Towards causal-patch physics in dS/CFT

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[with Lucas Hackl, Illan Halpern]

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Plan of the talk

- Intro to Higher-spin dS/CFT
- Folding de Sitter space in half
- Observer-dependent quantum mechanics
- Massless holography in dS_4/Z_2
- Higher-spin gravity on fixed geometry
- Bulk and CFT from twistor space
- Outlook

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Quantum Gravity in 1997-98

- AdS/CFT: first (by my standards) working models of quantum gravity.

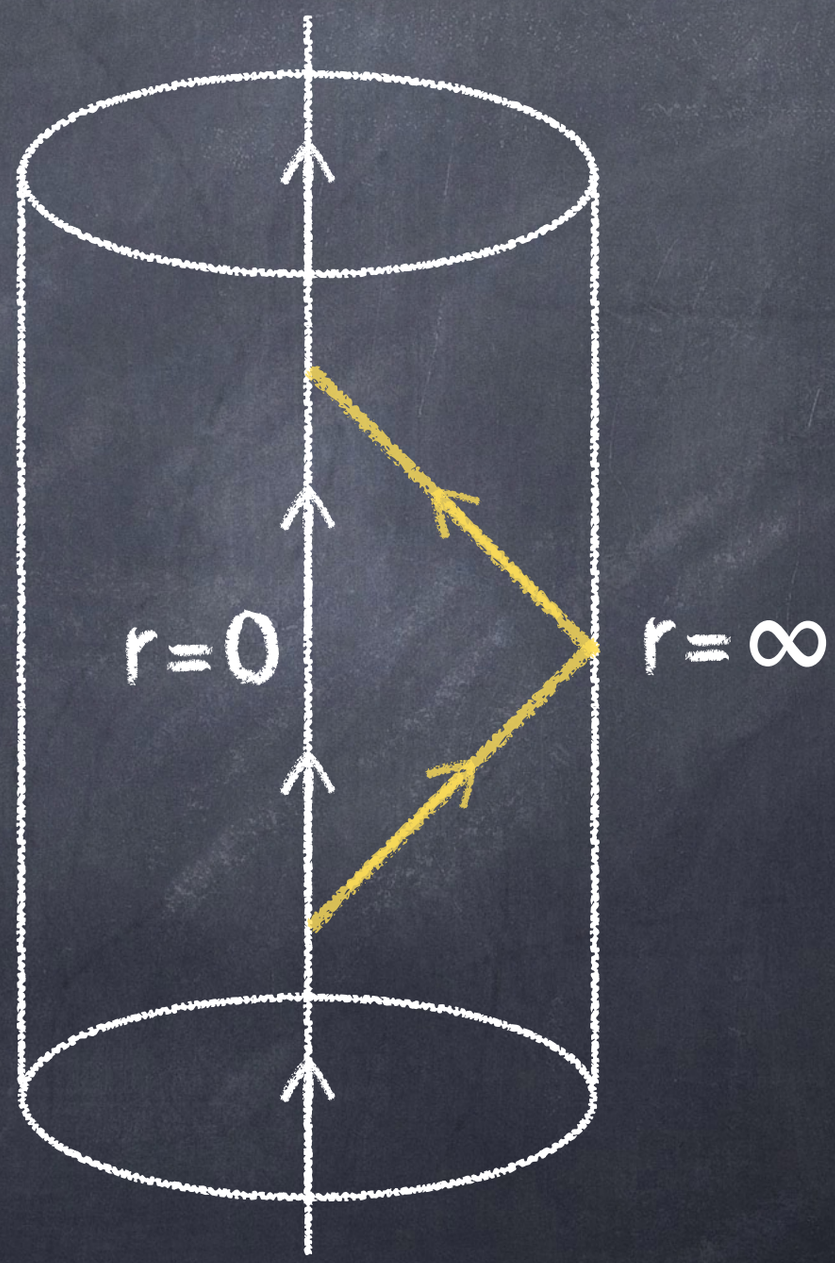
- Positive Λ: all such models observationally falsified.

- Contact with experiment at cosmological scales. Through holography, quantum gravity is not just about the Planck length!

The significance of Λ

Anti-de Sitter ($\Lambda < 0$):

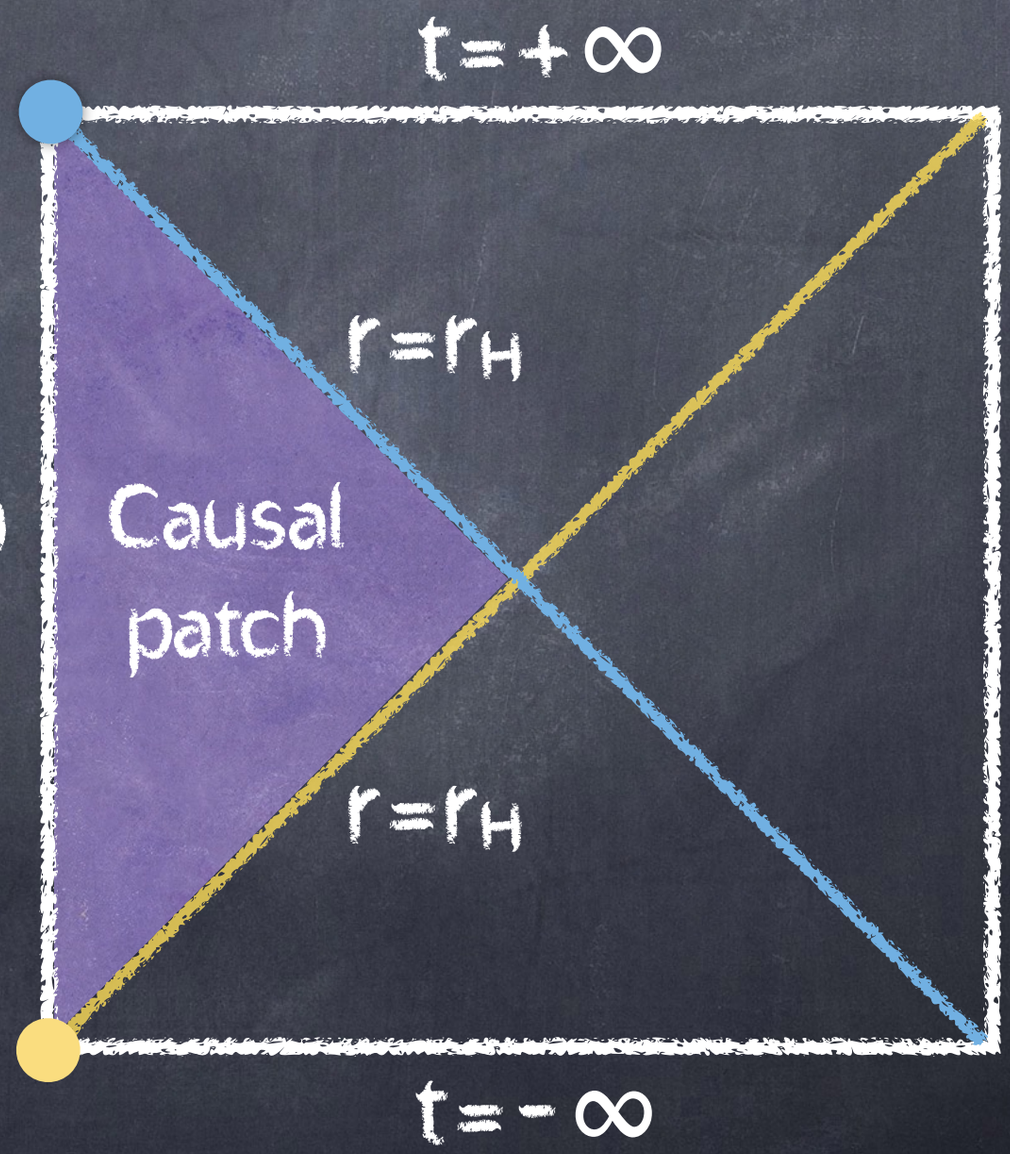
- $r=\infty$ is a physical place: Can get there and back within finite time.
- Can prepare and measure states at $r=\infty$.
- Outcomes calculated by CFT.



The significance of Λ

De Sitter ($\Lambda > 0$):

- Cosmological horizon around every observer.
- $r = \infty$ causally inaccessible, unphysical.
- Must tackle quantum gravity inside a finite region!



The big question

- AdS/CFT reconciles $G \neq 0$, $\hbar \neq 0$.

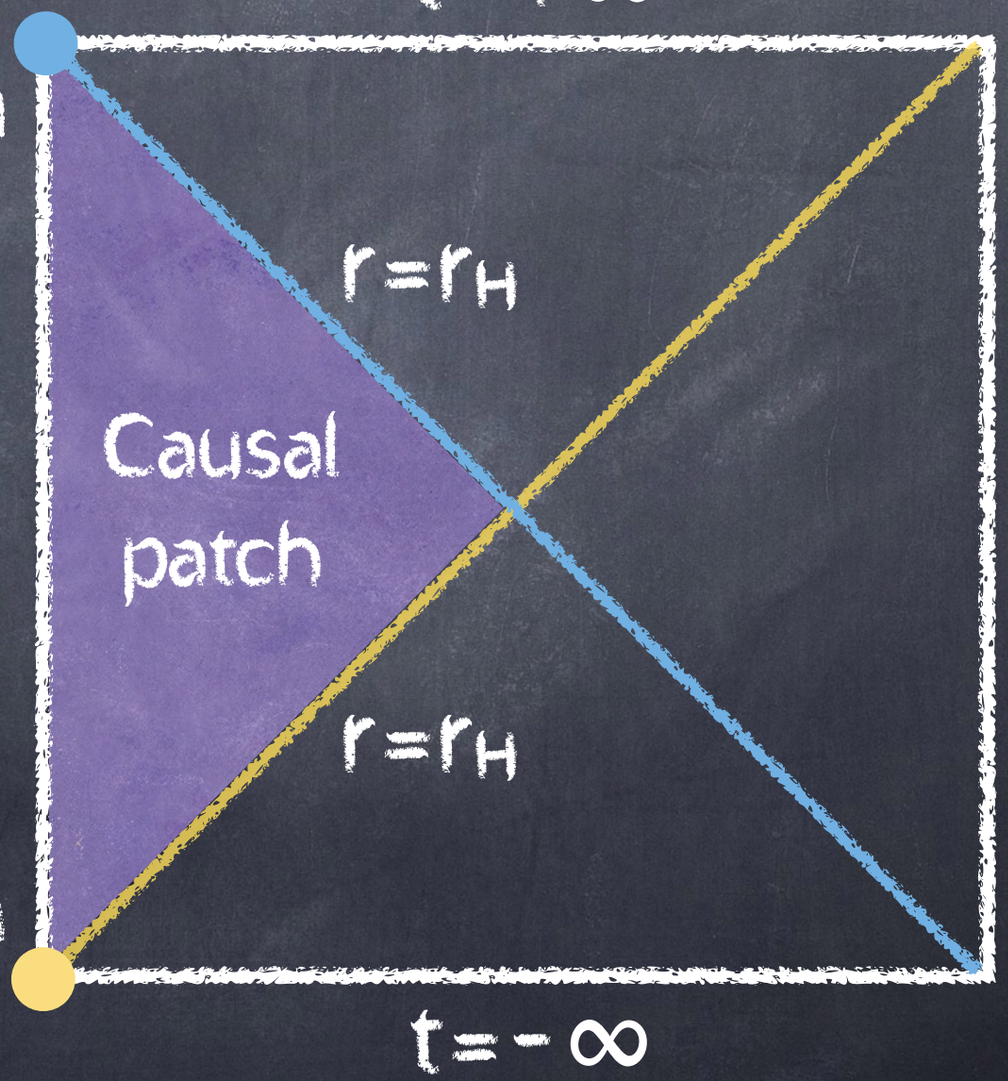
  Retreat to $r = \infty$ avoids many hard questions of quantum spacetime.

- Now, must reconcile $G \neq 0$, $\hbar \neq 0$, $\Lambda > 0$!

  Must venture into the bulk.
  Quantum spacetime head-on?

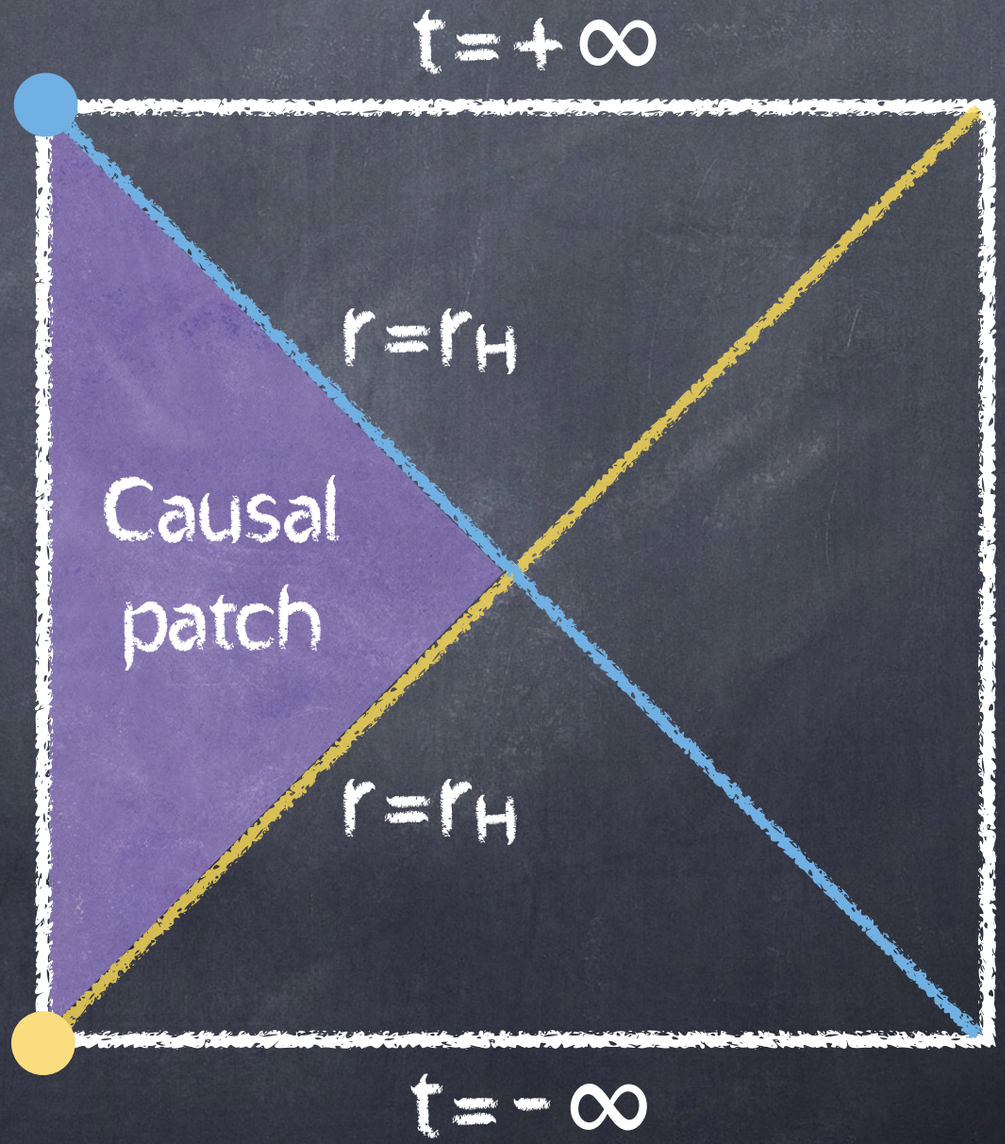
dS/CFT:
a conservative strategy

- Apply AdS wisdom to $\Lambda>0$.
- Use $t=\pm\infty$ instead of $r=\infty$.
- Take largest, simplest finite region: causal patch of cosmological observer.
- Patch is anchored by lightcones to $t=\pm\infty$: retaining some connection to the boundary.



dS/CFT: Conceptual difficulties

- Find dictionary between $t=\pm\infty$ and causal patch.
- CFT lives on spacelike boundary. Time must emerge holographically.
- How to handle the two boundaries?



dS/CFT:
Technical difficulties

String theory not understood in dS → no model.

- No unitary SUSY in de Sitter.

- Excited string modes → massive bulk fields
  → complex conformal weights in CFT.

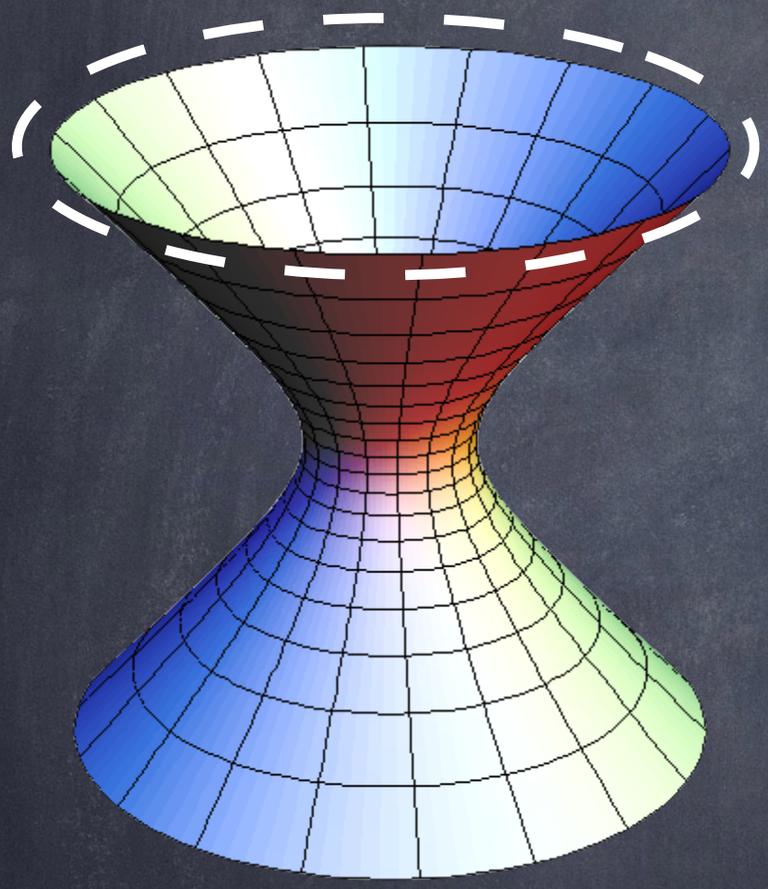
$$\phi(t, \vec{x}) \sim e^{-imt}$$

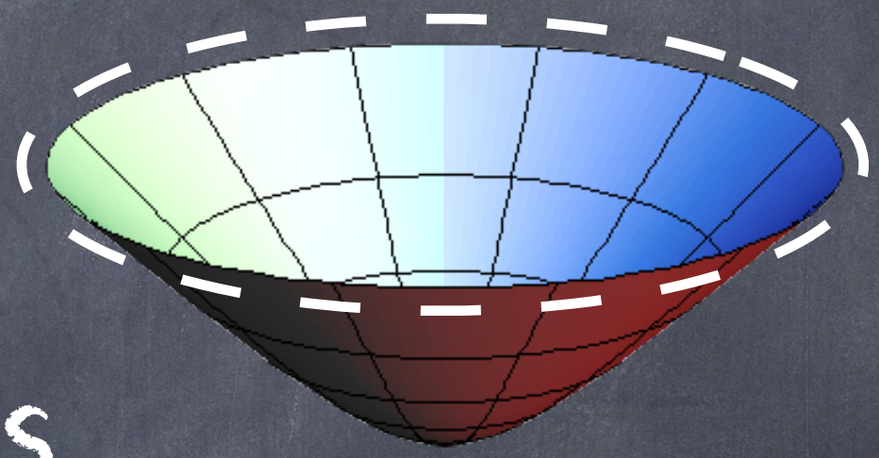
dS/CFT:
History in a nutshell

- 2001-2004: Many ideas proposed for the conceptual issues, with no reference model.

- String community "gives up", settles on a solid but insufficient picture. [Maldacena, 2001]

Maldacena's interpretation





dS ↔ Euclidean AdS

$t = \infty \leftrightarrow r = \infty$

Z_{CFT} [sources] = Path integral in Euclidean AdS [fields at $r = \infty$]

= Hartle-Hawking $\Psi_{H.H.}$ in dS [fields at $t = \infty$]

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- 2011: A working model.
  [Anninos, Hartman, Strominger]

The higher-spin model

4d Bulk: Vasiliev's Higher Spin gravity
3d Boundary: N free massless scalars

- Simplest AdS/CFT [Klebanov & Polyakov, 2001]

- Survives in dS.

- Bulk has ∞ many massless fields
  → non-local, no GR limit.

- However: d=4, Λ>0!

Higher-spin gravity

- One massless field of each spin (0,1,2,3,4,...)
- ∞-dim gauge symmetry.
- "The opposite of SUGRA": spacetime symmetries extended in a bosonic direction.
- Close relation to twistor theory.
- Unbroken ∞-dim symmetry → free CFT dual.

The Vasiliev field equations

$$\bar{s}_{\dot{\alpha}} \star \bar{s}^{\dot{\alpha}} - 2i = (s_\alpha \star s^\alpha - 2i) \star \kappa\bar{\kappa} \; ; \quad [s_\alpha, s_{\dot{\alpha}}]_\star = 0$$

$$ds_\alpha + [W, s_\alpha]_\star = 0 \; ; \quad d\bar{s}_{\dot{\alpha}} + [W, \bar{s}_{\dot{\alpha}}]_\star = 0$$

$$dW + W \star W = 0$$

- Like GR - Non-perturbative, diff-invariant.
- No action principle or quantization, except through holography.

Goals and attitude

- Study HS gravity as a working model of holography with $G \neq 0$, $\hbar \neq 0$, $\Lambda > 0$.

- Recover physics in a de Sitter causal patch.

- Shamelessly use unrealistic features of HS theory.

- Concrete goal: obtain finite-dimensional Hilbert space in de Sitter, in accordance with BH entropy.

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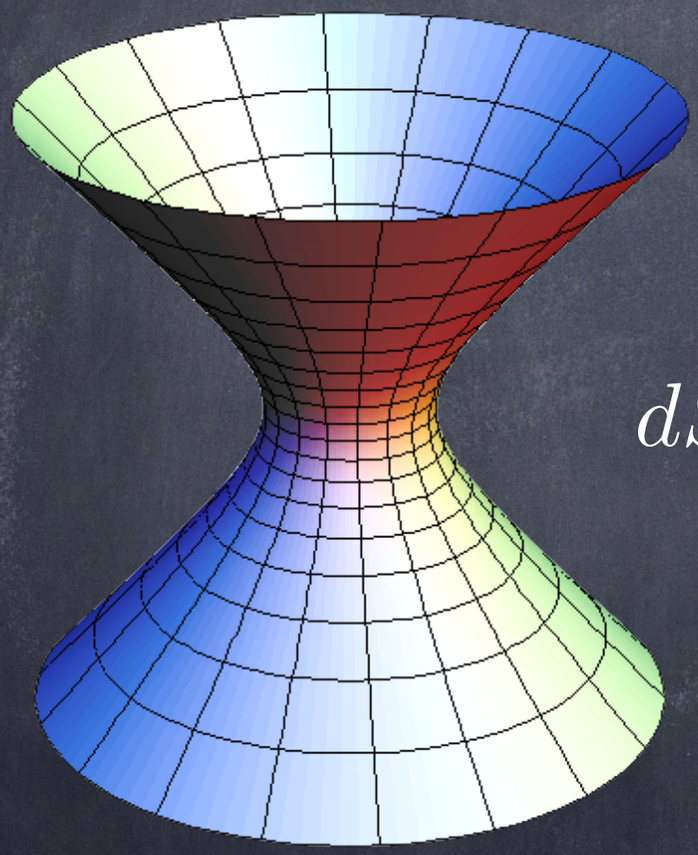
dS_4 embedded in $R^{1,4}$

$$\mathcal{I}^+ : \ \ell_\mu \ell^\mu = 0; \ \ell^\mu \cong \alpha \ell^\mu; \ \ell^0 > 0$$



$$dS_4 : \ x_\mu x^\mu = 1$$

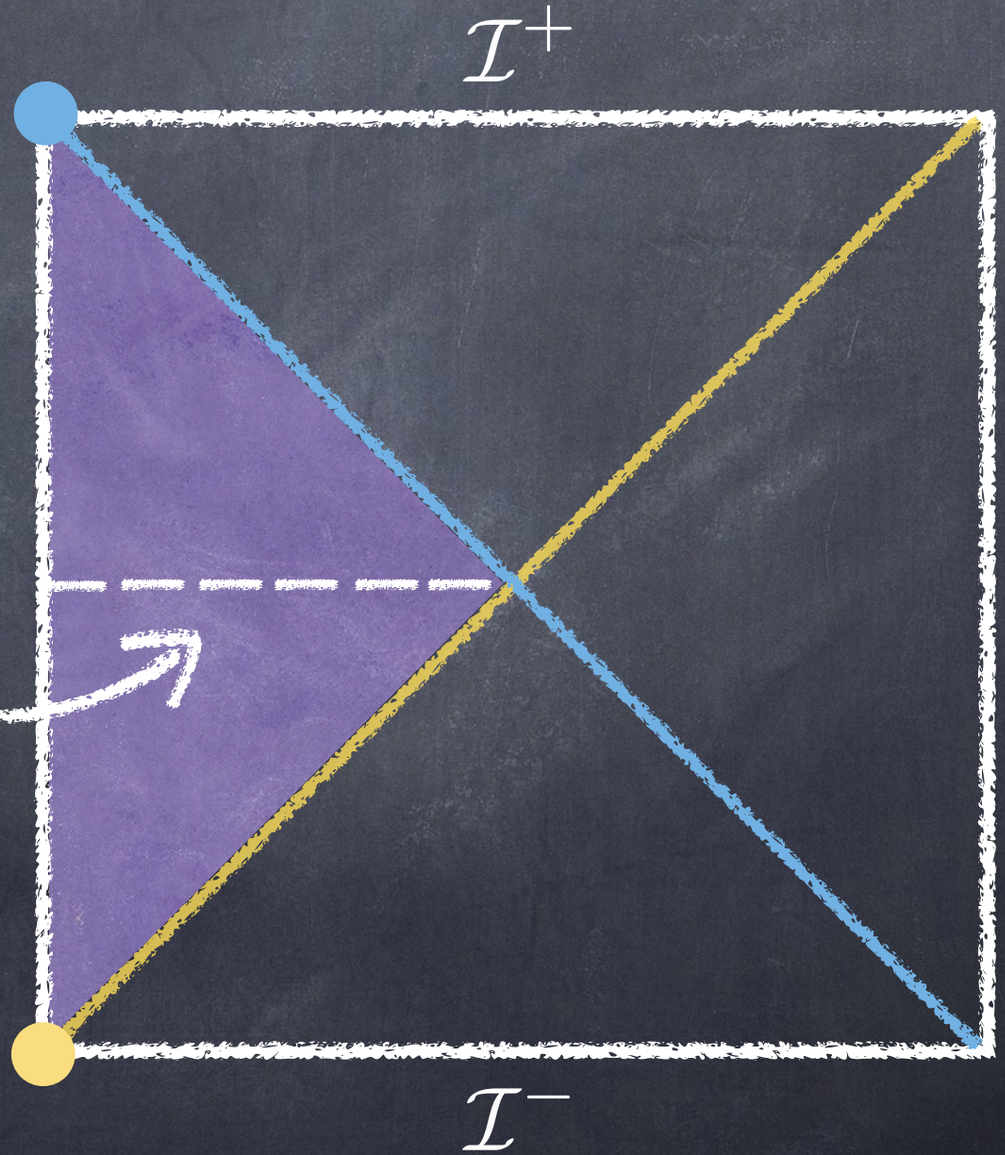
O(1,4) symmetry

- Bulk isometries
- Boundary conformal group

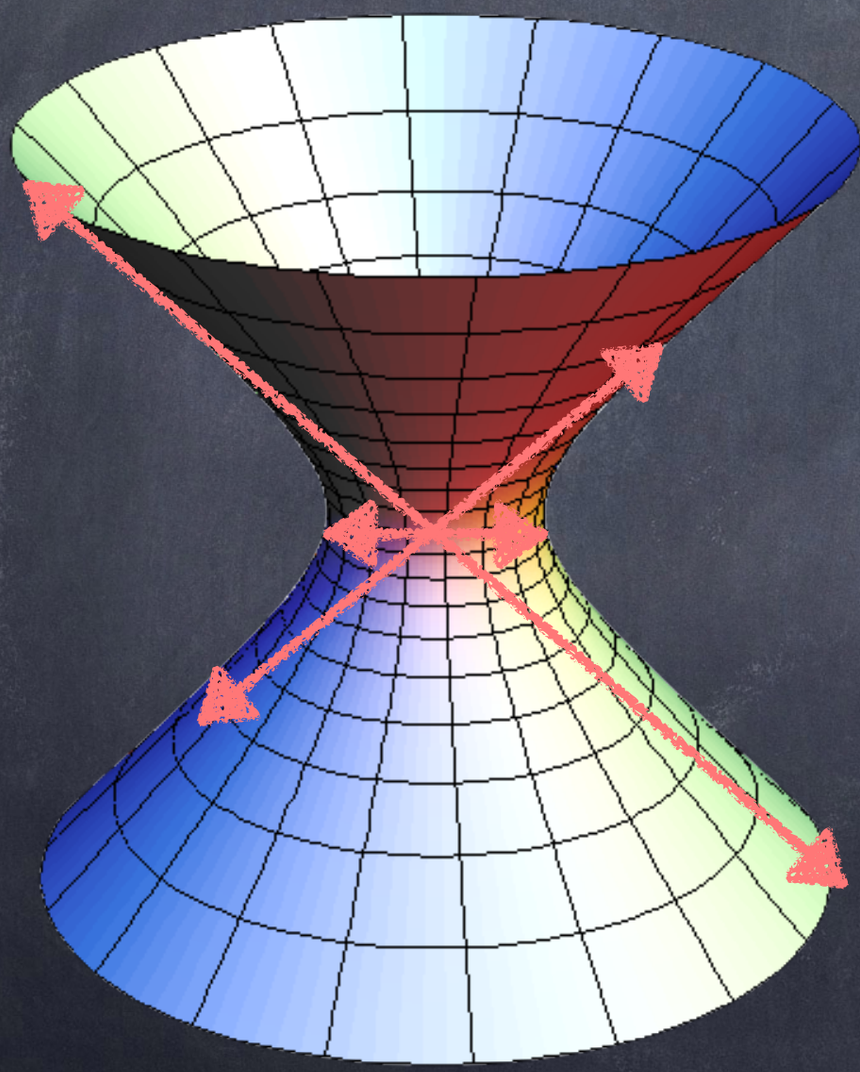
$$\mathcal{I}^- : \ \ell_\mu \ell^\mu = 0; \ \ell^\mu \cong \alpha \ell^\mu; \ \ell^0 < 0$$

de Sitter space is twice too big

- Two boundaries at $t=\pm\infty$.
- Every observer sees half of space.
- Only expanding patch is relevant for cosmology.



Schrodinger's "Elliptic de Sitter" dS_4/Z_2



Bulk:

$$x^\mu \leftrightarrow -x^\mu$$

Boundary:

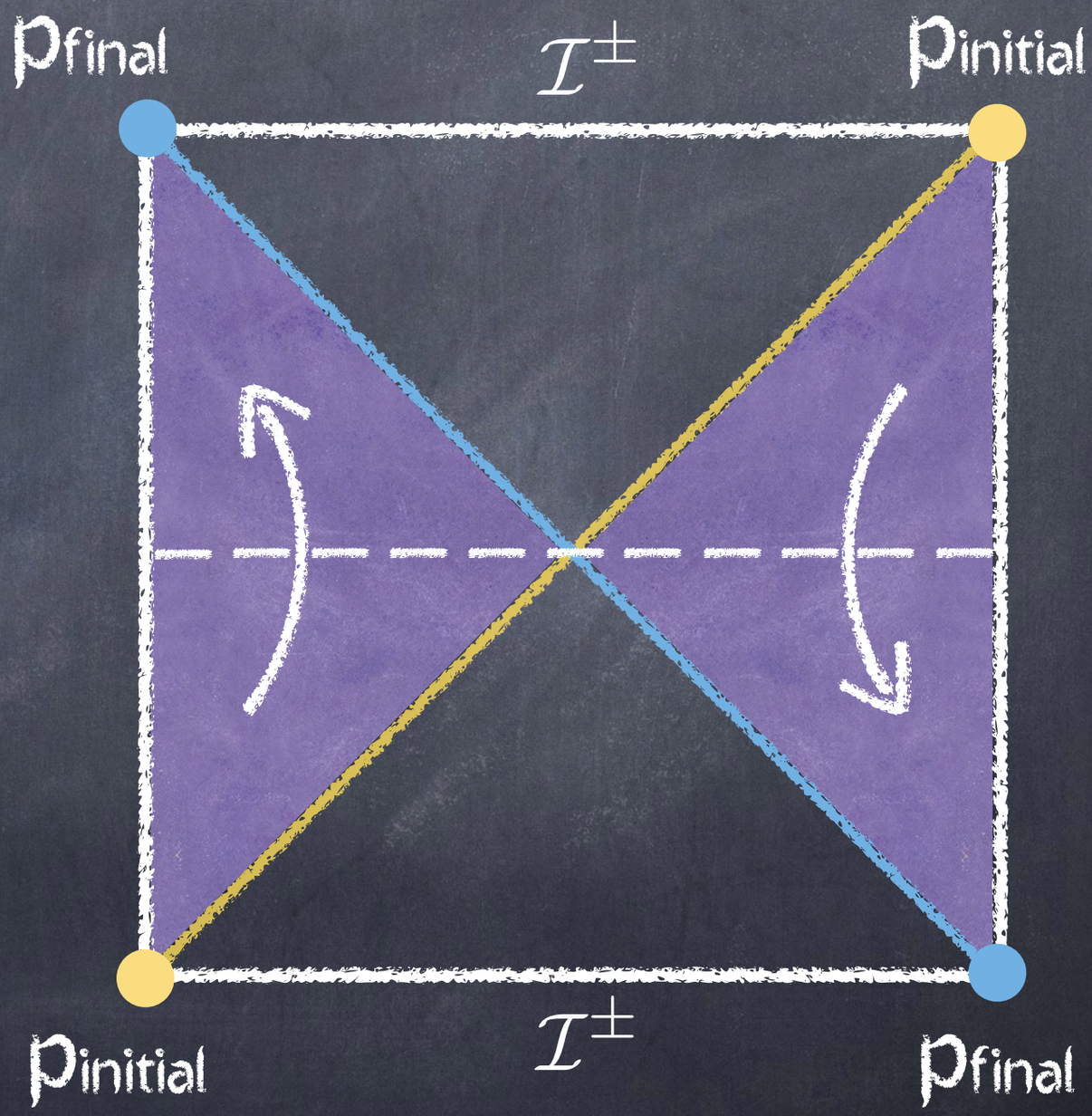
$$\mathcal{I}^+ \leftrightarrow \mathcal{I}^-$$

- Folding in half in space and time.
- O(1,4) symmetry preserved.

[Parikh, Savonije, Verlinde, 2002]

The promise of dS_4/Z_2

- Each observer sees all of space.
- Clasically, same information on the boundary and inside a causal patch!
- However, boundary is still unobservable. Still need a dictionary!



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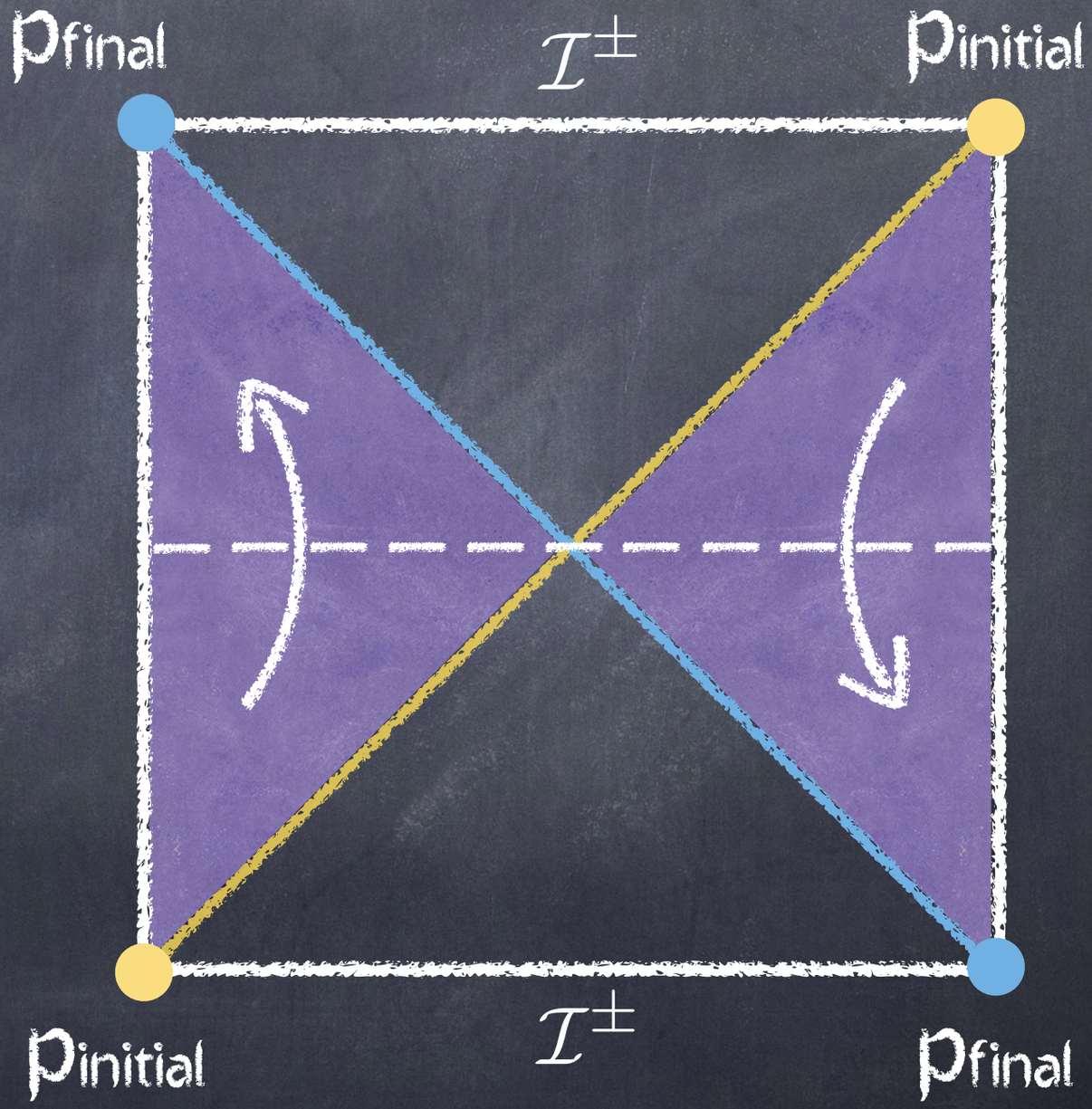
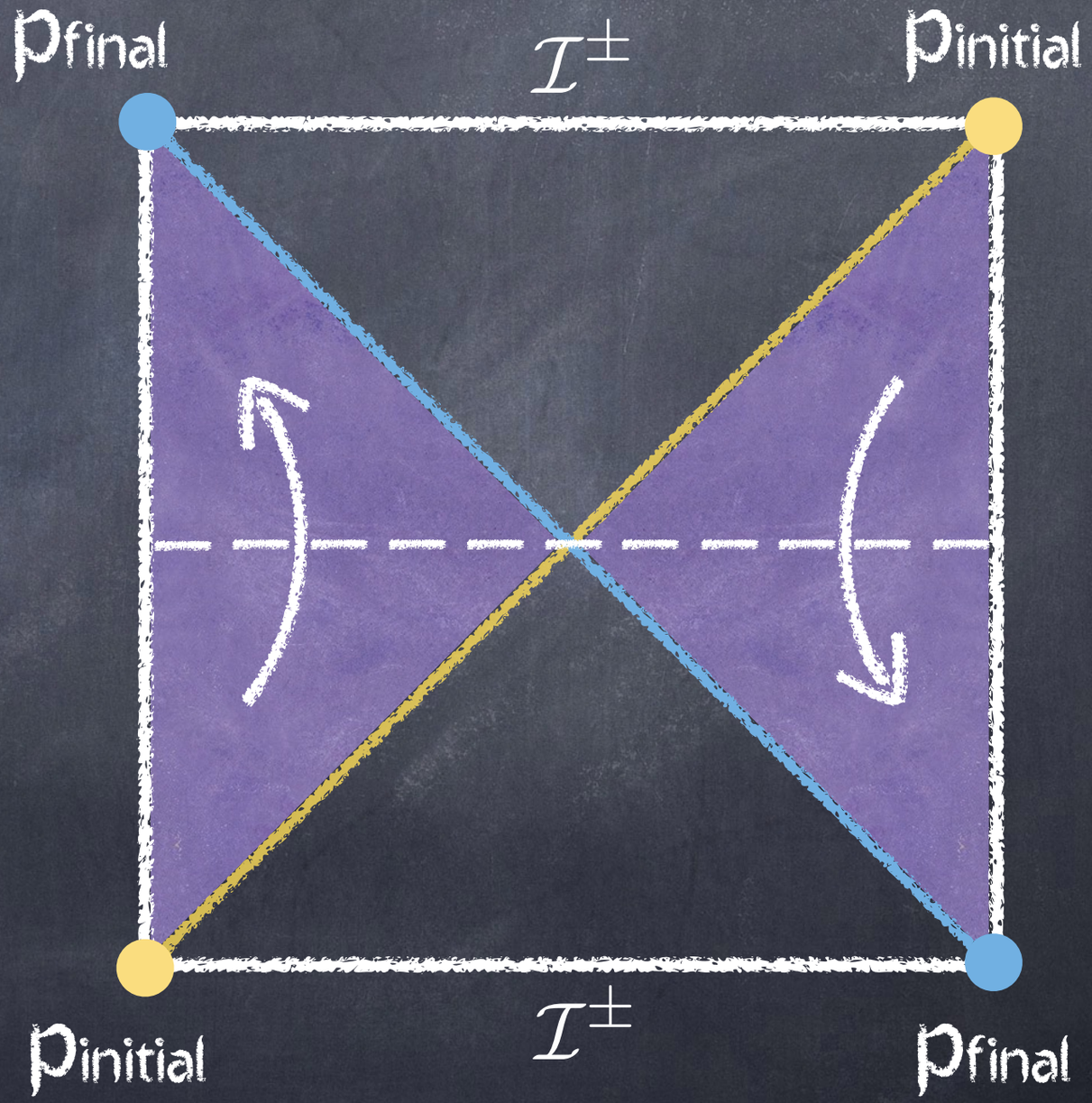
Causality in dS_4/Z_2

- Spacetime not time-orientable.
- However, no closed timelike curves.
- An observer's patch is orientable, based on ordering of her worldline endpoints.



Observer-dependent quantum mechanics

- Quantum mechanics requires time orientation for sign of [x,p].
- Exists only separately for each observer!



Horizon complementarity

['t Hooft, 1985; Susskind et al, 1993]

- Proposed solution to black hole information.
- No states of entire Universe, only of observable patches.
- Hilbert spaces seen by different observers are secretly isomorphic - same quantum info.
- A vague idea. Explicit realization?

Quantum field theory in dS/Z_2

- The geometry "forces" complementarity.
  - Classically, same information for all observers.
  - Quantum mechanically, have a prescription for translating info between observers.

[Halpern, YN, 2015; Hackl, YN, 2016]

Translation between observers

Unify the world-pictures of different observers through a global "meta" structure.

Hermitian operators in observer's picture

Antipodally symmetric wavefunctions in global dS

[Halpern, YN, 2015; Hackl, YN, 2016]

Quantum field theory in dS/Z_2

- Observers agree on physics wherever they agree on time orientation.

  - However, disagree on entanglement across horizons.

  - Nevertheless, no obvious paradoxes. Generalized complementarity?

[Halpern, YN, 2015; Hackl, YN, 2016]

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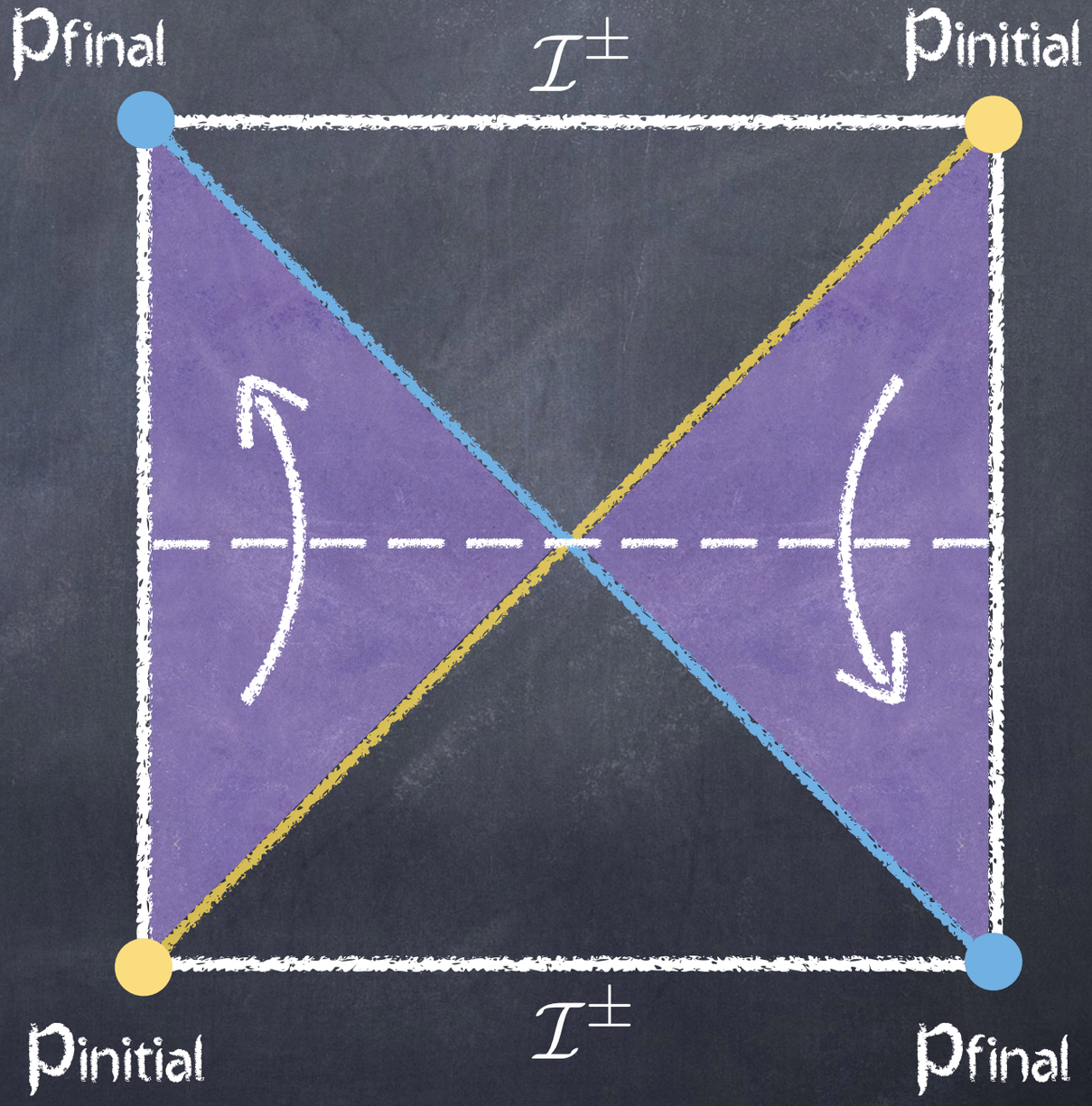
Holographic dictionary for free fields in dS_4/Z_2

- Express the operator algebra in a causal patch in terms of boundary fields.

- Use only the CFT partition function. No bulk calculations!



Solution
(scalar field example)

Boundary field modes: $c_{lm}(\omega)$

CFT partition function:

$$Z_{\rm CFT} = \exp\left(-\int \frac{d\omega}{2\pi} \sum_{lm} S_l(\omega)\, |c_{lm}(\omega)|^2\right)$$

Causal-patch operator algebra:

$$\left[c_{lm}(\omega), c^\dagger_{l'm'}(\omega')\right] = \frac{4\pi}{S_l(\omega)} \left(\tanh\frac{\pi\omega}{2}\right)^{(-1)^l} \delta_{ll'}\delta_{mm'}\delta(\omega-\omega')$$

What went into this result?

- $R \times O(3)$ symmetry of the causal patch.
- Linearity of the free field equations.
- Masslessness:
  Massless fields have a special relation between boundary data at ∞ and the dS/Z_2 topology.
  [YN, 2014]

Prospects for full higher-spin gravity

Good:

- dS/Z_2 holography likes massless fields.
  HS theory has only massless fields.

Scary:

- What to do without the pure dS geometry?
- How to handle the complicated bulk interactions?
- What do horizons mean in HS theory, anyway?

Cause for hope:
Higher spins are magic.

=

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Vasiliev's field equations are "Unfolded"

- Equations written in terms of "Master Fields".

- Contain not only fields of every spin, but also Taylor series of derivatives.

- The whole solution encoded at every point!

Different symmetry structure

- Since fields go together with derivatives, HS gauge group includes internal translations, separate from diffeomorphisms.

- The spin-2 field is tied to "internal" translations, Diffeomorphisms not essential to the dynamics!

The diff-invariant Vasiliev field equations

$$\bar{s}_{\dot\alpha} \star \bar{s}^{\dot\alpha} - 2i = (s_\alpha \star s^\alpha - 2i) \star \kappa\bar\kappa \; ; \quad [s_\alpha, s_{\dot\alpha}]_\star = 0$$

$$ds_\alpha + [W, s_\alpha]_\star = 0 \; ; \quad d\bar{s}_{\dot\alpha} + [W, \bar{s}_{\dot\alpha}]_\star = 0$$

$$dW + W \star W = 0$$

"Gravitational". Interesting to study.

Vasiliev field equations on fixed (A)dS geometry

$$\bar{s}_{\dot\alpha} \star \bar{s}^{\dot\alpha} - 2i = (s_\alpha \star s^\alpha - 2i) \star \kappa\bar\kappa \; ; \quad [s_\alpha, s_{\dot\alpha}]_\star = 0$$

$$\nabla s_\alpha + [W, s_\alpha]_\star = 0 \; ; \quad \nabla \bar{s}_{\dot\alpha} + [W, \bar{s}_{\dot\alpha}]_\star = 0$$

$$dW + W \star W = -\frac{i}{16} \left(dv^\alpha{}_{\dot\gamma}\, dv^{\beta\dot\gamma}\, s_\alpha \star s_\beta + dv_\gamma{}^{\dot\alpha}\, dv^{\gamma\dot\beta}\, \bar{s}_{\dot\alpha} \star \bar{s}_{\dot\beta} \right)$$

"Non-gravitational". Feasible to study.

[YN, 2015]

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Do we need the non-linear Vasiliev equations?

- We just want the "on-shell action" as a (non-linear) functional of boundary data.
  - Can always write it as a functional of linearized bulk solution.
  - Tightly constrained by HS symmetry.

[Colombo, Sundell; Didenko, Skvortsov, 2012]

Exploit the free CFT

- Bulk HS theory: interacting m=0 fields.
- Boundary CFT: free m=0 fields.
  n-point functions encode bulk interactions!
- The CFT can be used to solve the bulk indirectly.
- Only useful if bulk and CFT are expressed on common footing.

Taking the fight to twistor space

Twistors:

- Spinors of the spacetime symmetry group.
- Nonlocal, "maximally lightlike" shapes.
- Ideal for describing free massless fields.
- In 4d, the Penrose transform translates between twistor space and free massless fields.

Taking the fight to twistor space

Twistor Space

Linearized HS theory in 4d

Free CFT in 3d

Taking the fight to twistor space

Twistor Space

Penrose transform

Linearized HS theory in 4d

Free CFT in 3d

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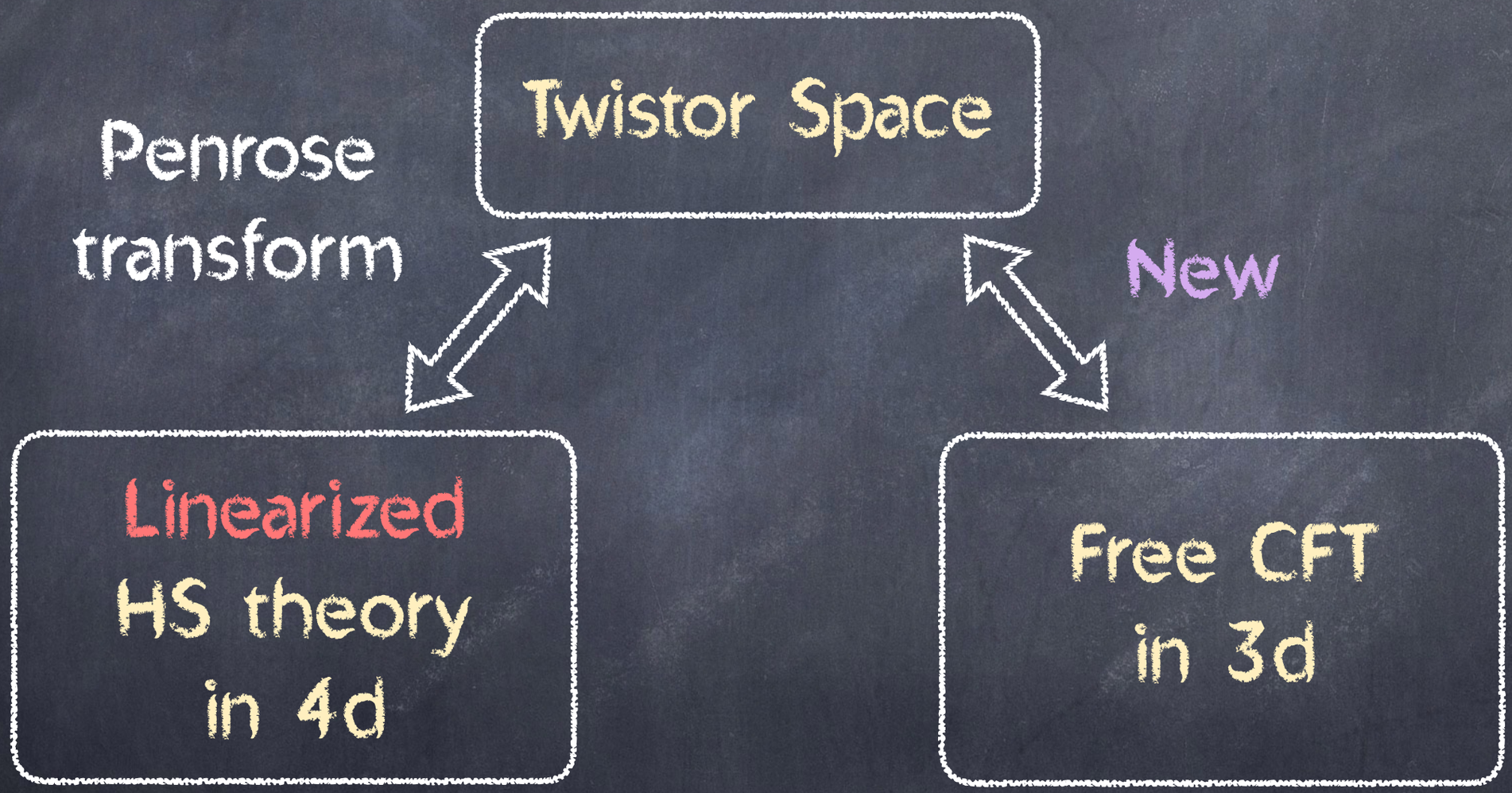

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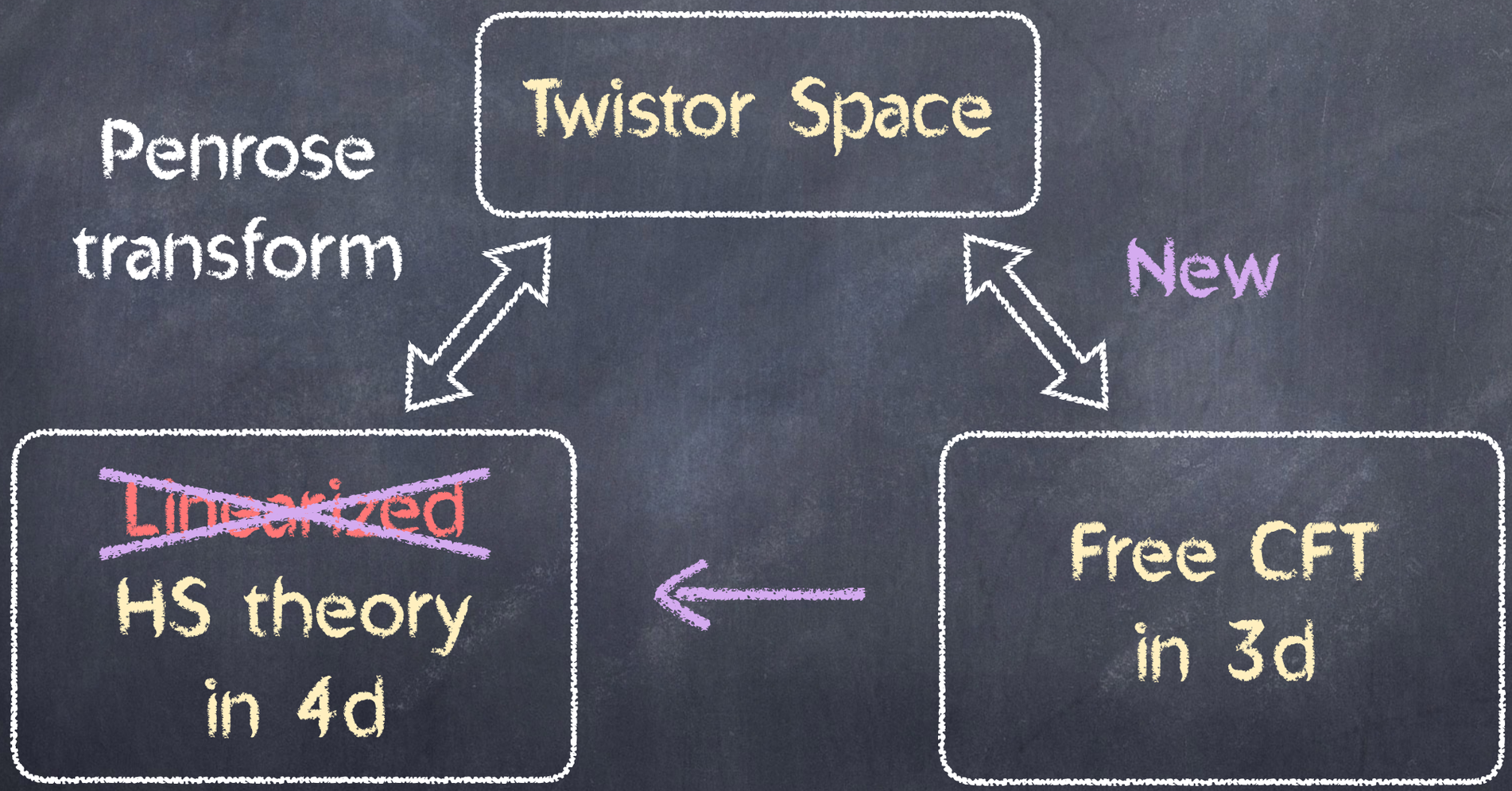


Penrose transfom:

Bulk ↔ Twistor space

$$f(Y) = C(x;Y) \star \delta_x(Y)$$

Twistor function

Fields and their derivatives at point x

Higher-spin algebra

Boundary ↔ Twistor space

CFT action with sources:

$$S[\Pi(\ell',\ell)] = -\int d^3\ell\, \bar{\phi}_i \Box \phi^i - \int d^3\ell'\, d^3\ell\, \bar{\phi}_i(\ell')\Pi(\ell',\ell)\phi^i(\ell)$$

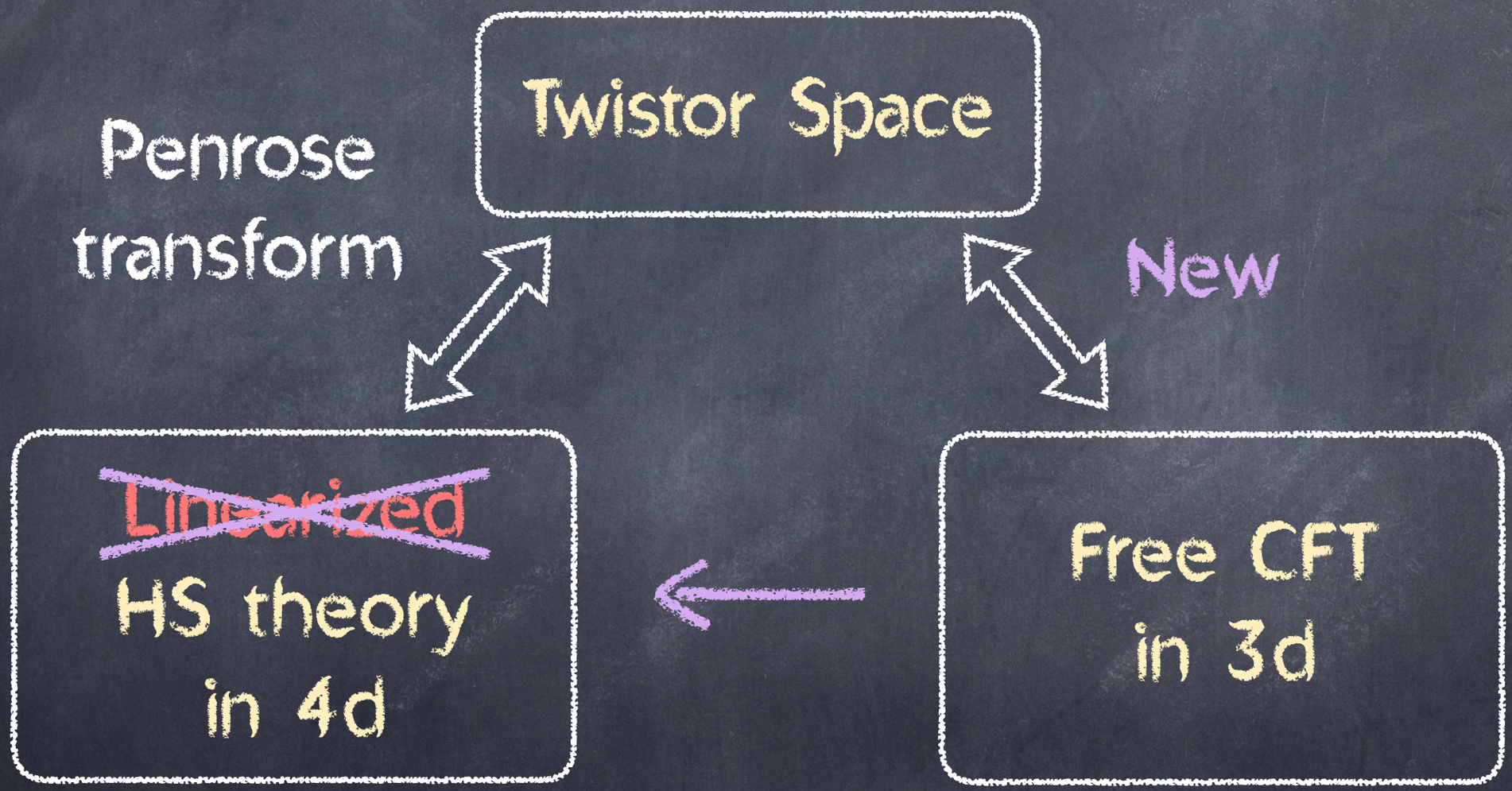
Transform into twistor space:

$$f(Y) = \int d^3\ell\, d^3\ell'\, \Pi(\ell',\ell)\sqrt{-\ell\cdot\ell'}\,\delta_\ell(Y)\star\delta_{\ell'}(Y)$$

Partition function:

$$Z[f(Y)] = \left(\det{}_\star\left[1+f(Y)\right]\right)^{-N/4}$$

Taking the fight to twistor space



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Outlook

- Translate the free-field dS/Z_2 holography into the language of higher-spin theory.
- Hope that the magic continues, and that the full non-linear result will follow.

- Obtain the first working model of quantum sort-of-gravity inside a cosmological horizon.
- Finite-dim. Hilbert space?
- Expected side benefit: a more explicit quantization of higher-spin theory.

OIST

Clifford algebra: $\{\gamma_\mu, \gamma_\nu\} = -2\eta_{\mu\nu}$

O(1,4) symmetry generated by: $\gamma_{[\mu}\gamma_{\nu]}$

Reflection (CPT): $\Gamma \rightarrow x\Gamma x \; ; \quad x = x^\mu \gamma_\mu$

"Square root" representation: $\Gamma \rightarrow x\Gamma$ Spinors!

Higher-spin algebra: $[Y_a, Y_b]_\star = I_{ab}$

O(1,4) symmetry generated by: $Y_a Y_b$

Reflection (CPT): $f(Y) \rightarrow \delta_x(Y) \star f(Y) \star \delta_x(Y)$

"Square root" representation: $f(Y) \rightarrow f(Y) \star \delta_x(Y)$

Penrose transform!

"Square root representation"
made manifest via boundary bilocal:

$$K(\ell,\ell';Y) \sim \sqrt{-\ell\cdot\ell'}\,\delta_\ell(Y)\star\delta_{\ell'}(Y)$$

$$K(\ell,\ell';Y)\star\delta_x(Y) = K(\ell,\tilde{\ell}';Y)$$

where $\tilde{\ell}' =$ reflection of ℓ' around x

The same bilocal transforms the CFT into twistor space:

$$f(Y) = \int d^3\ell\, d^3\ell'\, \Pi(\ell',\ell) K(\ell,\ell';Y)$$