Notes on Orientifolds

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Virtual meeting -1

Plan

- Bosonic string orientifolds:
- Short Mention of the boundary state formalism
- Superstrings
- Gepner Models (Tentatively)

Based on

- R. Blumenhagen, E. Plauschin
  Introduction to Conformal Field Theory.
  Springer, 2009
- C.Angelantonj, A.Sagnotti
  Open Strings,
  Phys. Rept. **371** (2002), 1-150, hep-th/0204089
- P. di Vecchia, A. Liccardo
  D Branes in String Theory, I
  hep-th/9912161

- Closed bosonic string $D = 26$, Torus amplitude.
- Line element

$$ds^2 = \frac{1}{\tau_2}|d\sigma_1 + \tau\, d\sigma_2|^2, \quad 0 \le \sigma_{1,2} \le 1$$

$$\omega = \sigma_1 + \tau\sigma_2, \quad ds^2 = \frac{d\omega\, d\bar{\omega}}{\tau_2}$$

- τ - complex structure - parametrizes unequivalent Tori.
- Coordinates on a Torus are periodically identified

$$\omega \sim \omega + m, \quad \omega \sim \omega + n\tau$$

  m, n are integer
- Modular group

$$T: \quad \tau \to \tau + 1$$

$$S: \quad \tau \to -\frac{1}{\tau}$$

- S and T form a modular group $SL(2, Z)$ with

$$S^2 = (ST)^3 = 1$$

- Put a point on a string on a horizontal axis. It propagates upwards in time for $\omega_0 = 2\pi\tau_2$. It shifts in the space by $\omega_1 = 2\pi\tau_1$, where $\tau = \tau_1 + i\tau_2$
- Time translations in CFT are generated by $H = L_0 + \tilde{L}_0 - 2$, space translations are generated by $P = L_0 + \tilde{L}_0$
- We have a path integral

$$Z = Tr[e^{-2\pi\tau_2 H}\, e^{2\pi i\tau_1 P}] = Tr[q^{L_0 - 1}\bar{q}^{L_0 - 1}], \quad q = e^{2\pi i\tau}$$

- Integrating over modular parameter and using $L_0 = \frac{1}{2}\sum_{m=-\infty}^{\infty} \alpha_{-m}\alpha_m$

$$Z = \int_{\mathcal{F}} d^2\tau \frac{1}{q\bar{q}} \int d^{24}p\, e^{-\pi\tau_2 p^2/2}\, Tr[q^N \bar{q}^{\tilde{N}}],$$

- Performing Gaussian integral over the p^2 and using

$$Tr[q^N] = Tr[q^{\sum_{n=1}^{\infty} \alpha_{-n}\alpha_n}] = \prod_{n=1}^{\infty} \frac{1}{1-q^n}$$

we finally get

$$Z = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \frac{1}{\tau_2^{12}(\eta(\tau)\overline{\eta(\tau)})^{24}}$$

where $\eta(\tau)$ is a Dedekind function

$$\eta(\tau) = q^{1/24} \prod_{n=1}^{\infty} (1-q^n)$$

- Under modular transformations

$$\eta(\tau+1) = e^{i\pi/12}\eta(\tau), \quad \eta\left(-\frac{1}{\tau}\right) = \sqrt{-i\tau}\,\eta(\tau)$$

- The fundamental domain $\mathcal{F}$: $|\tau| \geq 1$ and $-\frac{1}{2} \leq \tau_1 \leq \frac{1}{2}$

- From the partition function we can read the spectrum. Expanding
$$\frac{1}{(\eta\bar{\eta})^{24}} = \frac{1}{q\bar{q}}\left(1 + (24)^2 q\bar{q} + \ldots\right) \sim |0,\tilde{0}\rangle + \alpha^i_{-1}\bar{\alpha}^j_{-1}|0,\tilde{0}\rangle + \ldots$$
i.e. we have a tachyon (negative mass), (g_{mn}, B_{mn}, ϕ) -zero mass, + massive fields (positive powers of $q\bar{q}$)
- We have symmetry that interchanges left and right modes:
$$\Omega : q \Leftrightarrow \bar{q}, \quad \Omega^2 = 1$$
- This operation changes the orientation of the string. We can project onto symmetrical states using
$$P_+ = \frac{1+\Omega}{2}$$
- Tachyon survives the projection (no oscillators)
- On the massless level
$$(\alpha^i_{-1}\bar{\alpha}^j_{-1} + \alpha^j_{-1}\bar{\alpha}^i_{-1})|0,\,\tilde{0}\rangle \quad \rightarrow g_{mn}, \phi \quad \textit{present}$$
$$(\alpha^i_{-1}\bar{\alpha}^j_{-1} - \alpha^j_{-1}\bar{\alpha}^i_{-1})|0,\,\tilde{0}\rangle \quad \rightarrow B_{mn} \quad \textit{absent}$$

- Consider relevant partition function

$$Tr\left(q^{L_0-1}(\bar{q})^{\bar{L}_0-1}\left(\frac{1+\Omega}{2}\right)\right)=$$
$$=\frac{1}{2}Tr\left(q^{L_0-1}(\bar{q})^{\bar{L}_0-1}\right)+\frac{1}{2}Tr\left(q^{L_0-1}(\bar{q})^{\bar{L}_0-1}\Omega\right)=\frac{1}{2}\mathcal{T}+\frac{1}{2}\mathcal{K}$$

- A state for a closed string

$$|L,R\rangle=\prod_i\alpha^i\prod_j\bar{\alpha}^j|0,\tilde{0}\rangle,\quad \Omega|L,R\rangle=|R,L\rangle$$

- Therefore for the Klein bottle amplitude

$$\sum_{L,R}\langle L,R|q^{L_0-1}\bar{q}^{\bar{L}_0-1}|R,L\rangle=\sum_L\langle L,L|(q\bar{q})^{L_0-1}|L,L\rangle$$

- Finally since $q\bar{q}=e^{2\pi i(2i\tau_2)}$ and $Tr(q^{L_0-1})=\frac{1}{\eta^{24}(q)}$ we get

- Partition function

$$Z = \frac{1}{2}\int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \frac{1}{\tau_2^{12}(\eta\bar{\eta})^{24}} + \frac{1}{2}\int_0^{\infty} \frac{d\tau_2}{\tau_2^2} \frac{1}{\tau_2^{12}\eta^{24}(2i\tau_2)} = \frac{1}{2}\mathcal{T} + \frac{1}{2}\mathcal{K}$$

- Fundamental polygon of the Klein bottle is similar to Torus, (draw a rectangular with arrows on their sides: three clockwise, one vertical right counterclockwise). The vertical side is $i\tau_2$ -the time, the horizontal side is 1.
- For Torus polygon - bottom and the left sides are clockwise, top and the right sides counterclockwise
- There is an alternative option for the time a) take a Klein bottle polygon, keep the arrows. b) divide it in half using the vertical line at 1/2 c) Lift the right half up by $i\tau_2$. d) Flip the right half to the left so you match the arrows.
- The horizontal side has the length 1/2 the vertical one $2i\tau_2$. We have a cylinder with two crosscaps instead of the boundaries

Torus is a double cover of the Klein bottle when the lattice shift is accompanied with

$$z \to 1 - \bar{z} + i\tau_2$$

In order to obtain the transverse channel amplitude we first change variables $t = 2\tau_2$

$$\mathcal{K} = \int_0^\infty \frac{d\tau_2}{\tau_2^2} \frac{1}{\tau_2^{12}\eta^{24}(2i\tau_2)} = 2^{13} \int_0^\infty \frac{dt}{t^{14}} \frac{1}{\eta^{24}(it)}$$

Then we perform modular transformations $t = 1/l$

$$\tilde{\mathcal{K}} = 2^{13} \int_0^\infty dl \, \frac{1}{\eta^{24}(il)}$$

Consider divergences $\tau_2 \to \infty$ (which is $l \to 0$) and $\tau_2 \to 0$ (which is $l \to \infty$) The first limit is protected by the power of τ_2. We consider the second limit. It is convenient to consider it in the transverse channel. We have

$$\int_0^\infty dl \left(e^{2\pi l} + 24 + \sum_{n=1} a_n e^{-2\pi n l} \right)$$

- Here l is a Schwinger parameter
- Massive states give contribution

$$\int_0^\infty dl\, e^{-M^2 l} = \frac{1}{M^2}$$

- Their contribution is important when $l \leq \frac{1}{M^2}$ but here we are considering large l.
- Therefore we should worry about massless modes
- To this end we should include all diagrams with Euler number zero

$$\chi = 2 - 2h - b - c$$

where h- handles , b - boundaries, c - crosscaps

- Torus ($h = 1$), Klein bottle ($c = 2$), Mobius strip ($b = 1$, $c = 1$), cylinder ($b = 1$, $c = 1$)

Open Strings: Chan-Paton factors

- Open strings have extra (non-dynamical) degrees of freedom attached to the ends. The string state has the form

$$|\vec{m}, i, j\rangle = |\vec{m}\rangle \otimes |i, j\rangle, \quad i, j = 1, ..., \mathcal{N}$$

- The orientation operator Ω acts as follows

$$\Omega|i, j\rangle = \sum_{i,j=1}^{\mathcal{N}} \gamma_{ja}|a, b\rangle(\gamma^{-1})_{bi} \tag{1}$$

- From the requirement $\Omega^2 = 1$ and the relation $\langle a, b|i, j\rangle = \delta_{ai}\delta_{bj}$ we get

$$\gamma^T = \pm\gamma$$

  Therefore we can have only $SO(\mathcal{N})$ and $Sp(\mathcal{N})$ groups.

- Computing the contribution

$$\sum_{i,j}^{\mathcal{N}} \langle i, j||i, j\rangle = \mathcal{N}^2, \quad \sum_{i,j}^{\mathcal{N}} \langle i, j|\Omega|i, j\rangle = Tr(\gamma^T\gamma^{-1}) = \pm\mathcal{N}$$

  Where the upper sign is for $SO(\mathcal{N})$

- A cylinder can be obtained from its double-cover - a torus. Take rectangular with coordinates $(0,0),(0,i\tau_2),(1,i\tau_2),(1,0)$ - torus. Double it horizontally and mode it by

$$z \to -\bar{z}, \quad z \to 2-\bar{z}$$

- Both cylinder and Klein bottle have their modular parameter pure imaginary.
- Direct channel amplitude represents the open string

$$\mathcal{A} = \frac{\mathcal{N}^2}{2}\int_0^\infty \frac{d\tau_2}{\tau_2^{14}} Tr\left(q^{(N-1)/2}\right) = \frac{\mathcal{N}^2}{2}\int_0^\infty \frac{d\tau_2}{\tau_2^{14}}\frac{1}{\eta^{24}(i\tau_2/2)}$$

- The transverse channel represents the close string propagating between two boundaries. To get it first choose $t=\tau_2/2$ and then modular transform (S–transformation) $l=1/t$

$$\tilde{\mathcal{A}} = \frac{\mathcal{N}^2 2^{-13}}{2}\int_0^\infty dl \frac{1}{\eta^{24}(il)}$$

- Mobius strip has a double cover -Torus whose modular parameter has both real and imaginary parts

$$\tau = \frac{1}{2} + \frac{i\tau_2}{2}$$

- One can compute traces directly: Insert Ω in the cylinder amplitude, note that $-\sigma_{open} \sim \pi - \sigma_{open}$ means

$$\Omega \alpha_n \Omega^{-1} = \pm(-1)^n \alpha_n$$

- Often written in terms of modified characters

$$\hat{\chi}(i\tau_2 + \frac{1}{2}) = q^{h-c/24} \sum_k (-1)^k d_k q^k$$

with $q = e^{-2\pi\tau_2}$

$$\mathcal{M} = \frac{\epsilon \mathcal{N}}{2} \int_0^\infty \frac{d\tau_2}{\tau_2^{14}} \frac{1}{\hat{\eta}^{24}(i\tau_2/2 + 1/2)}, \quad \epsilon = \pm 1$$

- The transformation for transverse channel is done using

$$P = TST^2S$$

- The Mobius strip amplitude in the transverse channel

$$\hat{\mathcal{M}} = 2\frac{\epsilon\mathcal{N}}{2}\int_0^\infty dl \frac{1}{\hat{\eta}^{24}(il+1/2)}, \quad \epsilon = \pm 1$$

- For open string (direct channel) -analogous to the closed string

$$Z = \frac{1}{2}\mathcal{A} + \frac{1}{2}\mathcal{M}$$

- To summarize Transverse channel is always closed string propagating between two boundaries (cylinder), two crosscaps (Klein bottle) and boundary and crosscap (Mobius strip)
- Direct channel is closed string (Klein bottle) and open string (cylinder, Mobius strip)
- Tadpole cancellation: expanding η function and collecting the coefficients for singular (massless) terms we get

$$\hat{\mathcal{K}} + \hat{\mathcal{A}} + \hat{\mathcal{M}} \sim (2^{13} + 2^{-13}\mathcal{N}^2 - 2\epsilon\mathcal{N})$$

It is zero if $\epsilon = 1$ and $\mathcal{N} = 2^{13}$ i.e. the gauge group is $SO(2^{13})$.

- It is similar to the one we just described.
- Consider Dp brane. In terms of open string coordinates we have

$$\partial_\sigma X^a|_{\sigma=0} = 0, \quad a = 0, 1..., p \quad (N)$$

$$X^i|_{\sigma=0} = y^i, \quad i = p+1, ..., d-1 \quad (D)$$

- Let us go to the closed string channel. To this end $(\sigma, \tau) \Leftrightarrow (\tau, \sigma)$. Therefore we get for the closed string coordinate

$$\partial_\tau X^a|_{\tau=0}|B_X\rangle = 0, \quad a = 0, 1..., p$$

$$X^i|_{\tau=0}|B_x\rangle = y^i|B_x\rangle, \quad i = p+1, ..., d-1$$

Using

$$X^\mu = q^\mu + 2\alpha' p^\mu + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \left(\frac{\alpha_n^\mu}{n} e^{-2in(\tau-\sigma)} + \frac{\bar{\alpha}_n^\mu}{n} e^{-2in(\tau+\sigma)} \right)$$

- We get in terms of closed string oscillators

$$(\alpha_n^a + \bar{\alpha}_{-n}^a)|B_x\rangle = 0, \quad (\alpha_n^i - \bar{\alpha}_{-n}^i)|B_x\rangle = 0$$

$$p^a|B_x\rangle = 0, \quad (q^i - y^i)|B_x\rangle = 0$$

- Introducing

$$S^{\mu\nu} = (\eta^{ab}, -\delta^{ij})$$

We can write a solution

$$|B_x\rangle = N_p \delta^{(d-p-1)}(q^i - y^i) \prod_{n=1}^{\infty} e^{-\frac{1}{n}\alpha_{-n} S \bar{\alpha}_{-n}} |0, \tilde{0}\rangle$$

- Using the closed string propagator D one can write a cylinder amplitude

$$\langle B_x|D|B_x\rangle$$

Crosscap state

- Similarly for closed string coordinate we have

$$X(\tau,\sigma)|C\rangle = X(\tau,\sigma+\pi)|C\rangle$$

$$\partial_\sigma X(\tau,\sigma)|C\rangle = \partial_\sigma X(\tau,\sigma+\pi)|C\rangle, \quad \partial_\tau X(\tau,\sigma)|C\rangle = -\partial_\tau X(\tau,\sigma+\pi)|C\rangle$$

- This leads to

$$(\alpha_n + (-1)^n \bar{\alpha}_{-n})|C\rangle = 0$$

- It is solved by

$$|C\rangle = Nexp\left(-\sum_{k=1}^{\infty} \frac{(-1)^k}{k} \alpha_{-k}\bar{\alpha}_{-k}\right)|0,\tilde{0}\rangle$$

THANK YOU!!!