Supersymmetric Reducible Higher-Spin Multiplets in Various Dimensions

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- Motivation
- Field equations for reducible representations of the Poincare group: Bosons and Fermions
- BRST constructions, supersymmetry
- Supersymmetry, Lagrangians, lower spin examples
- Conclusions, open problems

Based on

- D. Sorokin, M.T., Nuclear Physics **B 929** 216, 2018

- Consistent and highly nontrivial interacting theory of massless Higher Spin fields
  On AdS_4: M.A. Vasiliev, Phys.Lett. **B 285** 225, 1992;
  On AdS_D: M.A. Vasiliev, Phys.Lett **B 567**, 139, 2003
- A connection with (Super)String Theory - a theory on massive higher spin fields on a flat background
- Supersymmetric Higher Spin Theories are very interesting but relatively less explored
- Higher Spin theories are already nontrivial at a free level
- Free theory is usually formulated on a constant curvature background: flat, de Sitter, anti de Sitter - one has a sufficient abelian gauge invariance of a free action for a field with $s \geq 3$

General set up

- BRST formalism borrowed from the Open String Field Theory
- Is off -shell, gauge invariant, leads to the required field equations, describes correct spectrum
- Leads to a description for reducible representations of the Poincare group
- The spectrum is “larger” than for the case of irreducible (Fronsdal) modes, but the BRST charge and therefore the Lagrangian are much simpler
- An analog of Virasoro constraints can be obtained by formally taking $\alpha' \to \infty$ limit in the free equations for the open superstring

Example: E-M Field

- The field $A_\mu(x)$ satisfies in the Lorentz gauge the equation $\partial^\mu A_\mu(x) = 0$ and the massless Klein - Gordon equation $\Box A_\mu(x) = 0$
- Let us introduce an auxiliary field $C(x)$

$$\partial^\mu A_\mu(x) = C(x)$$

- To make it gauge invariant under $\delta A_\mu(x) = \partial_\mu \lambda(x)$ we have

$$\delta C(x) = \Box \lambda(x)$$

- Finally a gauge invariant Klein - Gordon equation

$$\Box A_\mu(x) = \partial_\mu C(x)$$

- The field equations are Lagrangian

$$\mathcal{L} = -\frac{1}{2}(\partial^\mu A^\nu(x))(\partial_\mu A_\nu(x)) + C(x)\partial^\mu A_\mu(x) - \frac{1}{2}(C(x))^2$$

- After excluding of the field $C(x)$ we get $\mathcal{L} = -\frac{1}{4}F^{\mu\nu}(x)F_{\mu\nu}(x)$

Example: A linearized gravity and a scalar

- Similarly to the vector field we have for Klein - Gordon for $s = 2$

$$\Box g_{\mu\nu}(x) = \partial_\mu C_\nu(x) + \partial_\nu C_\mu(x)$$

which is gauge invariant under

$$\delta g_{\mu\nu}(x) = \partial_\mu \lambda_\nu(x) + \partial_\nu \lambda_\mu(x), \quad \delta C_\mu(x) = \Box \lambda_\mu(x)$$

- Transversality equation is now

$$\partial^\nu g_{\mu\nu}(x) - \partial_\mu D(x) = C_\mu(x)$$

where to make it gauge invariant we introduced a new field $D(x)$ which transforms as $\delta D(x) = \partial^\mu \lambda_\mu(x)$

- Finally a gauge invariant field equation for $D(x)$

$$\Box D(x) = \partial^\mu C_\mu(x)$$

- The field equations are Lagrangian again

$$\mathcal{L} = -\frac{1}{2}(\partial^\mu g^{\nu\rho})(\partial_\mu g_{\nu\rho}) + 2C^\mu \partial^\nu g_{\mu\nu} - C^\mu C_\mu + (\partial^\mu D)(\partial_\mu D) + 2D\partial^\mu C_\mu$$

- Describes two physical fields with spins 2 and 0, contained in $g_{\mu\nu}(x)$

A general case. Bosons

- We have always three fields $\phi^{(n)}(x)$, $C^{(n-1)}(x)$ and $D^{(n-2)(x)}$
- A gauge invariant description of spins $n, n-2, .., 1/0$. (all traces)
- The field equations

$$
\begin{aligned}
\Box\phi_{\mu_1,\ldots\mu_n}(x) &= \partial_{(\mu_1}C_{\mu_2,\ldots,\mu_n)}(x) \\
\partial^{\mu_n}\phi_{\mu_1,\ldots,\mu_{n-1}\mu_n}(x) - \partial_{(\mu_{n-1}}D_{\mu_1,\ldots,\mu_{n-2})}(x) &= C_{\mu_1,\ldots,\mu_{n-1}}(x) \\
\Box D_{\mu_1,\ldots\mu_{n-2}}(x) &= \partial^{\mu_{n-1}}C_{\mu_1,\ldots,\mu_{n-1}}(x)
\end{aligned}
$$

- The equations are gauge invariant with an unconstrained parameter $\lambda_{\mu_1,\ldots,\mu_{n_1}}(x)$

$$
\begin{aligned}
\delta\phi_{\mu_1,\ldots\mu_n}(x) &= \partial_{(\mu_1}\lambda_{\mu_2,\ldots,\mu_n)}(x) \\
\delta C_{\mu_1,\ldots,\mu_{n-1}}(x) &= \Box\lambda_{\mu_1,\ldots,\mu_{n-1}}(x) \\
\delta D_{\mu_1,\ldots\mu_{n-2}}(x) &= \partial^{\mu_{n-1}}\lambda_{\mu_1,\ldots,\mu_{n-1}}(x)
\end{aligned}
$$

- These equations are Lagrangian

- The fields are totally symmetrical: introduce one set of oscillators

$$[\alpha^\mu, \alpha^{\nu+}] = \eta^{\mu\nu}, \quad |\Phi^{\phi,D,C}\rangle = \frac{1}{k!}\varphi^{(\phi,D,C)}_{\mu_1,..\mu_k}(x)\alpha^{\mu_1+}...\alpha^{\mu_k}|0\rangle$$

- d'Alembertian, gradient and divergence operators are realised as

$$l_0 = p \cdot p, \quad l = \alpha \cdot p, \quad l^+ = \alpha^+ \cdot p$$

with $A \cdot B = A^\mu B_\mu$ and $p_\mu = -i\partial_\mu$

- Compute the algebra, introduce ghosts

$$\{c_0, b_0\} = \{c^+, b\} = \{c, b^+\} = 1,$$

and build a nilpotent BRST charge

$$Q = c_0 l_0 + c^+ l + c l^+ - c^+ c b_0$$

- Ghost number: All c have ghost number $+1$, all b ghost number -1, the rest have ghost number zero
- Oscillator number

$$N = \alpha^{+} \cdot \alpha + c^{+} b + b^{+} c, \quad [N, Q] = 0$$

- The general state has a form

$$|\Phi^{(n)}\rangle = |\phi^{(n)}\rangle + c_0 b^{+} |C^{(n-1)}\rangle + c^{+} b^{+} |D^{(n-2)}\rangle$$

- The BRST invariant Lagrangian

$$\mathcal{L} = \int dc_0 \langle \Phi | Q | \Phi \rangle, \quad \delta |\Phi\rangle = Q b^{+} |\lambda\rangle$$

- Eliminating ghost variables we get

$$\begin{aligned} \mathcal{L} &= \langle \phi | p \cdot p | \phi \rangle - \langle D | p \cdot p | D \rangle + \langle C || C \rangle - \\ &- \langle \phi | \alpha^{+} \cdot p | C \rangle - \langle C | \alpha \cdot p | \phi \rangle + \langle D | \alpha \cdot p | C \rangle + \langle C | \alpha^{+} \cdot p | D \rangle \end{aligned}$$

- The spin $\frac{3}{2}$ field $\Psi^a_\mu(x)$, where a is a spinorial index.
- It should satisfy transversality condition and to maintain gauge invariance we introduce an extra field $\chi^a(x)$

$$\partial^\mu \Psi_\mu(x) + \gamma^\nu \partial_\nu \chi(x) = 0$$

with

$$\delta\Psi_\mu(x) = \partial_\mu \tilde{\lambda}(x), \quad \delta\chi(x) = -\gamma^\nu \partial_\nu \tilde{\lambda}(x)$$

- The gauge invariant Dirac equation

$$\gamma^\nu \partial_\nu \Psi_\mu(x) + \partial_\mu \chi(x) = 0$$

- The equations are again Lagrangian

$$L_R = -i\bar{\Psi}^\nu \gamma^\mu \partial_\mu \Psi_\nu - i\bar{\Psi}^\mu \partial_\mu \chi + i\bar{\chi}\partial^\mu \Psi_\mu + i\bar{\chi}\gamma^\mu \partial_\mu \chi$$

- Describes spins $\frac{3}{2}$ and $\frac{1}{2}$ - gamma trace

- The fermionic Lagrangian

$$\begin{aligned} L_F &= -i\bar{\Psi}\gamma^\mu\partial_\mu\Psi - i\,n\bar{\Psi}\partial\chi + i\,n\bar{\chi}\partial\cdot\Psi + i\,n(n-1)\bar{\Sigma}\gamma^\mu\partial_\mu\Sigma \\ &+ i\,n\bar{\chi}\gamma^\mu\partial_\mu\chi - i\,n(n-1)\bar{\chi}\partial\Sigma + i\,n(n-1)\bar{\Sigma}\partial\cdot\chi\,. \end{aligned}$$

- Gauge transformations

$$\delta\Psi^{(n)} = \partial\,\tilde{\lambda}^{(n-1)}, \quad \delta\Sigma^{(n-2)} = \partial\cdot\tilde{\lambda}^{(n-1)}, \quad \delta\chi^{(n-1)} = -\gamma^\nu\partial_\nu\tilde{\lambda}^{(n-1)}$$

- Equations of motion

$$\begin{aligned} &\gamma^\nu\partial_\nu\Psi \;+\; \partial\chi = 0\,, \\ &\partial\cdot\Psi \;-\; \partial\Sigma + \gamma^\nu\partial_\nu\chi = 0\,, \\ &\gamma^\nu\partial_\nu\Sigma \;+\; \partial\cdot\chi = 0. \end{aligned}$$

- Fermionic “triplet” contains a physical field $\Psi^{(n)}(x)$ and two auxiliary fields $\Sigma^{(n-2)}(x)$ and $\chi^{(n-1)}(x)$.

- Observation: Fermions are again totally symmetric, we have only α^μ oscillators for them
- Crucial difference: Dirac operator brings a bosonic ghost zero mode
- Use open superstring field theory as a hint
- Write BRST charges in bosonic (B) and fermionic (F) sectors and find an operator (SUSY) that maps them into each other

$$Q_F \mathcal{Q} = \mathcal{Q} Q_B$$

- The sectors have bosonic α^μ -oscillators the same, but ψ^μ oscillators are different. SUSY operator only acts on the ψ^μ and on corresponding (bosonic) β, γ ghosts
- As a result we have mixed symmetry fields at least in one the sectors

- We have a Lagrangian

$$L_{tot.} = \langle \Phi_B | Q_B | \Phi_B \rangle + \langle \Phi_F | Q_F | \Phi_F \rangle$$

- Invariant under gauge transformations

$$\delta |\Phi_B\rangle = Q_B |\Lambda_B\rangle, \quad \delta |\Phi_F\rangle = Q_F |\Lambda_F\rangle$$

- Invariant under supersymmetry transformations

$$\delta \langle \Phi_B | = \langle \Phi_F | \epsilon \mathcal{Q}, \qquad \delta |\Phi_F\rangle = \epsilon \mathcal{Q} |\Phi_B\rangle.$$

  provided the SUSY generator $\mathcal{Q}$ satisfies

$$Q_F \, \mathcal{Q} = \mathcal{Q} \, Q_B$$

- Consideration in OSFT: Y.Kazama, A.Neveu, H.Nicolai,P.West Nucl.Phys. **B 278**, 833 1986 contains an infinite number of oscillators, fields and pictures.

A total set of operators

- The simplest choice: keep only α_μ and (b, c) oscillators in the fermionic sector. Also a pair of bosonic ghosts (for the Dirac operator)

$$[\gamma_0, \beta_0] = i$$

- In the bosonic sector again (α, b, c) plus one set of fermionic oscillators ψ_μ and the bosonic ghosts γ and antighosts β

$$\{\psi^\mu, \psi^{\nu +}\} = \eta^{\mu\nu}, \qquad [\gamma, \beta^+] = [\gamma^+, \beta] = i\,.$$

- Divergence operators

$$l = p \cdot \alpha, \quad g = p \cdot \psi$$

- Gradients symmetrized w.r.t "alpha" indexes and gradients antisymmetrized w.r.t "psi" indexes

$$l^+ = p \cdot \alpha^+, \quad g^+ = p \cdot \psi^+$$

- Dirac operator (in F sector) and the d'Alembertian

$$g_0 = \frac{1}{\sqrt{2}} \gamma \cdot p, \quad l_0 = p \cdot p$$

BRST charges,SUSY

- The nilpotent BRST charge in the B sector

$$Q_B = c_0 l_0 + \tilde{Q}_B - M_B b_0 \ ,$$

$$\tilde{Q}_B = c^+ l + c l^+ + \gamma^+ g + \gamma g^+, \quad M_B = c^+ \, c + \gamma^+ \, \gamma$$

- The nilpotent BRST charge in the F sector

$$Q_F \ = \ c_0 \, l_0 + \gamma_0 \, g_0 + \tilde{Q}_R - M_F b_0 - \frac{1}{2} \, \gamma_0^2 \, b_0$$

$$\tilde{Q}_F = c^+ l + c l^+, \quad M_F = c^+ \, c,$$

- The solution to $Q_F \mathcal{Q} = \mathcal{Q} Q_B$ has the form

$$\mathcal{Q} = {}_B\langle 0| \exp\left(\frac{1}{\sqrt{2}} \gamma \cdot \psi + \frac{i}{2} \gamma \beta - i \gamma \beta_0 \right) |\tilde{0}\rangle_F$$

- A comment on ghost zero modes: b_0 is always an annihilator, whereas

$$\beta_0 |0\rangle_F = \gamma_0 |\tilde{0}\rangle_F = 0$$

$$(|\tilde{0}\rangle_F)^+ = {}_F\langle 0|, \quad (|0\rangle_F)^+ = {}_F\langle \tilde{0}|, \quad {}_F\langle 0||0\rangle_F = {}_F\langle \tilde{0}||\tilde{0}\rangle_F = 1$$

- A field in the B sector

$$|\Phi^B\rangle \;=\; |\Phi_1^B\rangle \;+\; c_0|\Phi_2^B\rangle$$

- The Lagrangian in the B sector

$$L_B = \langle\Phi_1^B|l_0|\Phi_1^B\rangle - \langle\Phi_2^B|\tilde{Q}_B|\Phi_1^B\rangle - \langle\Phi_1^B|\tilde{Q}_B|\Phi_2^B\rangle + \langle\Phi_2^B|M_B|\Phi_2^B\rangle\,.$$

- One can use a part of the initial BRST symmetry and truncate a field in the F sector to

$$|\Phi^F\rangle \;=\; |\Phi_1^F\rangle \;+\; \gamma_0\,|\Phi_2^F\rangle \;+\; 2\,c_0\,g_0\,|\Phi_2^F\rangle$$

- The Lagrangian in the F sector

$$L_F = \langle\Phi_1^F|g_0|\Phi_1^F\rangle + \langle\Phi_2^F|\tilde{Q}_F|\Phi_1^F\rangle + \langle\Phi_1^F|\tilde{Q}_F|\Phi_2^F\rangle - 2\langle\Phi_2^F|M_F g_0|\Phi_2^F\rangle\,.$$

- Expanding $\mathcal{Q}(\beta_0)$ in power series of β_0 one can get rid of $|\tilde{0}_R\rangle$ and we finally obtain SUSY transformations

$$\delta|\Phi_1^B\rangle = U^+\,\epsilon^+|\Phi_1^F\rangle - \gamma^+\,U^+\,\epsilon^+|\Phi_2^F\rangle, \qquad \delta|\Phi_2^B\rangle = 2\,U^+ g_0\,\epsilon^+|\Phi_2^F\rangle,$$
$$\delta|\Phi_1^F\rangle = -2\epsilon\;g_0\,U|\Phi_1^B\rangle + \epsilon\,\gamma\,U|\Phi_2^B\rangle, \qquad \delta|\Phi_2^F\rangle = \epsilon\;U|\Phi_2^B\rangle$$

  with

$$U = {}_B\langle 0|\;\exp\left(\frac{1}{\sqrt{2}}\gamma\cdot\psi + \frac{i}{2}\gamma\beta\right)|0\rangle_F.$$

- SUSY closes on-shell in $D = 3, 4, 6, 10$, in both sectors. No pictures
- A comment: constraints on $|\Phi_{B/F}\rangle$:

$$N_{gh}|\Phi_{B/F}\rangle = 0$$

  GSO projection

$$P_B \;=\; \frac{1}{2}\left[1\;-\;(-1)^{(\psi^\dagger\,\psi + i\gamma^\dagger\,\beta - i\,\gamma\,\beta^\dagger)}\right]$$
$$P_F \;=\; \frac{1}{2}\left[1\;+\;\gamma_*\,(-1)^{i\,\gamma_0\,\beta_0}\right],$$

- The fields are of the type $|X^{(a,b)}\rangle$, where a is a number of α_μ^+ oscillators, and $b = 0, 1$ is a number of ψ_μ^+ oscillators.
- The fermionic sector

$$|\Phi_1^F\rangle = |\Psi^{(n,0)}\rangle + c^+ b^+ |\Sigma^{(n-2,0)}\rangle, \quad |\Phi_2^F\rangle = b^+ |\chi^{(n-1,0)}\rangle$$

- Gauge transformations with a parameter $|\Lambda_1^F\rangle = b^+|\tilde{\lambda}\rangle$
- The bosonic sector contains mixed symmetry fields

$$|\Phi_1^B\rangle = |\phi^{(n,1)}\rangle + c^+ b^+ |D^{(n-2,1)}\rangle + \gamma^+ b^+ |B^{(n-1,0)}\rangle + c^+ \beta^+ |A^{(n-1,0)}\rangle,$$

$$|\Phi_2^B\rangle = b^+ |C^{(n-1,1)}\rangle + \beta^+ |E^{(n,0)}\rangle + c^+ b^+ \beta^+ |F^{(n-2,0)}\rangle.$$

- Parameters of gauge transformations

$$|\Lambda_1^B\rangle = b^+ |\lambda^{(n-1,1)}\rangle + \beta^+ |\rho^{(n,0)}\rangle + \beta^+ c^+ b^+ |\xi^{(n-2,0)}\rangle,$$

$$|\Lambda_2^B\rangle = b^+ \beta^+ |\tau^{(n-1,0)}\rangle.$$

Examples ($n = 0$). $N = 1$ Maxwell supermultiplet

- Bosonic sector

$$|\phi\rangle = A_\mu(x)\psi^{\mu+}|0\rangle_B, \quad |E\rangle = \beta^+ E(x)|0\rangle_B$$

- Fermionic sector

$$|\Psi\rangle = \Psi(x)|0\rangle_F$$

- Total Lagrangian

$$L = -A^\mu \Box A_\mu + 2E\partial^\mu A_\mu + E^2 - i\bar{\Psi}\gamma^\mu\partial_\mu\Psi$$

- SUSY transformations

$$\delta A_\mu(x) = i\bar{\Psi}(x)\gamma_\mu\epsilon, \quad \delta\Psi(x) = -\epsilon\gamma^\nu\gamma^\mu\partial_\nu A_\mu(x) - \epsilon E(x), \quad \delta E(x) = 0$$

- Auxiliary field $E(x) = -\partial_\mu A^\mu(x)$. It is SUSY invariant due to fermionic e.o.m.

Examples (n=1). $N = 1$ SUGRAs (+ antisymmetric tensor supermultiplets)

- Bosonic

$$|\Phi_1^B\rangle = (\phi_{\nu,\mu}(x)\psi^{\nu+}\alpha^{\mu+} + ic^+\beta^+A(x) + b^+\gamma^+B(x))|0\rangle_B$$

$$|\Phi_2^B\rangle = (ib^+C_\nu(x)\psi^{\nu+} + \beta^+E_\mu(x)\alpha^{\mu+})|0\rangle_B$$

- Fermionic sector

$$|\Phi_1^F\rangle = \Psi_\mu(x)\alpha^{\mu+}|0\rangle_F, \quad |\Phi_2^F\rangle = \frac{b^+}{\sqrt{2}}\chi(x)|0\rangle_F$$

- Physical fields: $\phi_{\nu,\mu} = (g_{(\mu\nu)}, B_{[\mu\nu]}, \varphi)$ and $\Psi_\mu = (\Psi'_\mu, \chi')$ with $\gamma^\mu\Psi'_\mu = 0$
- An irreducible $N = 1,\ D = 10$ SUGRA supermultiplet
- $N = 1,\ D = 4$ SUGRA $(g_{\mu\nu}, \Psi'_\mu)$ + chiral $(B_{\mu\nu}, \varphi, \chi')$
- $N = (1,0), D = 6$ SUGRA $(g_{\mu\nu}, B^+_{\mu\nu}, \Psi'_\mu)$ + tensor $(B^-_{\mu\nu}, \varphi, \chi')$

Examples ($n = 1$). $N = 1$ SUGRAs (+ antisymmetric tensor supermultiplets)

- The Lagrangian in the bosonic sector

$$
\begin{aligned}
L_B \quad &= -\phi^{\nu,\mu}\Box\phi_{\nu,\mu} + B\Box A + A\Box B \\
&+E^{\mu}\partial_{\mu}B + C^{\nu}\partial^{\mu}\phi_{\nu,\mu} + C^{\nu}\partial_{\nu}A + E^{\mu}\partial^{\nu}\phi_{\nu,\mu} \\
&-B\partial_{\alpha}E^{\mu} - \phi^{\nu,\mu}\partial_{\mu}C_{\nu} - A\partial_{\nu}C^{\nu} - \phi^{\nu\mu}\partial_{\nu}E_{\mu} \\
&+C^{\nu}C_{\nu} + E^{\mu}E_{\mu}\,.
\end{aligned}
$$

- The Lagrangian in the fermionic sector

$$L_F = -i\bar{\Psi}^{\mu}\gamma^{\nu}\partial_{\nu}\Psi_{\mu} - i\bar{\Psi}^{\mu}\partial_{\mu}\chi + i\bar{\chi}\partial_{\mu}\Psi^{\mu} + i\bar{\chi}\gamma^{\nu}\partial_{\nu}\chi,$$

- SUSY transformations

$$\delta\phi_{\nu,\mu}(x) = i\bar{\Psi}_{\mu}(x)\gamma_{\nu}\,\epsilon, \quad \delta C_{\nu}(x) = -i(\partial_{\mu}\bar{\chi}(x))\gamma^{\mu}\gamma_{\nu}\,\epsilon, \quad \delta B(x) = -i\bar{\chi}(x)\,\epsilon,$$

$$\delta\Psi_{\mu}(x) = -\gamma^{\nu}\gamma^{\rho}\epsilon\,\partial_{\nu}\phi_{\rho,\mu}(x) - \epsilon E_{\mu}(x), \quad \delta\chi(x) = -\gamma^{\nu}\epsilon\, C_{\nu}(x)\,.$$

Examples ($n = 1$). $N = 1$ SUGRAs (+ antisymmetric tensor supermultiplets)

- The gauge transformations in the bosonic sector

$$\delta\phi_{\nu,\mu}(x) = \partial_\mu\lambda_\nu(x) + \partial_\nu\rho_\mu(x),$$

$$\delta A(x) = -\partial^\nu\rho_\nu(x) - \tau(x), \quad \delta B(x) = -\partial^\nu\lambda_\nu(x) + \tau(x),$$
$$\delta C_\nu(x) = -\Box\lambda_\nu(x) + \partial_\nu\tau(x), \quad \delta E_\mu(x) = -\Box\rho_\mu(x) - \partial_\mu\tau(x).$$

- The gauge transformations in the fermionic sector

$$\delta\Psi^a_\mu(x) = \partial_\mu\tilde{\lambda}^a(x), \qquad \delta\chi^a(x) = -(\gamma^\mu)^a{}_b\partial_\mu\tilde{\lambda}^b(x).$$

- Using this gauge freedom and equations of motion one can eliminate all auxiliary fields. As a result of the complete gauge fixing one is left with only transversal components of $\phi_{i,j}(x)$ and $\Psi_i(x)$

$$\Box\phi_{i,j}(x) = 0, \quad \gamma^\mu\partial_\mu\Psi_i(x) = 0$$

- The Bosonic Lagrangian (”alpha” - indexes are implicit)

$$\begin{aligned} L_B &= -\phi^\nu \Box \phi_\nu + n(n-1) D \Box D + n B \Box A + n A \Box B \\ &-2nB\partial\cdot E + 2n(n-1) D^\nu \partial \cdot C_\nu + 2n C^\nu \partial \cdot \phi_\nu \\ &-2n(n-1) F \partial \cdot B + 2n C^\nu \partial_\nu A + 2 E \partial^\nu \phi_\nu - 2n(n-1) F \partial^\nu D_\nu \\ &+ n C^\nu C_\nu + E^2 - n(n-1) F^2, \end{aligned}$$

- The fields

$$\phi^{(n,1)}(x)\ , D^{(n-2,1)}(x)\ , A^{(n-1,0)}(x)\ , B^{(n-1,0)}(x)$$
$$C^{(n-1,1)}(x)\ , E^{(n,0)}(x)\ , F^{(n-2,0)}(x)$$

- The Fermionic Lagrangian

$$\begin{aligned} L_F &= -i\bar\Psi\gamma^\mu\partial_\mu\Psi - in\bar\Psi\partial\chi + in\bar\chi\partial\cdot\Psi + in(n-1)\bar\Sigma\gamma^\mu\partial_\mu\Sigma \\ &+ in\bar\chi\gamma^\mu\partial_\mu\chi - in(n-1)\bar\chi\partial\Sigma + in(n-1)\bar\Sigma\partial\cdot\chi\,. \end{aligned}$$

- The fields

$$\Psi^{(n,0)}(x),\ \chi^{(n-1,0)}(x),\ \Sigma^{(n-2,0)}(x)$$

- SUSY transformations for the fermionic fields

$$\begin{aligned}
\delta\Psi_{\mu_1\ldots\mu_n}(x) &= -\gamma^\rho\gamma^\nu\,\epsilon\,\partial_\rho\phi_{\nu,\mu_1\ldots\mu_n}(x) - \epsilon E_{\mu_1\ldots\mu_n}(x),\\
\delta\Sigma_{\mu_1,\ldots,\mu_{n-2}}(x) &= -\gamma^\rho\gamma^\nu\,\epsilon\,\partial_\rho D_{\nu,\mu_1\ldots\mu_{n-2}}(x) - \epsilon F_{\mu_1\ldots\mu_{n-2}}(x),\\
\delta\chi_{\mu_1\ldots\mu_{n-1}}(x) &= -\gamma^\nu\,\epsilon\,C_{\nu,\mu_1,\ldots\mu_{n-1}}(x),
\end{aligned}$$

- SUSY transformations for the bosonic fields

$$\begin{aligned}
\delta\phi_{\nu,\mu_1\ldots\mu_n}(x) &= i\,\bar\Psi_{\mu_1,\ldots,\mu_n}(x)\gamma_\nu\,\epsilon,\\
\delta D_{\nu,\mu_1\ldots\mu_{n-2}}(x) &= i\,\bar\Sigma_{\mu_1\ldots\mu_{n-2}}(x)\gamma_\nu\,\epsilon,\\
\delta C_{\nu,\mu_1\ldots\mu_{n-1}}(x) &= -i\,\partial_\rho\bar\chi_{\mu_1,\ldots,\mu_{n-1}}(x)\gamma^\rho\gamma_\nu\,\epsilon,\\
\delta B_{\mu_1\ldots\mu_{n-1}}(x) &= -i\,\bar\chi_{\mu_1\ldots\mu_{n-1}}(x)\,\epsilon,\\
\delta A_{\mu_1\ldots\mu_{n-1}}(x) &= \delta E_{\mu_1\ldots\mu_n}(x) = \delta F_{\mu_1\ldots\mu_{n-2}}(x) = 0.
\end{aligned}$$

- Can be written in any dimension, but the algebra is closed only in $D = 3, 4, 6, 10$.

Conclusions, Open Problems

- Massive SUSY theories for the dimensions $D \geq 3$
- Deformation of these theories to (anti)de Sitter spaces
- Including interactions
- Further connection with the String Theory
- Many other questions

THANK YOU!!!