Symmetries At Null Boundaries

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Based on my recent papers with
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Outline

- Einstein GR and equivalence principle in presence of boundaries

- Null surfaces and boundaries as models for BH horizons

- Null boundary symmetries, 3d example

- Null boundary algebras

- Change of basis/slicing on solution phase space

- Summary and Outlook

Gauge theories in presence of boundaries

- Consider a gauge theory with generic fields Φ_α described by the action

$$S[\Phi_\alpha] = \int_{\mathcal{M}} d^D x\ \mathbf{L}(\Phi_\alpha)$$

where $\mathbf{L}$ is the Lagrangian which is a D-form.

- Φ_α belong to representation $\mathcal{R}_\alpha$ of the gauge Lie algebra $\mathcal{A}$,

$$\Phi_\alpha \to \tilde{\Phi}_\alpha = \mathcal{R}_\alpha \cdot \Phi_\alpha.$$

- In the above $\mathcal{R}_\alpha$ is a function over the spacetime and

$$S[\Phi_\alpha] = S[\tilde{\Phi}_\alpha]$$

- In gauge theories fields are defined up to gauge equivalence classes and physical observables are gauge invariant quantities.

- Gauge symmetry is in fact a redundancy of description which should be removed by gauge fixing, but yet, there may be nontrivial gauge transformations in presence of boundary in spacetime.

- Variation principle stipulates that

$$\delta S = \int_{\mathcal{M}} d^D x \, \mathbf{E}_{\Phi_\alpha} \delta\Phi_\alpha + \int_{\partial\mathcal{M}} \boldsymbol{\theta}(\Phi_\alpha, \delta\Phi_\alpha) := 0, \quad \forall \text{physically allowed } \delta\Phi_\alpha.$$

- On-shell $\mathbf{E}_{\Phi_\alpha} = 0$, and variation of the action is a surface term which should vanish.

- In presence of boundaries $\partial\mathcal{M} \neq \emptyset$ this may lead to interesting, non-trivial physics, depending on what "physically allowed $\delta\Phi_\alpha$" at the boundary are.

- Variation principle may require adding appropriate boundary terms or restrict $\delta\Phi_\alpha$ at the boundary.

- In our analysis we do not require vanishing of the boundary term, as it can always be guaranteed choosing appropriate surface terms.

- In a different viewpoint, we may define our boundary/initial value problem by specifying the behavior of Φ_α at the boundary:

$$\Phi_\alpha\Big|_{\partial\mathcal{M}} := \varphi_\alpha, \qquad \delta\Phi_\alpha\Big|_{\partial\mathcal{M}} := \delta\varphi_\alpha$$

- φ_α need not be invariant under a part of gauge transformations at $\partial\mathcal{M}$. These may be called boundary large gauge transformations.

- Boundary gauge transformations defined on the codimension one surface $\partial\mathcal{M}$, are a measure zero subset of gauge transformations and are not necessarily gauge reduendancies.

■ Here I advocate the viewpoint that

- there are boundary degrees of freedom (b.d.o.f.) which are labelled by boundary gauge transformations, i.e. b.d.o.f fall into coadjoint orbits of the physical residual gauge transformations.

- There is a maximal boundary phase space (MBPS) associated with boundary fields, fields on a codimension one surface.

- The residual/boundary gauge transformations are a handy and powerful method to identify and formulate b.d.o.f without invoking addition of extra d.o.f by hand.

- Imposing boundary conditions typically reduce this MBPS to a sub phase space.

- Requiring variational principle on the original theory yields different Hamiltonians on the Boundary Phase Space and/or reduction over the maximal boundary phase space.

▶ As an example one may consider Maxwell theory in a box,

- Besides the photons in the box we have b.o.d.f.

- Their response to the EM fields in the box is the boundary currents.

- Boundary currents are specified, choosing boundary conditions.

- This gives a macroscopic formulation of b.d.o.f and fixes the boundary/bulk interactions.

■ Einstein GR and its local (gauge) symmetry

- Einstein GR is based on Equivalence Principle which stipulates that

  all observers should give (exactly) the same description of local events in regions of spacetime to which they have causal access.

- Each observer is specified by a coordinate system and vice versa.

- Equivalence Principle at theory level is made manifest through general covariance, invariance of the action under diffeomorphisms.

- Physical observables in the Einstein GR are all defined through local diffeomorphism invariant quantities.

- In particular, any two metric tensors related by diffeomorphisms are physically equivalent:

$$x^\mu \to x^\mu + \xi^\mu(x), \quad g_{\mu\nu} \to g_{\mu\nu} + \delta g_{\mu\nu}, \; \delta g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$$

- The above is shared between all theories with local gauge symmetries: Action and physical observables should be gauge invariant.

- We typically fix the diff. invariance through choice of observers.

Equiv. Princ. needs amendment in presence of boundaries

- Einstein field equations define a well-posed dynamical problem.

- Metric is completely specified, giving the values of metric and its "time" derivative over a constant time slice, a Cauchy surface.

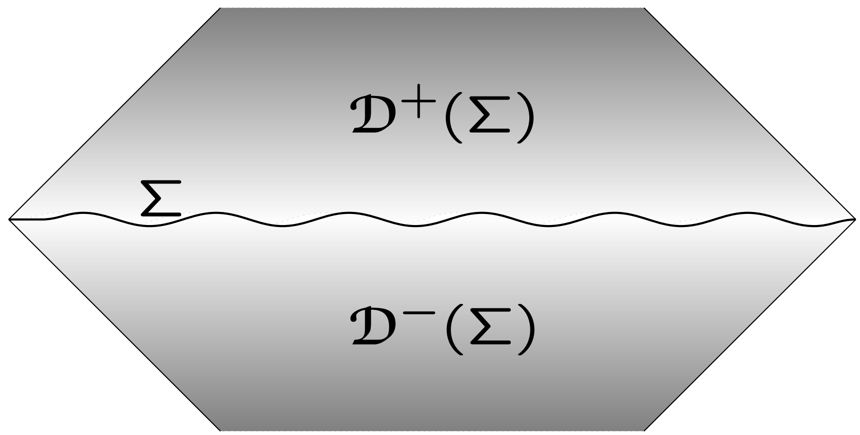
- Field equations are local and locally specifying this "Cauchy data," determines the evolution in the future lightcone of a given element on the Cauchy surface.

- In a D dimensional spacetime, there are
  - $D(D+1)/2$ metric components,
  - $D(D-3)/2$ propagating gravitons,
  - D diffeos.

  and

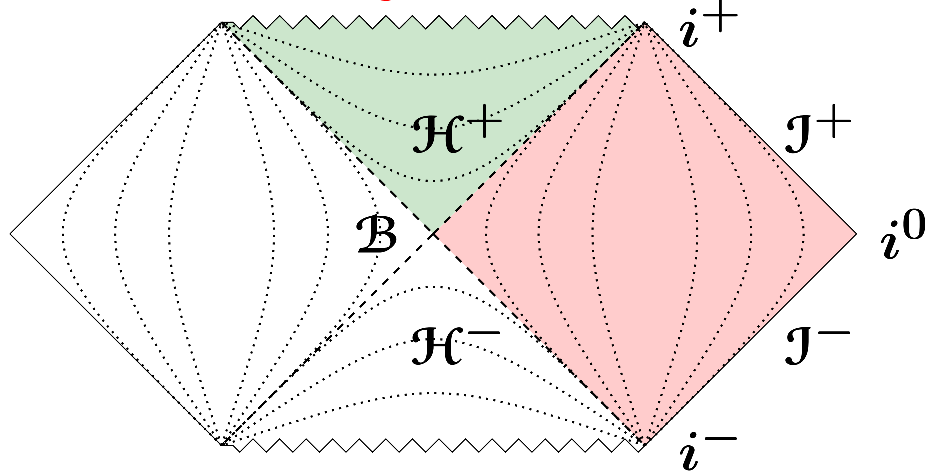
  - $D(D+1)/2$ field equations, $G_{\mu\nu} = 8\pi G T_{\mu\nu}$, out of which
  - $D(D-3)/2$ are second order diff.eq.,
  - D constraints ($\nabla^{\mu} G_{\mu\nu} = 0$) and D first order equations.
- D functions on codimension one surface Σ ($D-1$ dimensional spacelike or partially null surface) are to be specified by the initial data.

- Information in the initial data, by definition, is conserved as we move away from the (Cauchy) surface. Alternatively, the information about this D functions over Σ is propagated by the EoM.



- One may encode this information in a symmetry/charge language.

- In a black hole setup, horizon is the boundary of outside observers.



- Motivated by problems in BHs, we choose Σ to be a null surface, sitting at $r = 0$:

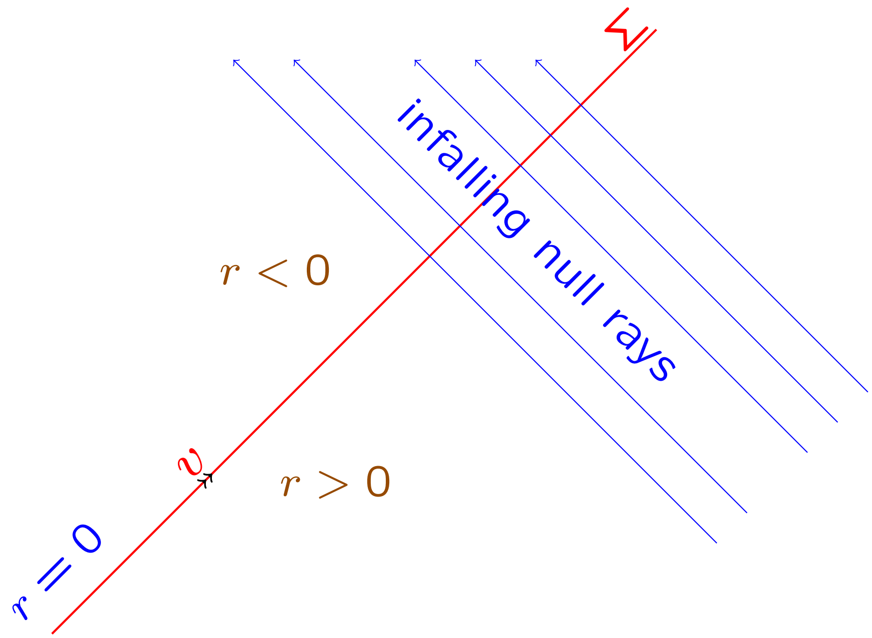
$$ds^2 = -F dv^2 + 2\mu dr dv + 2g_i dv dx^i + h_{ij} dx^i dx^j \tag{1}$$

F, μ, g_i, h_{ij} are functions of r, v, x^i, $i = 1, 2, \cdots, D-2$ and

$$g^{rr}\Big|_{r=0} = 0 \quad \Longrightarrow \quad (Fh + g^2)\Big|_{r=0} = 0,$$

where $h := \det h_{ij}, g^2 := h^{ij} g_i g_j$.

Depiction of a null surface



The b.d.of. are residing on Σ. We can see how a null surface is special for defining the surface charges on.

They interact with themselves and with infalling flux. Interaction with infalling flux is fixed by diff invariance (Bondi news/balance equation).

Solution space

- Metric (1) has $1 + 1 + (D-2) + (D-1)(D-2)/2$ functions in it.

- These may be decomposed into

  - three scalars (F, μ, h),
  - one vector g_i and
  - one symmetric-traceless tensor $H_{ij} := h_{ij}/h^{1/(D-2)}$,

  from the viewpoint of codimension two surface Σ_v, (constant v slice on Σ).

- These functions are subject to field equations, which determine their r dependence.

- The r dependence of the tensor mode H_{ij} is determined through

  $H_{ij}^{(0)}(v, x^i) := H_{ij}(r = 0; v, x^i)$, $H_{ij}^{\prime(0)}(v, x^i) := \partial_r H_{ij}(r = 0; v, x^i)$.

- The r dependence of the vector mode obeys first order eq. in r and is completely specified by $g_i^{(0)}(v, x^i) := g_i(r = 0; v, x^i)$.

- Among the three scalars only two are independent (e.g. one may set $\mu = 1$ by a rescaling of r).

- The r dependence of the other two are determined in terms of

  $F_0(v, x^i) := F(r = 0; v, x^i)$, $h_0(v, x^i) := h(r = 0; v, x^i)$.

- We have only assumed smoothness and Taylor-expandibility,
- but no particular behavior (falloff condition), around $r = 0$.
- We have imposed EoM perturbatively around $r = 0$.
- We have not required variation principle.
- $r = 0$ is not a special place in spacetime and can be any (null) $D - 1$ dimensional hypersurface.
- Solution space is determined by
  "tensor modes" (gravitons) D(D-3) functions over Σ,
  2 scalars modes over Σ &
  (D-2) vector modes over Σ.
- This is the maximal solution space. By construction there can't be any solution geometry which is smooth around $r = 0$ and is not in the form (1).

Residual diffeos over the null surface Σ

- We have used diffeos to fix the null surface Σ at $r = 0$.

- There is a measure zero subset of them which keep $r = 0$ intact remained unfixed:

$$
\begin{aligned}
v &\to v + T(v, x^i) + \mathcal{O}(r) \\
r &\to \Big(\partial_v T(v, x^i) - W(v, x^i)\Big) r + \mathcal{O}(r^2) \\
x^i &\to x^i + Y^i(v, x^i) + \mathcal{O}(r)
\end{aligned}
\tag{2}
$$

- Subleading terms in r may be fixed order-by-order requiring that (2) keep the form of metric in solution space (1).

- Residual diffeos are specified by two scalar functions $T(v, x^i), W(v, x^i)$ and one vector $Y^i(v, x^i)$ over $r = 0$ null surface.

Symmetries of the solution space

- Upon (2) metric (1) keep its form but with transformed functions:

$$\begin{gathered} F_0 \to F_0 + \delta F_0, \qquad \mu \to \mu + \delta\mu, \qquad h_0 \to h_0 + \delta h_0, \\ g_i^{(0)} \to g_i^{(0)} + \delta g_i^{(0)}, \qquad H_{ij}^{(0)} \to H_{ij}^{(0)} + \delta H_{ij}^{(0)}, \end{gathered} \tag{3}$$

  where δX are linear in residual diffeo functions T, W, Y^i.

- Besides dynamical, propagating gravitons, there are $2 + (D-2)$ functions over Σ in our solution space.

- There are $2 + (D-2)$ functions over Σ in our residual diffeos.

- Residual diffeos rotate us within the solution space. They are hence symmetry generators.

- There are two classes of fields/states in our solution space:
  - $D(D-3)$ propagating tensor modes $H_{ij}^{(0)}, H_{ij}^{\prime(0)}$, one may call them hard modes,
  - D scalar and vector modes, one may call them "soft modes".
- Soft modes are boundary modes, only reside on $D-1$ dimensional hypersurface Σ and do not propagate into the bulk (away from $r=0$).
- In our example we have chosen Σ to be null surface, like future horizon of a BH.

Solution Phase Space

- One may use covariant phase space method (CPSM) to show that our solution space indeed forms a phase space:

there is a well-defined symplectic structure and a Poisson bracket on the solution space

- This solution phase space has two distinct parts:
soft modes & hard modes.

- If we turn off the hard modes, when there are no gravitons in the bulk, the soft sector forms a phase space on its own.

- There is a one-to-one correspondence between the soft modes in the solution space and the symmetry generators.

■ Symmetries of the solution phase space

- Using CPSM one may associate surface charges to symmetry generators (the non-trivial diffeos).

- These surface charges are given by integrals over codimension-2 compact spacelike surfaces, constant v slices on Σ, Σ_v.

- Surface charges are linear in symmetry generators $T(v,x^i), W(v,x^i)$ and $Y^i(v,x^j)$, but may have different field/states dependence, i.e.

- integrands of the surface charge integrals may have different functional dependence on $F_0, h_0, g_i^{(0)}$.

▶ Detour to CPSM

- To extract the non-trivial diffeo's and the associated surface charges we may use covariant phase space method (CPSM):

  i) All field configurations (histories) may form a Phase Space,

  ii) with the symplectic structure systematically constructed from the action of the theory:

- Consider a field configuration Φ and perturbations around it $\delta\Phi$.

- On-shell field configurations $\bar{\Phi}$ satisfy field equations and on-shell perturbations $\delta\Phi$ satisfy linearized field equations.

- Set of Φ and $\delta\Phi$ may be viewed as a phase space and one-forms in the corresponding cotangent space.

- On-shell cotangent space includes two important directions:
  - $\delta\Phi$ generated by gauge and/or diffeo's transformations on Φ;
  - parametric variations, generated by moving in the parameter space of the solutions Φ, e.g. the difference between two Sch'd solutions with masses m and $m + \delta m$.

▶ Symplectic structure

- Symplectic current $\boldsymbol{\omega}$ is a *finite*, *closed*, *nondegenerate*, it is a $(d-1;2)$-form, i.e. a $d-1$ form in space time and a two-form over the phase space:

$$\boldsymbol{\omega} = \boldsymbol{\omega}[\delta_1\Phi, \delta_2\Phi; \Phi]$$

- Symplectic structure $\boldsymbol{\Omega}_\Sigma$ is defined through integration of $\boldsymbol{\omega}$ over a Cauchy surface Σ:

$$\boldsymbol{\Omega}_\Sigma\ [\delta_1\Phi, \delta_2\Phi; \Phi] = \int_\Sigma \boldsymbol{\omega}[\delta_1\Phi, \delta_2\Phi; \Phi]$$

$\boldsymbol{\Omega}_\Sigma$ is a $(0;2)$-form.

- We build $\boldsymbol{\omega}$ within the *covariant phase space method*, constructed in [Lee-Wald '1990, Wald '1993] and refined in [Barnich-Brandt '2002, Barnich-Compère '2008].

▶ **Construction of the symplectic current**

- Presymplectic potential $\boldsymbol{\theta}[\delta\Phi;\Phi]$: $\boldsymbol{\omega} = \delta\boldsymbol{\theta}$, or

$$\boldsymbol{\omega}[\delta_1\Phi, \delta_2\Phi;\Phi] = \delta_1\boldsymbol{\theta}[\delta_2\Phi;\Phi] - \delta_2\boldsymbol{\theta}[\delta_1\Phi;\Phi]$$

- The Lee-Wald contribution to $\boldsymbol{\theta}$:

$$\delta\boldsymbol{L}\big|_{on-shell} = d\boldsymbol{\theta}_{(LW)}.$$

- $\boldsymbol{L}$ is a $(d;0)$-form and presympelctic structure $\boldsymbol{\theta}$ a $(d-1;1)$-form.

- Consistency of symplectic structure may require addition of *boundary terms* $\mathbf{Y}$:

$$\boldsymbol{\theta} = \boldsymbol{\theta}_{(LW)} + d\mathbf{Y}.$$

$\boldsymbol{Y}$ is a $(d-2; 1)$-form.

- Consistency of symplectic structure means its

  – Conservation:

  $d\boldsymbol{\omega}[\delta_1\Phi, \delta_2\Phi; \Phi] \approx 0$ for all on-shell fields and perturbations.

  – Non-degeneracy: $\boldsymbol{\Omega}_\Sigma$ has no degenerate directions, is conserved and is independent of Σ.

Surface charges

- Fundamental Theorem of Covariant Phase Space Method

$$\boldsymbol{\omega}[\delta\Phi, \delta_\chi\Phi; \Phi] \approx \boldsymbol{d}\boldsymbol{K}_\chi[\delta\Phi; \Phi]$$

- $\delta_\chi\Phi$ is a specific transformation generated by a symmetry χ and $\boldsymbol{K}_\chi$ is a $(d-2; 1)$-form.

- Given $\boldsymbol{K}$ one can define charge variations:

$$\not{\delta} Q_\chi = \oint_{\partial\Sigma} \boldsymbol{K}_\chi[\delta\Phi; \Phi]$$

$\not{\delta} Q_\chi$ is a $(0; 1)$-form.

- Charge Q_χ is integrable if

$$\delta_1 \not\delta_2 Q_\chi - \delta_2 \not\delta_1 Q_\chi = 0$$

Integrability [Lee-Wald '1991]:

$$\oint_{\partial\Sigma} \chi\cdot\boldsymbol{\omega}[\delta_1\Phi, \delta_2\Phi; \Phi] = 0, \quad \forall \chi, \delta\Phi$$

There usually exists $\boldsymbol{Y}$ terms which guarantee the above.

- Using integrability one can define surface charges Q_χ:

$$Q_\chi[\Phi] = \int_\gamma \oint_{\partial\Sigma} \boldsymbol{K}_\chi[\delta\Phi; \Phi] + N_\chi[\Phi]$$

where N is the zero point charge.

- If $\not\delta Q_\chi$ is zero everywhere on the phase space, χ is called pure gauge transformation. These are the "real gauge d.o.f".

- Surface integrals over the boundary of Σ, $\partial\Sigma$, in our case Σ_v.

- Algebra of charges:

$$\{Q_\chi, Q_\xi\} = Q_{[\chi,\xi]} + \text{possible central terms} \tag{4}$$

- Notes:

  - Charges are functions over the solution phase space,

  - the bracket is Poisson bracket among these functions, and

  - $[\chi, \xi]$ is the Lie bracket of generators.

- Charges Q_ξ may be used to label soft states/configurations in the phase space, and hence how to account for them.

End of detour ◀

Null Boundary Symmetries for $3d$ gravity

- $2d, 3d$ examples are special as there are no hard modes (gravitons) in the game, we just have the soft modes.

- Details of the $2d$ Einstein-Dilaton gravity and $3d$ Einstein-Λ theory examples may be found in [arXiv:2007.12759 [hep-th]].

- Here we present the $3d$ example with the action

$$S = \frac{1}{16\pi G}\int \mathrm{d}^3x \sqrt{-g}\,(R-2\Lambda), \qquad \mathcal{E}_{\mu\nu} := R_{\mu\nu} - 2\Lambda g_{\mu\nu} = 0. \tag{5}$$

- Depending on Λ, $\Lambda < 0, \Lambda = 0, \Lambda > 0$ we respectively have AdS_3, flat or dS_3 gravities.

- All solutions to the respective theories are locally AdS_3, flat or dS_3.

- Maximal solution space near $r = 0$ null surface has the form

$$ds^2 = -F dv^2 + 2\eta dvdr + 2f\, dvd\phi + h d\phi^2\,, \tag{6}$$

$$F(v, r, \phi) = F_0(v, \phi) + rF_1(v, \phi) + \mathcal{O}(r^2) \tag{7a}$$

$$f(v, r, \phi) = f_0(v, \phi) + rf_1(v, \phi) + \mathcal{O}(r^2) \tag{7b}$$

$$h(v, r, \phi) = \Omega(v, \phi)^2 + rh_1(v, \phi) + \mathcal{O}(r^2) \tag{7c}$$

- Since $r = 0$ is a null hypersurface, $g^{rr}|_{r=0} = 0$:

$$F_0 = -\left(\frac{f_0}{\Omega}\right)^2 . \tag{8}$$

- Irrespective of what Λ is, EoM relate f_1, h_1, F_1 to three function F_0, Ω, μ which parametrize the solution space.

Symmetry generators

$$\xi^v = T$$
$$\xi^r = r(\partial_v T - W) + \frac{r^2 \partial_\phi T}{2\Omega^2}\left(f_1 + \partial_\phi \eta - \frac{f_0 h_1}{\Omega^2}\right) + \mathcal{O}(r^3) \tag{9}$$
$$\xi^\phi = Y - \frac{r\eta\partial_\phi T}{\Omega^2} + \frac{r^2 \eta h_1 \partial_\phi T}{2\Omega^4} + \mathcal{O}(r^3)$$

where T, Y and W are some functions of v and ϕ.

Algebra of symmetry generators:

$$[\xi(W_1, T_1, Y_1), \xi(W_2, T_2, Y_2)]_{\text{adj. Lie bracket}} = \xi(W_{12}, T_{12}, Y_{12}) \tag{10}$$

where

$$T_{12} = T_1 \partial_v T_2 - T_2 \partial_v T_1 + Y_1 \partial_\phi T_2 - Y_2 \partial_\phi T_1$$
$$W_{12} = T_1 \partial_v W_2 - T_2 \partial_v W_1 + Y_1 \partial_\phi W_2 - Y_2 \partial_\phi W_1 + \partial_v Y_1 \partial_\phi T_2 - \partial_v Y_2 \partial_\phi T_1$$
$$Y_{12} = Y_1 \partial_\phi Y_2 - Y_2 \partial_\phi Y_1 + T_1 \partial_v Y_2 - T_2 \partial_v Y_1 \,,$$

- Null Boundary Symmetry (NBS) algebra (10) is $Diff(C_2) \oplus Weyl$, where C_2 is the null cylinder spanned by v, ϕ and $Diff(C_2)$ is generated by T, Y and Weyl scaling is generated by W.

- Repeating this analysis for a D dimensional gravity, we obtain $Diff(C_{D-1}) \oplus Weyl$, where C_{D-1} is the null cylinder spanned by v, x^i and $Diff(C_{D-1})$ is generated by T, Y^i and Weyl scaling is generated by W.

- It is more convenient to describe the solution space in terms three other functions $\Gamma(v, \phi), \Upsilon(v, \phi), \mathcal{P}(v, \phi)$. Their explicit expressions in terms of metric functions may be found in our paper.

- It is straightforward algebra to compute field variations like $\delta_\xi \eta, \delta_\xi \Omega, \delta_\xi \Gamma, \delta_\xi \Upsilon, \delta_\xi \mathcal{P}$.

Surface charges and their algebra

- Standard computations yields the following surface charge variations associated with the symmetry generators ξ

$$\not\delta Q_\xi = \frac{1}{16\pi G}\int_0^{2\pi} d\phi \left[W\delta\Omega + Y\delta\Upsilon + T\not\delta\mathcal{A}\right] , \qquad (12)$$

where $\not\delta\mathcal{A}$ is a combination of the solution space fields which is not a closed one-form, i.e. $\delta\not\delta\mathcal{A} \neq 0$ and hence $\not\delta\mathcal{A} \neq \delta\mathcal{A}$.

- The charge associated to T ("supertranslations" along v direction) is not integrable, while the charges associated with the "scaling in r" generated by W and the "superrotations" generated by Y are integrable.

NBS algebra in the integrable slicing

- Consider the field-dependent changes of basis (change of slicing in the solution phase space):

$$T = -\frac{\mathcal{P}}{\chi}\hat{T}, \quad Y = \hat{Y} + \frac{f_0\mathcal{P}}{\chi\Omega^2}\hat{T}, \quad W = \hat{W} + \frac{\Gamma\mathcal{P}}{\chi}\hat{T} - \frac{f_0}{\Omega^2}\partial_\phi\left(\frac{\mathcal{P}\hat{T}}{\chi}\right) \ ,$$

where

$$\mathcal{P} := -\ln\left(\frac{\chi^2}{\eta}\right), \qquad \chi := \partial_v\Omega - \partial_\phi(\frac{f_0}{\Omega}).$$

- In this basis the charge variation takes the form

$$\delta Q_\xi = \frac{1}{16\pi G}\int_0^{2\pi} d\phi \left(\hat{W}\delta\Omega + \hat{Y}\delta\Upsilon + \hat{T}\delta\mathcal{P}\right) \tag{13}$$

- It is manifestly integrable.

- Using EoM all functions in the solution phase space may be solved in terms of Ω, $\mathcal{P}$, Υ, up to two v-dependent integration constants.

- The two v-dependent functions do not appear in the charge expressions. So, our solution phase space is completely parametrized in terms of Ω, $\mathcal{P}$, Υ.

- The charges Ω, $\mathcal{P}$, Υ are arbitrary functions of v, ϕ.

- The integrable basis or integrable slicing of the solution phase space is not unique. There are (infinitely) many more.

Integrable NBS charge algebra

- Fourier expand the charges ($Z = \Omega, \mathcal{P}, \Upsilon$)

$$Z(v,\phi) := 8G \sum_n Z_n(v) e^{-in\phi}.$$

- Going through the standard charge algebra analysis (noting the field dependence of the symmetry generators), and upon quantisation $i\{,\} \to [,]$, we have

$$[\Omega_m(v), \Omega_n(v)] = 0\,, \;\; [\mathcal{P}_m(v), \mathcal{P}_n(v)] = 0\,,$$
$$[\Omega_m(v), \mathcal{P}_n(v)] = \frac{i}{8G}\, \delta_{m+n,0}$$
$$[\Upsilon_m(v), \Upsilon_n(v)] = (m-n)\, \Upsilon_{m+n}(v)$$
$$[\Upsilon_m(v), \Omega_n(v)] = -n\, \Omega_{m+n}(v)$$
$$[\Upsilon_m(v), \mathcal{P}_n(v)] = -(m+n)\, \mathcal{P}_{m+n}(v) + \frac{n}{4G}\, \delta_{m+n,0}$$

- While the charges are in general v dependent, their algebra is not; the algebra takes the same form for all v.

- The algebra is independent of the cosmological constant Λ.

- $\Omega_m(v), \mathcal{P}_n(v)$ form a Heisenberg algebra with effective $\hbar = \frac{1}{8G}$.

- $\Upsilon_m(v)$ for a Witt algebra $(Diff(S^1))$.

- $\Omega_m(v)$ is a field of weight one and $\mathcal{P}_n(v)$ is of weight zero in this Witt algebra. (Fields on the null cylinder spanned by v, ϕ.)

Fundamental slicing for NBS charge algebra

- Consider the simple change of slicing

$$\widehat{W} = \tilde{W} - 2\partial_\phi \tilde{Y} + \tilde{Y}\partial_\phi \mathcal{P}\,, \;\; \hat{T} = \tilde{T} - \partial_\phi(\Omega\tilde{Y})\,, \;\; \hat{Y} = \tilde{Y}$$

$$\Upsilon = -2\partial_\phi \Omega - \Omega\partial_\phi \mathcal{P} + 16\pi G\,\mathcal{S}\,,$$

assume $\tilde{W}, \tilde{Y}, \tilde{T}$ to be independent of charges in new slicing $\Omega, \mathcal{P}, \mathcal{S}$.

- Υ and $\mathcal{S}$ differ in the 'orbital superrotation part'.

- The algebra of charges in the fundamental NBS slicing is

$$[\Omega_m(v), \Omega_n(v)] = 0\,, \;\; [\mathcal{P}_m(v), \mathcal{P}_n(v)] = 0\,, \;\; [\Omega_m(v), \mathcal{P}_n(v)] = \frac{i}{8G}\,\delta_{m+n,0}$$

$$[\mathcal{S}_m(v), \mathcal{S}_n(v)] = (m-n)\,\mathcal{S}_{m+n}(v)\,, [\mathcal{S}_m(v), \Omega_n(v)] = [\mathcal{S}_m(v), \mathcal{P}_n(v)] = 0$$

- This algebra is $Heisenberg \oplus Diff(S^1)$.

Discussion, Concluding Remarks and Outlook

⊛ Presence of Boundaries brings in new 'boundary d.o.f.'.

- The b.d.o.f. may be classified and labelled by nontrivial diffeos.

- Using CPSM one can construct the boundary phase space which govern b.d.o.f.

- Motivated by identification and formulation of BH microstates we discussed null boundaries Σ.

- $\Sigma \sim R_v \times \Sigma_v$, where Σ_v is a codim. two compact surface.

- Σ may be viewed as the null limit of the stretched horizon.

- Physics in the outside horizon region is then described by

$$\text{b.d.o.f} \oplus \text{bulk d.o.f.}$$

- The Hilbert space of b.d.o.f, $\mathcal{H}_{bdof}$ may be labeled by the surface charges associated with nontrivial diffeos on Σ_v.

- We have shown in appropriate slicing, these surface charges satisfy

$$NBS - algebra = Heisenberg + Diff(\Sigma_v)$$

  To be more precise, $Diff(\Sigma_v)$ is the area-preserving Diff of Σ_v.

- Besides our b.d.o.f for asymptotic flat spacetimes there are usual BMS-like diffeos/charges/states.

- Our proposal is that the BH microstates are in $\mathcal{H}_{bdof}$ and are labeled by these charges.

- The interactions of these microstates and the bulk dof is also fixed by the diff. invariance:

Boundary d.o.f interact with bulk d.of. through the *Bondi news* through the horizon.

- In essence, we have extended the Equivalence Principle to the cases with horizon, which can hopefully account for BH microstates too.

- The analysis so far is classical and we should quantize the system.

- It should be possible to perform a semiclassical analysis in which the boundary d.o.f are quantized while the bulk is classical.

⊛ To fully formulate the above proposals, one should study

- The relation between Barnich-Troessart modified bracket and the Wald-Zoupas method.
- Formulate in full generality the NBS analysis in generic theories in diverse dimensions.
- Quantization of the boundary phase space and the 'semiclassical' description mentioned above.
- Full theory of change of slicing on the solution phase space and general boundary conditions.
- Relation between the theory of deformation of algebras and the change of slicing.

⊛ Having these tools and results one may

- Tackle the BH microstate problem.
- Relation between our approach and the Hawking-Perry-Strominger soft hair proposal.
- Resolving the BH unitarity problem?!
- Connection to our horizon fluff proposal [Afshar, Grumiller, MMShJ, 2016; MMShJ, Yavartanoo, 2016 & Afshar, Grumiller, MMShJ, Yavartanoo, 2017].
- Relation to membrane paradigm.

My view on BH microstates & information puzzle:

BH microstates are certain states among the near horizon soft hair
and
are indistinguishable (degenerate) from the asymptotic symmetry viewpoint.

This Heisenberg algebra arises as a result of Rindler wedge, ubiquitously found in any nonextreme NH geometry.

Membrane paradigm may be providing the way to identify BH microstates.

Thank You For Your Attention