Notes on Orientifolds

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Virtual meeting -2

Plan

- Superstrings
- Brief mention of Gepner Models

Based on

- R. Blumenhagen, E. Plauschinn
  Introduction to Conformal Field Theory
  Springer, 2009
- C.Angelantonj, A.Sagnotti
  Open Strings,
  Phys. Rept. **371** (2002), 1-150, hep-th/0204089
- J.Polchinski
  String Theory, vol 2
  Cambridge University Press, 1998
- E.Kiritsis
  Introduction to Superstring Theory
  hep-th/9709062

- Superstring in NSR formulation

$$S = \frac{1}{4\pi\alpha'} \int d^2\xi (\partial_- X^\mu \; \partial_+ X_\mu - i\psi_-^\mu \partial_+ \psi_{-,\mu} - i\psi_+^\mu \partial_- \psi_{+,\mu})$$

with $\xi^\pm = \tau \pm \sigma$ and $\partial_\pm = \frac{\partial}{\partial \xi^\pm}$. Critical dimension $D = 10$, i.e, $\mu = 0, ..9$

- Boundary conditions for open string : $\psi_+^\mu(\tau, 0) = \psi_-^\mu(\tau, 0)$ and

$$\psi_+^\mu(\tau, \pi) = \psi_-^\mu(\tau, \pi) \quad (R)$$

$$\psi_+^\mu(\tau, \pi) = -\psi_-^\mu(\tau, \pi) \quad (NS)$$

- Solved by

$$\psi_\pm^\mu = \frac{1}{\sqrt{2}} \sum_r \psi_r^\mu e^{-ir(\tau \pm \sigma)}$$

where r is integer for R sector and half integer for NS sector.

- For closed string we impose boundary conditions independently for ψ_+^μ and ψ_-^μ
$$\psi_+^\mu = \sum_r \bar{\psi}_r^\mu e^{-2ir(\tau+\sigma)}, \quad \psi_-^\mu = \sum_r \psi_r^\mu e^{-2ir(\tau-\sigma)}$$
Again r is integer for R sector, half integer for NS sector.
- For open string NS sector generates bosons, and R sector generates fermions.
- For closed string (NS-NS) and (R-R) sectors generate bosons, and (R-NS) and (NS-R) sectors generate fermions.
- GSO projection in NS sector (projects out the tachyon)
$$P_{GSO} = \frac{1}{2}(1-(-1)^{N_F}), \quad N_F = \sum_{r>0} \psi^\mu_{-r}\psi_{r\,\mu}$$
- GSO projection in R sector (a choice of chirality)
$$P_{GSO} = \frac{1}{2}(1 \pm \gamma^{11}(-1)^{N_F}), \quad N_F = \sum_{r>0} \psi^\mu_{-r}\psi_{r\,\mu}$$

- The world-sheet geometry is the same as for the bosonic string.
- We need to compute a sum over 4 spin structures - all periodicities in time and space: NS -antiperiodic (in time), NS- periodic, R- antiperiodic, R- periodic.
- These are often denoted as $(-,-)$, $(-,+)$, $(+,-)$ and $(+,+)$
- Periodicity in time is achieved by inserting of the operator $(-1)^F$ into pass integral- GSO projection
- The modular transformation is $SL(2,Z)$,

$$(\sigma_1, \sigma_2) \to (a\sigma_1 + b\sigma_2, c\sigma_1 + d\sigma_2)$$

  Therefore none of the spin structures (except $(+,+)$) can be modular invariant.

- We use for bosons and for fermions

$$Tr[q^{\sum_{n=1}^{\infty}\alpha_{-n}\alpha_n}] = \prod_{n=1}^{\infty}\frac{1}{1-q^n}, \quad Tr[q^{\sum_{n=1}^{\infty}\psi_{-n}\psi_n}] = \prod_{n=1}^{\infty}(1+q^n)$$

- Similarly to the bosonic string in NS sector

$$Tr_{NS}[q^{N_B+N_F-\frac{1}{2}}] = \frac{\prod_{n=1}^{\infty}(1+q^{n-1/2})^8}{q^{\frac{1}{2}}\prod_{n=1}^{\infty}(1-q^n)^8}$$

- In the R sector

$$Tr_R[q^{N_B+N_F}] = 16\,\frac{\prod_{n=1}^{\infty}(1+q^n)^8}{\prod_{n=1}^{\infty}(1-q^n)^8}$$

16 – because the vaccum is 16 component Majorana - Weyl spinor in $D = 10$

- Let us insert the GSO projection into the NSR sector

$$Tr\left(\frac{1-(-1)^F}{2}q^{N_B+N_F-\frac{1}{2}}\right)=\frac{\prod_{n=1}^{\infty}(1+q^{n-1/2})^8-\prod_{n=1}^{\infty}(1-q^{n-1/2})}{q^{\frac{1}{2}}\prod_{n=1}^{\infty}(1-q^n)^8}$$

- GSO in the R sector just halves the vacuum degeneracy
- Introducing Jacobi theta functions as

$$\frac{\theta_2^4(0|\tau)}{\eta^{12}(\tau)}=16\,\frac{\prod_{n=1}^{\infty}(1+q^n)^8}{\prod_{n=1}^{\infty}(1-q^n)^8}$$

$$\frac{\theta_3^4(0|\tau)}{\eta^{12}(\tau)}=\frac{\prod_{n=1}^{\infty}(1+q^{n-1/2})^8}{q^{\frac{1}{2}}\prod_{n=1}^{\infty}(1-q^n)^8},\quad \frac{\theta_4^4(0|\tau)}{\eta^{12}(\tau)}=\frac{\prod_{n=1}^{\infty}(1-q^{n-1/2})^8}{q^{\frac{1}{2}}\prod_{n=1}^{\infty}(1-q^n)^8}$$

$SO(2n)$ characters

- Let us introduce orthogonal decompositions. In NS sector

$$O_{2n} = \frac{\theta_3^n + \theta_4^n}{2\eta^n}, \qquad V_{2n} = \frac{\theta_3^n - \theta_4^n}{2\eta^n}$$

- In R sector

$$S_{2n} = \frac{\theta_2^n + i^{-n}\theta_1^n}{2\eta^n}, \qquad C_{2n} = \frac{\theta_2^n - i^{-n}\theta_1^n}{2\eta^n}$$

- They have expansions

$$O_{2n} = q^{h_o - n/24}(1 + n(2n-1)q + ...), \qquad V_{2n} = q^{h_v - n/24}(2n + ...)$$

$$S_{2n} = q^{h_s - n/24}(2^{n-1} + ...), \qquad C_{2n} = q^{h_c - n/24}(2^{n-1} + ...)$$

  with $(h_o, h_v, h_s, h_c) = (0, 1/2, n/8, n/8)$

- Recall for the Dedekind $\eta(\tau)$ function

$$\frac{1}{\eta^n} = q^{-n/24}(1 + nq + ...)$$

SO(2n) characters modular transformations

- Under T- duality transformations

$$\begin{pmatrix} O_{2n} \\ V_{2n} \\ S_{2n} \\ C_{2n} \end{pmatrix} = e^{-\frac{in\pi}{12}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & e^{\frac{in\pi}{4}} & 0 \\ 0 & 0 & 0 & e^{\frac{in\pi}{4}} \end{pmatrix} \begin{pmatrix} O_{2n} \\ V_{2n} \\ S_{2n} \\ C_{2n} \end{pmatrix}$$

- Under S- duality tranformations

$$\begin{pmatrix} O_{2n} \\ V_{2n} \\ S_{2n} \\ C_{2n} \end{pmatrix} = e^{-\frac{in\pi}{12}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & i^{-n} & -i^{-n} \\ 1 & -1 & -i^{-n} & i^{-n} \end{pmatrix} \begin{pmatrix} O_{2n} \\ V_{2n} \\ S_{2n} \\ C_{2n} \end{pmatrix}$$

- Recall also that for the Dedekind $\eta(\tau)$ function

$$T : \eta(\tau+1) = e^{\frac{i\pi}{12}} \eta(\tau)$$

$$S : \eta\left(-\frac{1}{\tau}\right) = \sqrt{-i\tau}\,\eta(\tau)$$

- Left and right fermions have opposite chirality

$$Z_{IIA} = \frac{1}{(\sqrt{\tau_2}\eta\bar{\eta})^8}(\bar{V}_8 - \bar{S}_8)(V_8 - C_8)$$

- Left and right fermions have the same chirality

$$Z_{IIB} = \frac{1}{(\sqrt{\tau_2}\eta\bar{\eta})^8}(\bar{V}_8 - \bar{S}_8)(V_8 - S_8)$$

- No tachyons. Massless spectra:
- Both theories contain an universal multiplet from $\bar{V}_8 \cdot V_8$ (NS-NS)

$$g_{\mu\nu}, B_{\mu\nu}, \phi$$

- Type IIA/IIB contains odd/even forms -potentials- $A_\mu, A_{\mu\nu\rho}/$ $A, A_{\mu\nu}, A^+_{\mu\nu\rho\sigma}$ in the RR sector from $\bar{S}_8 \cdot C_8/\bar{S}_8 \cdot S_8$,

$$\lambda^1_a\lambda^2_{\bar{b}} \sim (\gamma^{\mu\nu})_{a\bar{b}}F_{\mu\nu} + (\gamma^{\mu\nu\rho\sigma})_{a\bar{b}}F_{\mu\nu\rho\sigma}$$

$$\lambda^1_a\lambda^2_b \sim (\gamma^{\mu})_{ab}F_{\mu} + (\gamma^{\mu\nu\rho})_{ab}F_{\mu\nu\rho} + (\gamma^{\mu\nu\rho\sigma\tau})_{ab}F^+_{\mu\nu\rho\sigma\tau}$$

- Type IIA contains two gravitini of opposite chirality to each other, and two spin 1/2 fields with opposite chirality to each other from $\bar{V}_8 \cdot C_8$ and form $\bar{S}_8 \cdot V_8$ (NS-R and (R-NS) sectors)

$$\psi^1_{a,\mu}, \psi^2_{\bar{a},\mu}, \lambda^1_{\bar{a}}, \lambda^2_a$$

- Type IIB contains two gravitini of the same chirality, and two spin 1/2 fields with the chirality (which is opposite to the one of the gravitini) from $\bar{V}_8 \cdot S_8$ and form $\bar{S}_8 \cdot V_8$ (NS-R and (R-NS) sectors)

$$\psi^1_{a,\mu}, \psi^2_{a,\mu}, \lambda^1_{\bar{a}}, \lambda^2_{\bar{a}}$$

- Type II A is trivially anomaly free, type IIB is anomaly free in a nontrivial way
- There are two more ten dimensional supersymmetric string theories - heterotic strings with gauge groups $E_8 \otimes E_8$ and $SO(32)$. Their massless spectra corresponds to $D = 10$ $N = 1$ SUGRA + SYM

- Type IIB is left - right invariant, therefore we can mode it out by the action of the Ω operator
- In the massless spectrum we have for bosons

$$g_{\mu\nu}, \phi, A_{\mu\nu}$$

- For fermions

$$\psi'_{a,\mu}, \lambda'_{\bar{a}}$$

- This theory will be anomalous, unless we add Super Yang Mills i.e. open string sector (see the part I)
- We have for the Klein bottle, cylinder Mobius strip amplitudes

$$\mathcal{K} = \int_0^\infty \frac{d\tau_2}{\tau_2^2} \frac{(V_8 - S_8)(2i\tau_2)}{\tau_2^4 \eta^8(2i\tau_2)}$$

$$\mathcal{A} = \mathcal{N}^2 \int_0^\infty \frac{d\tau_2}{\tau_2^2} \frac{(V_8 - S_8)(i\tau_2/2)}{\tau_2^4 \eta^8(i\tau_2/2)}$$

$$\mathcal{M} = \epsilon\mathcal{N} \int_0^\infty \frac{d\tau_2}{\tau_2^2} \frac{(\hat{V}_8 - \hat{S}_8)(i\tau_2/2 + 1/2)}{\tau_2^4 \hat{\eta}^8(i\tau_2 + 1/2)}$$

- As in the case of the bosonic string we can transform to the transverse channel, using modular transformations

$$\hat{\mathcal{K}} = 2^5 \int_0^\infty dl \, \frac{(V_8 - S_8)(il)}{\eta^8(il)}, \quad \hat{\mathcal{A}} = 2^{-5} \mathcal{N}^2 \int_0^\infty dl \, \frac{(V_8 - S_8)(il)}{\eta^8(il)}$$

$$\hat{\mathcal{M}} = 2\epsilon \mathcal{N} \int_0^\infty dl \, \frac{(\hat{V}_8 - \hat{S}_8)(il + 1/2)}{\hat{\eta}^8(il + 1/2)}$$

- We have NS-NS tadpole and R-R tadpole generated by a nondynamical 10-form $a_{[10]}$. The later can have a term in the action

$$\mu_{10} \int a_{[10]}$$

- Tadpole cancellation: expanding the characters and collecting the coefficients for singular (massless) terms we get

$$\hat{\mathcal{K}} + \hat{\mathcal{A}} + \hat{\mathcal{M}} \sim (2^5 + 2^{-5} \mathcal{N}^2 + 2\epsilon \mathcal{N})$$

It is zero if $\epsilon = -1$ and $\mathcal{N} = 2^5$ i.e. the gauge group is $SO(32)$.

- World – sheet and space – time supersymmetries are connected. For $D = 4,\ N = 1$ SUSY algebra

$$\{Q_\alpha, Q_{\bar\alpha}\} = (\sigma^\mu)_{\alpha\bar\alpha} P_\mu, \quad \{Q_\alpha, Q_\beta\} = \{Q_{\bar\alpha}, Q_{\bar\beta}\} = 0,$$

- These supecharges can be realized in terms of space time spinor operators S_α, $C_{\bar\alpha}$, Internal CFT operators Σ, and ghosts φ

$$Q_\alpha = \frac{1}{2\pi i}\oint dz\, e^{-\varphi/2} S_\alpha \Sigma, \quad Q_{\bar\alpha} = \frac{1}{2\pi i}\oint dz\, e^{-\varphi/2} C_{\bar\alpha}\bar\Sigma$$

with

$$: e^{q_1\varphi(z)} e^{q_2\varphi(w)} := (z-w)^{-q_1 q_2} : e^{-(q_1+q_2)\varphi(w)} : + \ldots$$

$$S_\alpha(z) C_{\bar\alpha}(w) = \frac{1}{\sqrt{2}} (\sigma^\mu)_{\alpha\bar\alpha}\psi_\mu(w) + \mathcal{O}(z-w),$$

$$S_\alpha(z) S_\beta(w) = \frac{\epsilon_{\alpha\beta}}{\sqrt{z-w}} + \mathcal{O}(\sqrt{z-w}),$$

$$\Sigma(z)\bar\Sigma(w) = \frac{1}{(z-w)^{3/4}} + (z-w)^{1/4} J(w) + ..$$

where $J(x)$ is $U(1)$ current (R-symmetry)

- Requirement of the BRST invariance ($\oint dz e^{-\varphi/2} G$, where G is a supercurrent) of the fermion vertex operators Q_α and $Q_{\bar{\alpha}}$ means

$$G_{int.}(z)\Sigma(w) \sim (z-w)^{-1/2}, \quad G_{int.}(z)\bar{\Sigma}(w) \sim (z-w)^{-1/2}$$

- Finally, introducing an operator $X(z)$

$$\langle X(z)X(w)\rangle = -\log(z-w)$$

and expressing

$$J = i\sqrt{3}\partial X, \quad \Sigma = e^{i\sqrt{3}X/2}, \quad T_{int} = -\frac{(\partial X)^2}{2}$$

one can show that, J, $G_{int.} = G^+_{int.} + G^-_{int.}$ and T_{int} form $N=2$ superconformal algebra

- $D=4$, $N=1$ space-time SUSY implies $N=2$ world-sheet SUSY

- Internal CFT has the central charge equal to $9 = 6_{bos.} + 3_{Ferm.}$
- Representations of $N = 2$ superconformal algebra; there exist series of models with

$$c = \frac{3k}{k+2}, \quad k \geq 1$$

- For each k there exists a finite number of HW representations, specified by the weight h and $U(1)$ charge q

$$h^l_{m,s} = \frac{l(l+2) - m^2}{4(k+2)}, \quad q_{m,s} = -\frac{m}{k+2} + \frac{s}{2}$$

where, l, m and s are integers, constrained as

$$0 \leq l \leq k, \quad 0 \leq |m - s| \leq l, \quad s = 0, 2/\pm 1 \quad (NS)/R$$

- Gepner construction: Take a tensor product several such models so that

$$\sum_{i=1}^{r} c_i = \sum_{i=1}^{r} \frac{3k_i}{k_i + 2} = 9$$

- We'll be very sketchy and give some essential ingredients
- Recall the fusion rules for conformal families $[\phi_i]$

$$[\phi_i] \times [\phi_j] = \sum_k N_{ij}^k [\phi_k]$$

- These are obtained from the S matrix of the particular model and the Verlinde formula (we have χ_i characters $i = 0, ..N - 1$)

$$N_{ij}^k = \sum_{m=0}^{N-1} \frac{S_{im} S_{jm} S_{mk}^\star}{S_{om}}$$

- Simple currents: They are primary fields whose OPE with other primary fields involves one other particular primary field

$$[J_a] \times [\phi_i] = [\phi_j]$$

- Simple currents allow to build modular invariant partition functions form "diagonal" partition functions

$$Z = \sum \chi_i(\tau) M_{ij}^{(a)} \overline{\chi}_j(\tau)$$

- The boundary state in RCFT must be conformally invariant and invariant under extended symmetries
- In general they have form

$$|B_\alpha\rangle = \sum_i B_\alpha^i |\mathcal{B}_i\rangle$$

B_α^i are called reflection coefficients restricted as (Cardy condition)

$$n_{\alpha\beta}^j = \sum_i B_\alpha^i B_\beta^i S_{ij} = 0, 1, 2, \ldots$$

- With simple current extension we have boundary states

$$|B_\alpha^j\rangle = \sum_{k=0}^{L-1} |J^k B_\alpha\rangle$$

L -length of the orbit of the simple current $[J]^L = [1]$.

- Similarly for croscap states.
- Need to impose tadpole cancellation condition.

THANK YOU!!!