Calculation of Gauge Thresholds in Heterotic Compactifications

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Plan of the talk

- Motivation
- Torus amplitude, Partition Functions
- Calculation of Gauge Thresholds
- NonSUSY heterotic $SO(16) \otimes SO(16)$ superstring
- Conclusions

Based on

- V. S. Kaplunovsky, NPB **307**, 1988, 145
- E. Kiritsis, C. Kounnas, M. Petropoulos, J. Rizos, NPB **483**, 1997, 141
- C. Angelantonj, I .Florakis, M. T., PLB **736**, 2014, 365; NPB **900**, 2015, 170

Motivation

- Quantum Theory of Gravity, Unification of Fundamental Interactions.
- Particle Physics: MSSM still has unanswered questions like origin of values of coupling constants, of masses etc.
- We have bosonic string (open or closed). Lives in $D = 26$ space-time dimensions. Contains a tachyon. Has no fermions.
- Superstrings: Perturbatively five of them. They live in $D = 10$. They are connected by various types of dualities. Vacua include nonperturbative objects.
- These theories are supersymmetric. SUSY removes tachyon and has many other nice features.
- But SUSY (if exists) is spontaneously broken. How? A stringy mechanism is extremely hard to implement.

- In non SUSY vacua one generically has a nonzero dilaton tadpole (one loop).
- To cure this problem one has to take a back reaction on the metric into account at two-loop level: W. Fischler, L.Susskind (1984). Hard to implement in practice.
- Alternatively one can try to work around a “wrong” flat vacuum: E.Dudas, M.Nicolosi, G.Pradisi, A.Sagnotti (2004). Also hard. No conventional perturbation theory in this case.
- We consider NonSUSY heterotic $SO(16) \otimes SO(16)$ string. Nontachyonic vacuum (stable). No Wilson lines.
- Gauge threshold corrections. General approach: V.Kaplunovsky (1988). Valid either for SUSY/Non SUSY without tachyons.

- In field theory one loop partition function - spectrum. Example of a massive scalar field

$$S = \int d^D x (\partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi)$$

- Vacuum energy

$$e^{-\Gamma} = \int D\phi e^{-S_E}, \quad \Gamma \sim V \int_\epsilon^\infty \frac{dt}{t^{D/2+1}} e^{-tm^2}$$

where V is a volume of space–time, t is a Schwinger parameter and and ϵ is an ultraviolet cut off.

- When we have many particles then

$$e^{-tm^2} \to Str \left(e^{-tm^2} \right)$$

- Closed bosonic string $D = 26$, Torus amplitude.
- Line element

$$ds^2 = \frac{1}{\tau_2}|d\sigma_1 + \tau d\sigma_2|^2, \quad 0 \le \sigma_{1,2} \le 1$$

$$\omega = \sigma_1 + \tau\sigma_2, \quad ds^2 = \frac{d\omega d\bar{\omega}}{\tau_2}$$

- τ - complex structure - parametrizes unequivalent Tori.
- Coordinates on a Torus are periodically identified

$$\omega \sim \omega + m, \quad \omega \sim \omega + n\tau$$

  m, n are integer
- Modular group

$$T: \quad \tau \to \tau + 1$$
$$S: \quad \tau \to -\frac{1}{\tau}$$

- S and T form a modular group $SL(2, Z)$ with

$$S^2 = (ST)^3 = 1$$

- Put a point on a string on a horizontal axis. It propagates upwards in time for $\omega_0 = 2\pi\tau_2$. It shifts in the space by $\omega_1 = 2\pi\tau_1$, where $\tau = \tau_1 + i\tau_2$
- Time translations in CFT are generated by $H = L_0 + \tilde{L}_0 - 2$, space translations are generated by $P = L_0 + \tilde{L}_0$
- We have a path integral

$$Z = Tr[e^{-2\pi\tau_2 H}\, e^{2\pi i\tau_1 P}] = Tr[q^{L_0-1}\bar{q}^{L_0-1}], \quad q = e^{2\pi i\tau}$$

- Integrating over modular parameter and using $L_0 = \frac{1}{2}\sum_{m=-\infty}^{\infty}\alpha_{-m}\alpha_m$

$$Z = \int_{\mathcal{F}} d^2\tau \frac{1}{q\bar{q}} \int d^{24}p\, e^{-\pi\tau_2 p^2/2} Tr[q^N \bar{q}^{\tilde{N}}],$$

Torus Amplitude

- Performing Gaussian integral over the p^2 and using

$$Tr\left[q^N\right] = Tr\left[q^{\sum_{n=1}^{\infty}\alpha_{-n}\alpha_n}\right] = \prod_{n=1}^{\infty}\frac{1}{1-q^n}$$

we finally get

$$Z = \int_{\mathcal{F}}\frac{d^2\tau}{\tau_2^2}\frac{1}{\tau_2^{12}(\eta(\tau)\overline{\eta(\tau)})^{24}}$$

where $\eta(\tau)$ is a Dedekind function

$$\eta(\tau) = q^{1/24}\prod_{n=1}^{\infty}(1-q^n)$$

- Under modular transformations

$$\eta(\tau+1) = e^{i\pi/12}\eta(\tau), \quad \eta\left(-\frac{1}{\tau}\right) = \sqrt{-i\tau}\,\eta(\tau)$$

- The fundamental domain $\mathcal{F}$: $|\tau| \geq 1$ and $-\frac{1}{2} \leq \tau_1 \leq \frac{1}{2}$

$SO(2n)$ characters

- Let us introduce orthogonal decompositions. In NS sector

$$O_{2n} = \frac{\theta_3^n + \theta_4^n}{2\eta^n}, \quad V_{2n} = \frac{\theta_3^n - \theta_4^n}{2\eta^n}$$

- In R sector

$$S_{2n} = \frac{\theta_2^n + i^{-n}\theta_1^n}{2\eta^n}, \quad C_{2n} = \frac{\theta_2^n - i^{-n}\theta_1^n}{2\eta^n}$$

- They have expansions

$$O_{2n} = q^{h_o - n/24}(1 + n(2n-1)q + \ldots), \quad V_{2n} = q^{h_v - n/24}(2n + \ldots)$$

$$S_{2n} = q^{h_s - n/24}(2^{n-1} + \ldots), \quad C_{2n} = q^{h_c - n/24}(2^{n-1} + \ldots)$$

with $(h_o, h_v, h_s, h_c) = (0, 1/2, n/8, n/8)$

- Recall for the Dedekind $\eta(\tau)$ function

$$\frac{1}{\eta^n} = q^{-n/24}(1 + nq + \ldots)$$

- Under T- duality transformations

$$(O_{2n}, V_{2n}, S_{2n}, C_{2n}) = e^{-\frac{in\pi}{12}} diag(1, -1, e^{\frac{in\pi}{4}}, e^{\frac{in\pi}{4}})\ (O_{2n} V_{2n} S_{2n} C_{2n})$$

- Under S- duality transformations

$$\begin{pmatrix} O_{2n} \\ V_{2n} \\ S_{2n} \\ C_{2n} \end{pmatrix} = e^{-\frac{in\pi}{12}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & i^{-n} & -i^{-n} \\ 1 & -1 & -i^{-n} & i^{-n} \end{pmatrix} \begin{pmatrix} O_{2n} \\ V_{2n} \\ S_{2n} \\ C_{2n} \end{pmatrix}$$

- Recall also that for the Dedekind $\eta(\tau)$ function

$$T : \eta(\tau + 1) = e^{\frac{i\pi}{12}} \eta(\tau)$$

$$S : \eta\left(-\frac{1}{\tau}\right) = \sqrt{-i\tau}\eta(\tau)$$

- When compactifying on a circle with radius R we get a lattice partition function

$$\Gamma_{m,n} = \frac{1}{\eta\bar{\eta}} \sum_{m,n} q^{\alpha' p_L^2/4} \bar{q}^{\alpha' p_L^2/4}, \quad p_{L,R} = \frac{m}{R} \pm \frac{nR}{\alpha'}.$$

- Generically: We have a theory $\mathcal{M}$ and its low energy counterpart $\mathcal{N}$.
- One loop correction to the gauge coupling constant can be obtained from

$$\mathcal{L} = \int_0^\infty \frac{dt}{2t} C_\Lambda(t)\ Str\, e^{-tL}$$

  $C_\lambda(t)$ is an ultraviolet regulator, t is a proper time
- The background solves the classical equations of motion
- To obtain the one -loop correction to the coupling constant we expand the integral up to the second order in $A_\mu(x)$ and put $F_{\mu\nu}(x) = const$. We get

$$W = \int_0^\infty \frac{dt}{t} C_\Lambda(t)\mathcal{B}(t), \quad \mathcal{B}(t) = str(Q^2 \left(\frac{1}{12} - \chi^2\right) e^{-tM^2})$$

  Q- a generator of gauge group, χ - helicity operator

- We use torus partition function
- Compute a correlator on a torus

$$\int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \int d^2 z \epsilon_1^{\mu} \epsilon_2^{\mu} \langle V_\mu(z) V_\nu(0) \rangle.$$

- $V_\mu(z)$ vertex operator for a gauge boson

$$V^{\mu,A} = (\partial_z X^\mu + i(p \cdot \psi)\psi^\mu) \overline{J}^A e^{ip\cdot X}.$$

- Use torus propagators for noncompact bosons, fermions, and Kac-Moody currents

$$\langle X(z) X(0) \rangle, = -\log|(\theta_1(z))|^2 + 2\pi \frac{Imz^2}{\tau_2}, \quad \langle \psi(z)\psi(0)\rangle|^a_b = S(z)\left[{a \atop b}\right]$$

$$\langle J^A(\bar z) J^B(0) \rangle = \delta^{AB} \left(\frac{k}{4\pi^2} \bar\partial^2 \log \bar\theta_1(\bar z) + trQ^2 \right)$$

- Pick up the quadratic part in momenta of what we get.
- The result: for $4d$ partition function

$$Z = \frac{1}{\tau_2 \eta^2 \overline{\eta}^2} \sum_{a,b=0}^{1} \frac{\theta[^a_b]}{2\eta} C^{int}[^a_b],$$

we get threshold corrections

$$\Delta_{\mathcal{G}} = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} (\mathcal{B}_{\mathcal{G}}(\tau) - b_{\mathcal{G}})$$

$$\Delta_{\mathcal{G}} = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \left(\frac{1}{\eta^2 \overline{\eta}^2} \sum_{even} \frac{i}{\pi} \partial_\tau \left(\frac{\theta[^a_b]}{2\eta} \right) Tr_{int}(Q^2_{\mathcal{G}} - \frac{k_{\mathcal{G}}}{4\pi\tau_2}) C^{int}[^a_b] - b_{\mathcal{G}} \right)$$

where $b_{\mathcal{G}} = \lim_{\tau_2 \to \infty} \mathcal{B}(\tau)$

- Need of infrared regularization

- Partition function for $E_8 \otimes E_8$ heterotic string

$$Z_{E_8\times E_8} = \frac{(V_8 - S_8)(\overline{O}_{16} + \overline{S}_{16})(\overline{O}_{16} + \overline{S}_{16})}{(\sqrt{\tau_2}\eta\bar{\eta})^8}.$$

- Perform SUSY breaking orbifold Z_2

$$S_8 \to -S_8, \quad \overline{S}_{16}^{1,2} \to -\overline{S}_{16}^{1,2}, \quad \overline{C}_{16}^{1,2} \to -\overline{C}_{16}^{1,2}.$$

- Leads to the partition function for $SO(16) \otimes SO(16)$ tachyon free heterotic string

$$\begin{aligned} Z \quad &= \quad \frac{V_8(\overline{O}_{16}\overline{O}_{16} + \overline{S}_{16}\overline{S}_{16}) + O_8(\overline{V}_{16}\overline{C}_{16} + \overline{C}_{16}\overline{V}_{16})}{(\sqrt{\tau_2}\eta\bar{\eta})^8} - \\ &- \quad \frac{S_8(\overline{O}_{16}\overline{S}_{16} + \overline{S}_{16}\overline{O}_{16}) + C_8(\overline{V}_{16}\overline{V}_{16} + \overline{C}_{16}\overline{C}_{16})}{(\sqrt{\tau_2}\eta\bar{\eta})^8}. \end{aligned}$$

Massless spectrum

- Untwisted bosonic $g_{\mu\nu}, B_{\mu\nu}, \phi$ and $A_\mu((120,1) \oplus (1,120))$.
- Untwisted fermionic $\psi^\alpha((128,1) \oplus (1,128))$.
- Twisted fermionic $\xi_\alpha(16,16)$.
- The partition function (again)

$$
\begin{aligned}
Z &= \frac{V_8(\overline{O}_{16}\overline{O}_{16} + \overline{S}_{16}\overline{S}_{16}) + O_8(\overline{V}_{16}\overline{C}_{16} + \overline{C}_{16}\overline{V}_{16})}{(\sqrt{\tau_2}\eta\bar{\eta})^8} - \\
&- \frac{S_8(\overline{O}_{16}\overline{S}_{16} + \overline{S}_{16}\overline{O}_{16}) + C_8(\overline{V}_{16}\overline{V}_{16} + \overline{C}_{16}\overline{C}_{16})}{(\sqrt{\tau_2}\eta\bar{\eta})^8}.
\end{aligned}
$$

- Start with heterotic $E_8 \otimes E_8$ and compactify it on an orbifold

$$T^6/\mathbb{Z}_N \times \mathbb{Z}_2', \quad N = 2, 3, 4, 6.$$

- A discrete $\mathbb{Z}_N$ acts on T^4 as (breaks $\mathcal{N} = 4$ to $\mathcal{N} = 2$)

$$\mathbb{Z}_N : z_1 \to e^{2\pi/N} z_1, \quad z_2 \to e^{-2\pi/N} z_2.$$

- Extra $\mathbb{Z}_2'$ is freely acting (breaks $\mathcal{N} = 2$ to $\mathcal{N} = 0$)

$$\mathbb{Z}_2 : (-1)^{F_{st}+F_1+F_2} \delta.$$

  where F_{st} is a space-time fermion number, $F_{1,2}$ gauge group "fermion numbers" , and δ is an order two shift along T^2.
- We have Scherk-Schwarz spontaneous SUSY breaking on $K3 \times T^2$.

- In terms of characters (example $\mathbb{Z}_2 \times \mathbb{Z}_2'$)

$$V_8 - S_8 = V_4 O_4 + O_4 V_4 - S_4 S_4 - C_4 C_4,$$

$$\overline{O}_{16} + \overline{S}_{16} = \overline{O}_{12}\overline{O}_4 + \overline{V}_{12}\overline{V}_4 + \overline{S}_{12}\overline{S}_4 + \overline{C}_{12}\overline{C}_4,$$

- Under the first $\mathbb{Z}_2$

$$V_4^{(2)} \to -V_4^{(2)}, \quad S_4^{(2)} \to -S_4^{(2)}, \quad \overline{V}_4^{(2)} \to -\overline{V}_4^{(2)}, \quad \overline{S}_4^{(2)} \to -\overline{S}_4^{(2)}.$$

- Under the second $\mathbb{Z}_2'$: $\Gamma_{mn} \to (-1)^m \Gamma_{mn}$, and

$$(S_4^{(1)}, C_4^{(1)}) \to -(S_4^{(1)}, C_4^{(1)}), \quad (\overline{S}^{(2)}_{16/12}, \overline{C}^{(2)}_{16/12}) \to -(\overline{S}^{(2)}_{16/12}, \overline{C}^{(2)}_{16/12}).$$

- The gauge group is $SO(16) \otimes SO(12) \otimes SO(4)$.

- The threshold corrections have a form

$$\Delta_{\mathcal{G}} = \Delta_{\mathcal{G}}^{(u+)} + \Delta_{\mathcal{G}}^{(u-)} + \Delta_{\mathcal{G}}^{(t+)} + \Delta_{\mathcal{G}}^{(t-)},$$

- where

$$\Delta_{\mathcal{G}}^{(t+)} = S\Delta_{\mathcal{G}}^{(u-)}, \quad \Delta_{\mathcal{G}}^{(t-)} = T\Delta_{\mathcal{G}}^{(t+)}$$

- It is easier to computer the differences

$$\Delta_{SO(16)}^{u+} - \Delta_{SO(12)}^{u+} = -36\Gamma_{mn} : \quad Universal \quad contribution$$

L. Dixon, V.Kaplunovsky, J. Louis (1991); E.Kiritsis, C.Kounnas, M.Petropoulos, J.Rizos (1996).

- Torus moduli $T = T_1 + iT_2$ and $U = U_1 + iU_2$

$$\int_{\mathcal{F}} \frac{d\tau}{\tau_2^2} \Gamma_{m,n}(T,U) = -\log\left(T_2 U_2 |\eta(T)\eta(U)|^4\right).$$

- Non BPS contributions

$$\int_{\mathcal{F}_0(2)} \frac{d\tau}{\tau_2^2} (-1)^m \Gamma_{m,n}(T,U)(hol.) \times (anti - hol.).$$

- Written in terms of $SO(2n)$ characters

$$12(O_8^2 V_8 + 3V_8^3)(\overline{O}_8^2 \overline{V}_8 - \overline{V}_8^3).$$

- Holomorphic because of identities: I.Florakis, C.Kounnas (2009)

$$\overline{O}_8^2 \overline{V}_8 - \overline{V}_8^3 = 8.$$

- The one can take the integral using the technique of C.Angelantonj, I.Florakis, B.Pioline (2012,2013).

- One gets for differences

$$\Delta_{SO(16)} - \Delta_{SO(12)} =$$

$$= 36 \log \left(T_2 U_2 |\eta(T)\eta(U)|^4\right) - \frac{4}{3} \log \left(T_2 U_2 |\theta_4(T)\theta_2(U)|^4\right)$$

$$+ \frac{1}{3} \log |\hat{j}_2(T/2) - \hat{j}_2(U)|^4 ||j_2(U) - 24|,$$

where

$$j_2(U) - 24 = \frac{\eta^{24}(\tau)}{\eta^{24}(2\tau)}, \quad \hat{j}_2(\tau) - 24 = \frac{\theta_2^{12}}{\eta^1 2}.$$

- This form is the same for orbifolds with $N = 2, 3, 4, 6$ up to numerical constants.

- Remarkable Universality
- Symmetries of nonsupersymmetric compactifications
- Gravitational Thresholds:
  I. Florakis NPB **916**, 2017, 484
- Possible semi realistic models realistic models. Recent work:
  M. Blaszczyk, S. Groot-Nibbelink, O. Loukas, S. Ramos-Sanchez, JHEP **10**, 2014, 119
  S. Abel, K. R. Dienes, E. Mavroudi, PRD **91**, 2015, 126014
  A. E. Faraggi, C. Kounnas, H. Partouche, NPB **899**, 2015, 328
  I. Florakis, J. Rizos, NPb **913**, 2016, 495
  C. Angelantonj, I. Florakis, PLB **789**, 2019, 496
- More detailed study.