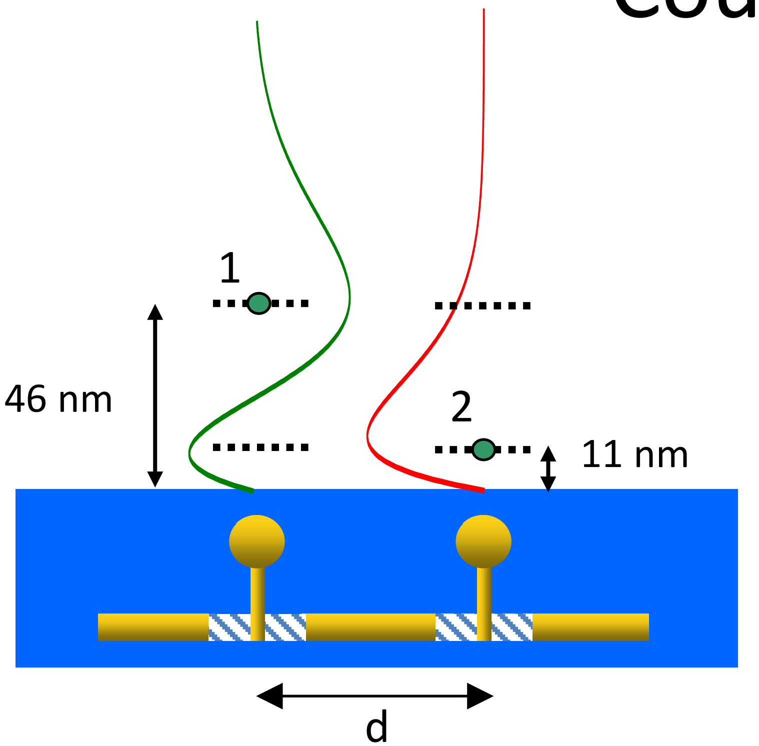


$$\chi_e' = n\frac{(ez_{12})^2}{\hbar}\frac{(\omega - \omega_0)}{(\omega - \omega_0)^2 + \gamma^2 + \gamma\tau\Omega^2}$$

$$\chi_e'' = n\frac{(ez_{12})^2}{\hbar}\frac{\gamma}{(\omega - \omega_0)^2 + \gamma^2 + \gamma\tau\Omega^2}$$

- Decoherence
- Qubit coupling
- Read out

Coupling between qubits



Coulomb interaction between qubits:

$$V_{e-e} = \frac{e^2}{|\mathbf{r}_1 - \mathbf{r}_2|} \approx \frac{e^2}{d}\left(1 - \frac{(z_1 - z_2)^2}{2d^2}\right),$$

state-dependent part

For two unscreened qubits:

$$\Delta f_C = \frac{e^2}{hd^3}\left(\langle z_0\rangle - \langle z_1\rangle\right)^2 = \frac{(4.5a_B)^2 e^2}{hd^3}$$

$E_\perp = 0$

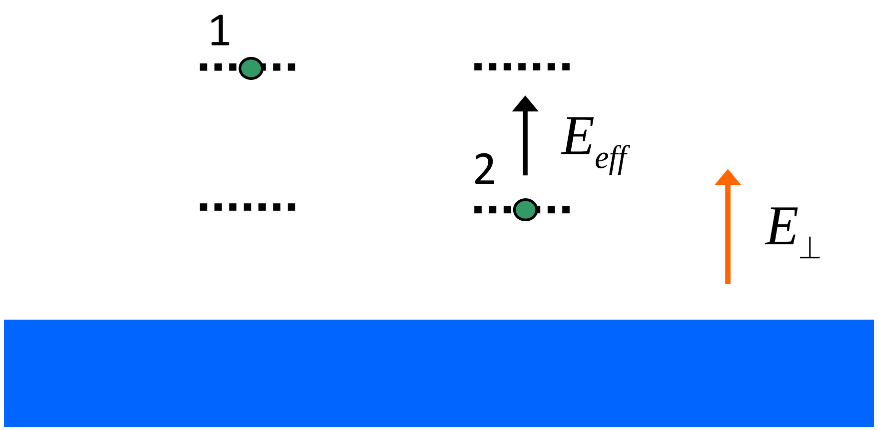
$E_\perp = 0$

$d = 0.5\ \mu\text{m}$

$\Delta f_C = 3\ \text{GHz}$

Coulomb shift

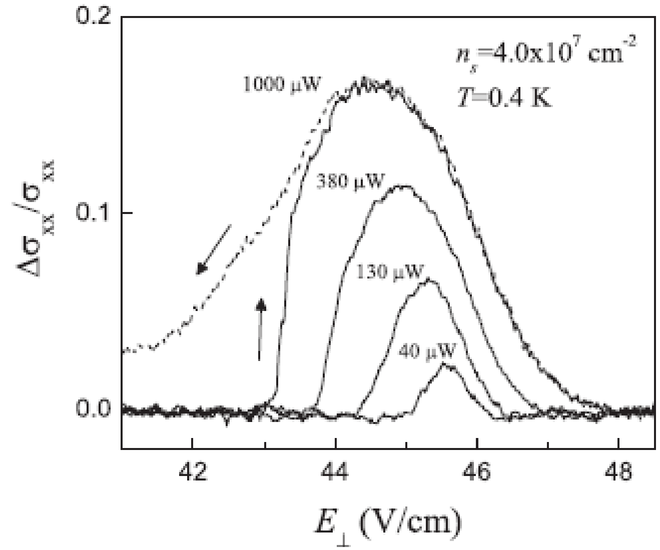
Stark shift due to effective E-field

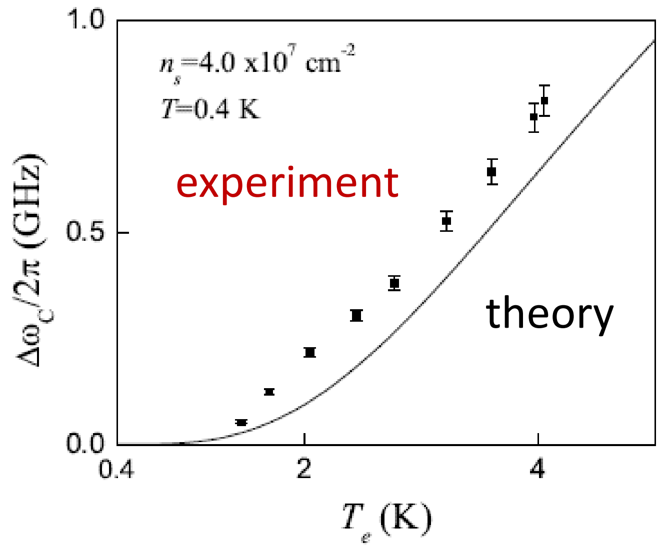


Estimate using mean-field approximation:

$$\Delta\omega_C = \frac{1}{\hbar}\left(z_{22} - z_{11}\right)\times$$

$$\times\left[\sum_n \rho_n z_{nn} - z_{11}\right]\sum_{i\neq j}\frac{e^2}{\left|\vec{r}_i - \vec{r}_j\right|^3}$$

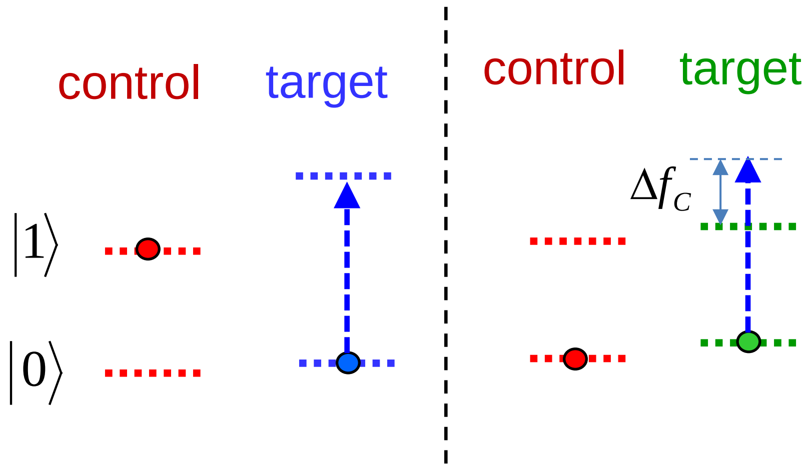




DK et al. PRL (2009)

Two-qubit gate

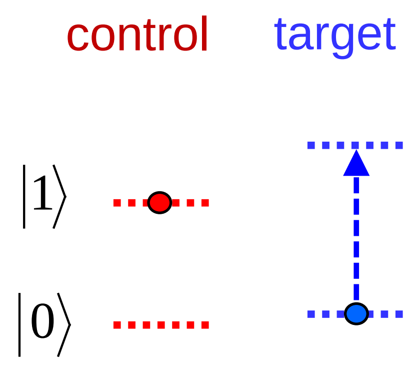
Two-qubit logic gate: depending on the state of control, target will be either excited or not (CNOT-gate)



Tangled state: control qubit is in the superposition state $\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$

apply Rabi π-pulse to target $\Rightarrow$

generate $\frac{1}{\sqrt{2}}\left(|0\rangle_c |0\rangle_t - |1\rangle_c |1\rangle_t\right)$

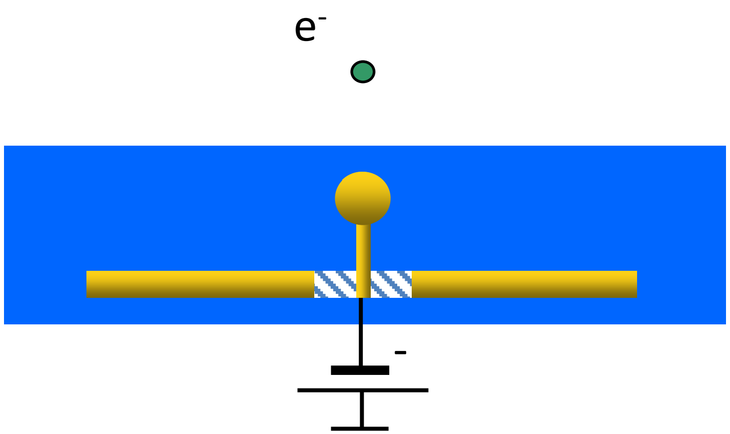


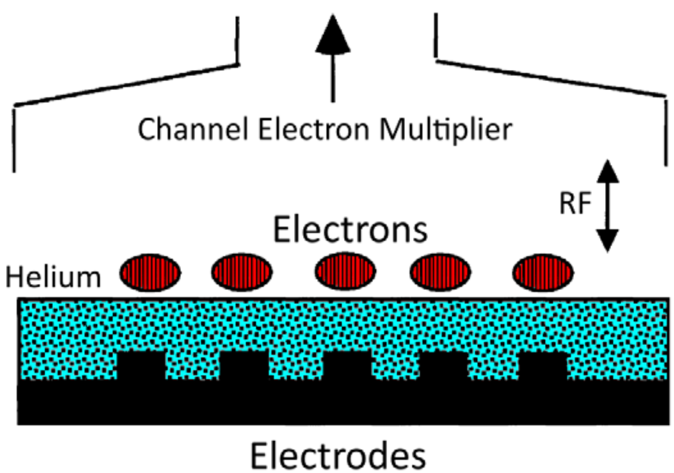
and etc.

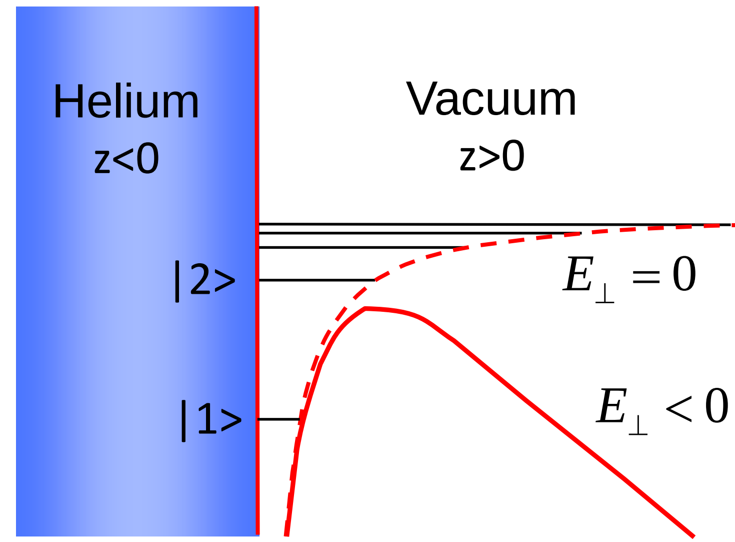
- Decoherence
- Qubit coupling
- Read out

Read-out

Originally proposed destructive readout



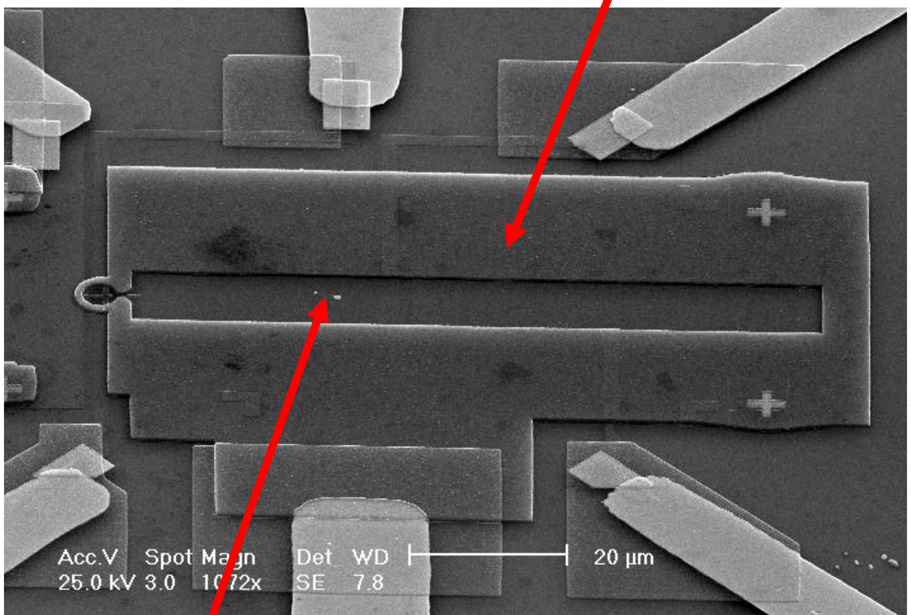


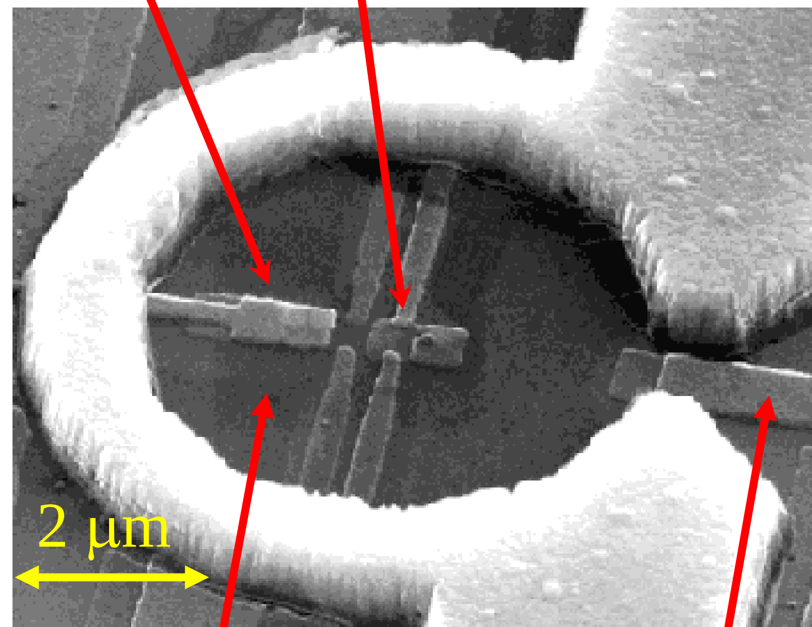


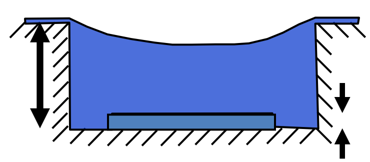
$E_\perp = -13$ V/cm

Signal amplification by secondary emission

Single-electron transistor



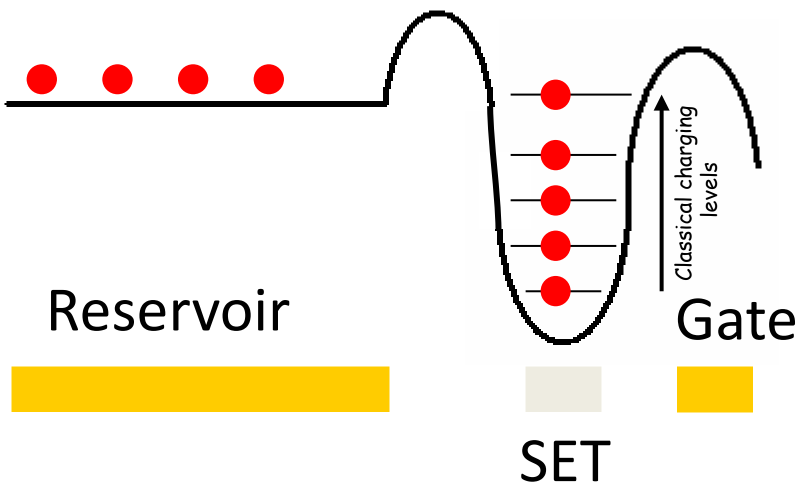


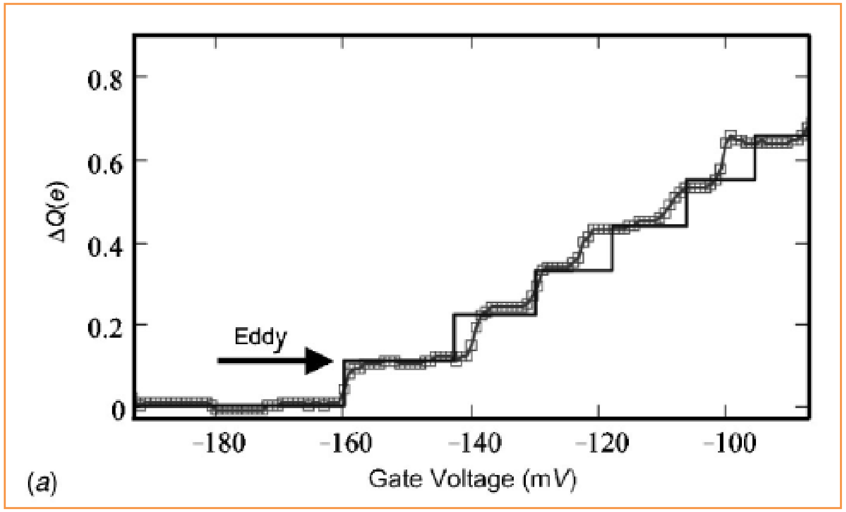


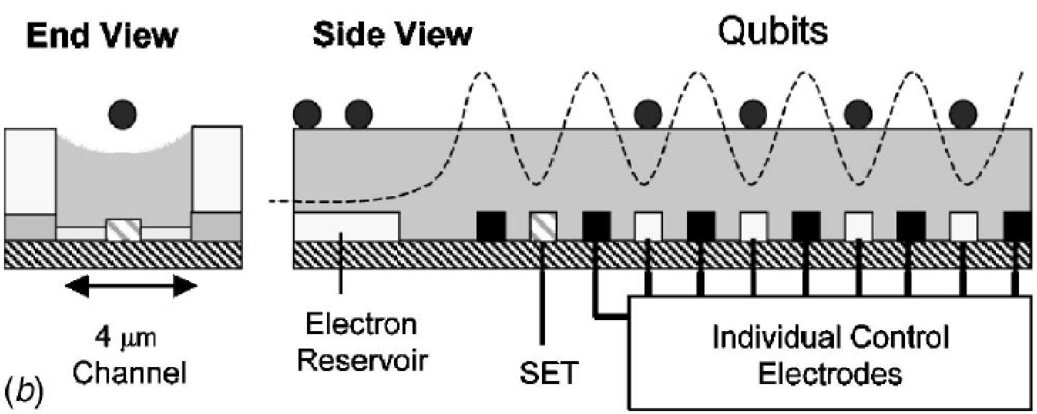
Royal Holloway
University of London

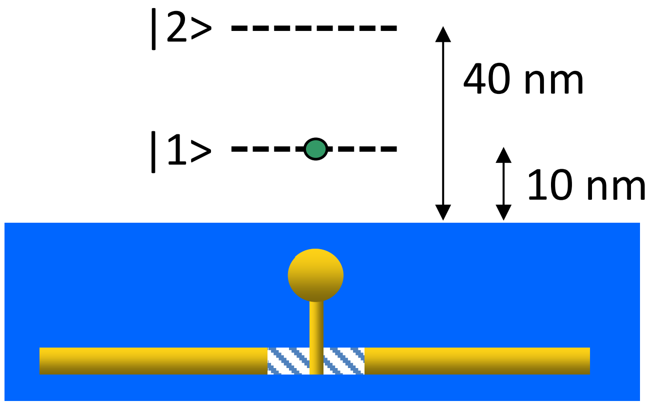
Read-out using SET

Papageorgiou et al APL (2005)









Proposal for spin qubits

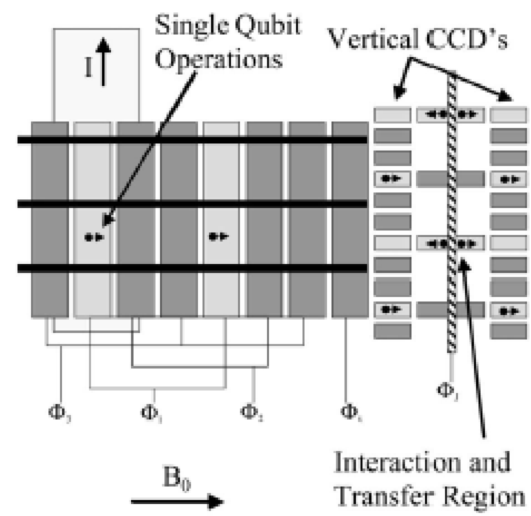
PHYSICAL REVIEW A **74**, 052338 (2006)

Spin-based quantum computing using electrons on liquid helium

S. A. Lyon
Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544, USA

(Received 17 September 2006; published 30 November 2006)

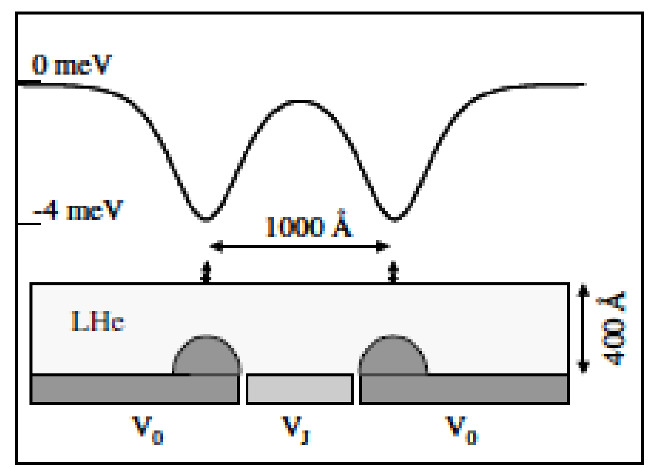
Numerous physical systems have been proposed for constructing quantum computers, but formidable obstacles stand in the way of making even modest systems with a few hundred quantum bits (qubits). Several approaches utilize the spin of an electron as the qubit. Here it is suggested that the spin of electrons floating on the surface of liquid helium will make excellent qubits. These electrons can be electrostatically held and manipulated much like electrons in semiconductor heterostructures, but being in a vacuum the spins on helium suffer much less decoherence. In particular, the spin-orbit interaction is reduced so that moving the qubits with voltages applied to gates has little effect on their coherence. Remaining sources of decoherence are considered, and it is found that coherence times for electron spins on helium can be expected to exceed 100 s. It is shown how to obtain a controlled-NOT operation between two qubits using the magnetic dipole-dipole interaction.



Fluctuating magnetic field due to Rashba effect (spin-orbit interaction):

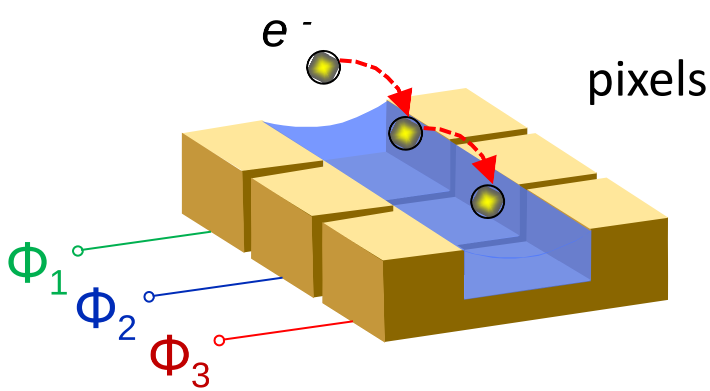
$$H_{s-o} = \alpha\left(\mathbf{p}_{\parallel} \times \mathbf{E}_{\perp}\right)\cdot \hat{S}$$

- T_2 exceeding 100 sec
- Qubit coupling by dipole interaction



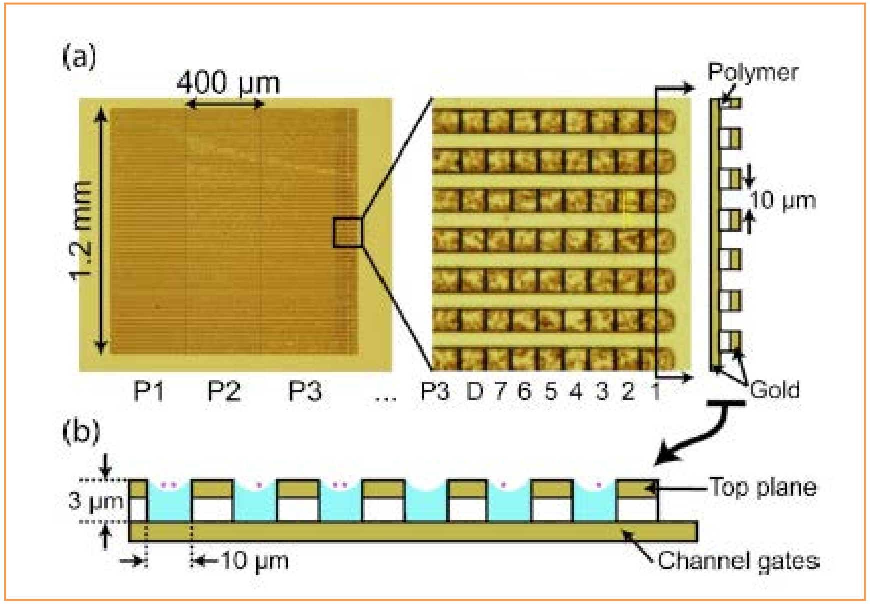
[S. A. Lyon, Phys. Rev. A, 74, 052338]

Mobile spin qubits!



- Electrons confined in microchannels
- Capacitive coupling to metal electrode
- Possibility to build a CCD

- Clocking on a 2D array of pixels
- 120 channels
- Efficiency of 99.99999999%
- Down to one electron per pixel



Steve Lyon, Princeton University, USA

F. R. Bradbury et. al., Phys. Rev. Lett. 107, 266803 (2011)

Towards hybrid systems!

PRL 105, 040503 (2010) PHYSICAL REVIEW LETTERS week ending 23 JULY 2010

Proposal for Manipulating and Detecting Spin and Orbital States of Trapped Electrons on Helium Using Cavity Quantum Electrodynamics

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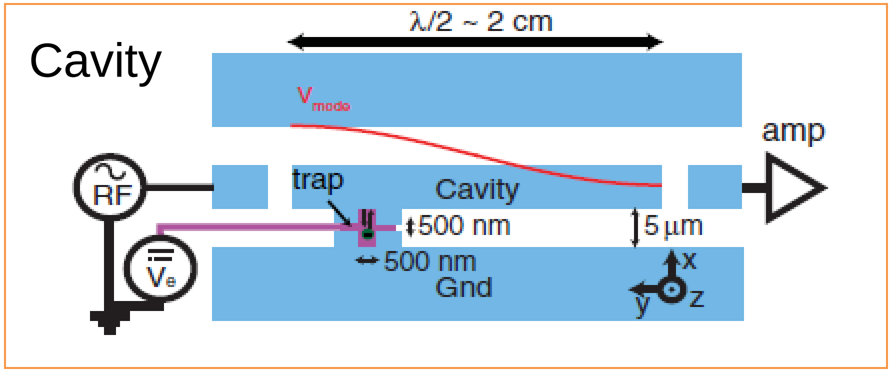
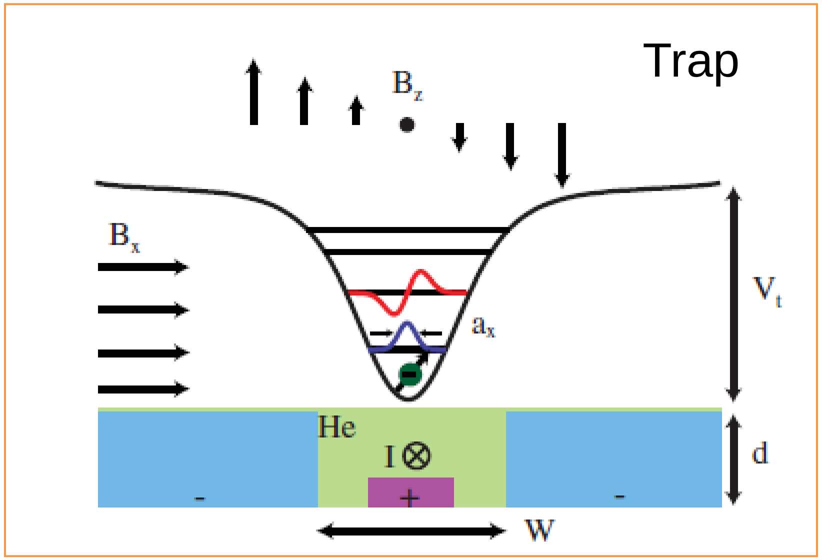
[1]*Department of Applied Physics and Physics, Yale University, New Haven, Connecticut 06511, USA*
[2]*Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824-2320, USA*
[3]*Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544, USA*





Progress: APS March Meeting 2012

- Strong coupling to RF cavity
- Electron-electron coupling via a single photon
- Manipulation of spin states via spin-orbit coupling

Summary

- Electrons on helium: unique model system

- Promising candidate for qubit implementation

- Some remarkable progress in quantum engineering

Steve Lyon, Princeton: CCD device
Mike Lee, University of London Royal Holloway and
Yuriy Moukharskii, Sacley: SET
David Rees, NCTU-RIKEN Joint Laboratory: Point Contact
David Shuster, University of Chicago and
Andreas Fragner,Yell University: Cavity QED

end more..

- A lot of work still needs to be done!

Quantum Dynamics Unit at OIST