SPECIAL RELATIVITY – MIDTERM EXAM

Exercise 1. *In the lecture, we described a relativistic particle in a (electromagnetic) vector field $A_\mu(x)$. Now, let's add in a scalar field $\phi(x)$. Recall that the rest energy m of a massless particle can be thought of as a potential energy. Let's take this completely seriously, and promote m into a position-dependent "potential" $\phi(x)$. Thus, consider the action:*

$$S = -\int \phi(x)\sqrt{-dx_\mu dx^\mu} + \int A_\mu(x)dx^\mu \ , \tag{1}$$

where we set the electric charge to 1, and assume $\phi(x) > 0$.

1. *Write this action as $S = \int L(x^\mu, \dot{x}^\mu)d\lambda$, where $\dot{x}^\mu \equiv dx^\mu/d\lambda$. What is the Lagrangian L?*

2. *What is the 4-momentum p^μ of a particle with position x^μ and 4-velocity u^μ?*

3. *What is the 4-acceleration α^μ of a particle with position x^μ and 4-velocity u^μ? Check that α^μ is orthogonal to u^μ.*

Exercise 2. *In the lecture, we mentioned that Lorentz contraction of a moving object should not be confused with what an observer would actually see. Let us explore this on a specific example. Consider an observer at rest at at $\mathbf{x} = (0, 0, 0)$, looking at a square object whose center is at $\mathbf{x} = (0, 0, R)$ and whose vertices are at $\mathbf{x} = (\pm a/2, 0, R)$ and $\mathbf{x} = (0, \pm a/2, R)$, also at rest. Assume $a \ll R$, and answer all questions to first order in a.*

1. *Consider the light seen by the observer at $t = 0$. What are the spacetime coordinates of the square's center and vertices when this light was emitted?*

2. *At $t = 0$, the observer suddenly accelerates to a relativistically large velocity $\mathbf{v} = (v, 0, 0)$. What are the spacetime coordinates of the emitted light from Part 1 (square's center and vertices), in the observer's new reference frame?*

3. *What is the new spherical position (θ, ϕ) at which the observer sees the square's center? What are the new angular sizes $\Delta\theta, \sin\theta\Delta\phi$ of the square's diagonals?*

4. *Explain in words: what happened to the Lorentz contraction?*