Enhanced gauge symmetry and suppressed cosmological constant in heterotic interpolating models

- will talk about the series of work done with Sota Nakajima
  - arXiv: 1905.10745, PTEP INkjm1
  - arXiv: 2003.1121, NPB INkjm 2
  - arXiv: 2101.10619, PLB INkjm 3
  - I-Koga-Nkjm in preparation

I) Introduction

- **Q:** How can string theory provide a chance to interrelate the unification of forces and the prob. of cosmological const?
- no SUSY in multi TeV scale according to the LHC experiment
- even in 10D, under modular inv.,

#(theories with SUSY) < #(theories without SUSY)

- Type IIB
- Type IIA
- Type I
- Heterotic $SO(32)$
- Heterotic $E_8 \times E_8$

call M_1

- Type 0B
- Type 0A
- Heterotic $SO(32)$
- Heterotic $SO(16) \times E_8$

call M_2

- Heterotic $SO(16) \times SO(16)$
- Heterotic $E_7^2 \times SU(2)^2$
- Heterotic $SO(24) \times SO(8)$
- …

today

- **possible A:** interpolation by a radius ($a = \sqrt{\alpha'}/R$) or in general radii of M_1 and M_2 upon compactification

'86: HI-Taylor

- choices:

  M_2 = SO(16) x SO(16) tachyon free '86 I-T & INkjm 1, 2

  Dixon-Harvey 1986, Alvarez-Gaume et al. 1986, Faraggi-Tsulaia '09, ...

  or

  = tachyonic ones should be allowed in SUSY restoring region

  cf. Faraggi '19, Faraggi, Matyas, Percival '19

- warning: consider all marginal deformations of the world sheet action

  $\Longrightarrow$
  - full set of Wilson lines should be turned on

    Narain-Sarmadi-Witten

  - generically spoil the nonabelian gauge group

  extrema $\leftrightarrow$ points of sym. enhancement &
  the stable 9D perturbative vacuum can be determined

• formula for one-loop cosm. const in SUSY res. region:

$$\Lambda^{(D)} = \xi(n_F - n_B)a^D + O(e^{-1/a})$$

H.I.-Taylor ('86)

M_1 0 ⟷ ∞ M_2

a

cf Abel-Dienes-Mavroudi

. . .

n_B, n_F; # of massless bosons & fermions in D dim.

- $n_B = n_F$ models (by now more than several existing) enjoy exponential suppression of $\Lambda^{(D)}$

e.g. Kounnas-Partouche, Abel-Stewart …

- In this setup, mass splitting due to broken SUSY is $\alpha' M_s^2 = a^2$.

e.g. $a \approx 0.01$ interesting possibility

More on the exponential suppression   I-T, INkjm2 app.

- The integrand of the τ_2 integration involves

$$(*) = \tau_2^{\#}(\Lambda_{0,0} - \Lambda_{1/2,0})e^{-m\pi\tau_2}$$

SUSY restoring factor   generic level from $\prod(\text{characters})$

- apply the Jacobi imaginary transf. $(*) = (\text{const})\tau_2^{\#'}\sum_{n=1}^{\infty} e^{-\frac{1}{4\tau_2}(2n-1)^2\pi\tilde{a}^2 - m\pi\tau_2}$

  $\text{i)}\ m \neq 0:$ the sum bdd at least by $\dfrac{e^{-\pi\tilde{a}\sqrt{m}}}{1-e^{-2\pi\tilde{a}\sqrt{m}}}$

  & can integrate over $[1,\infty]$

  $\text{ii)}\ m = 0:$ term by term integ. over $[1,\infty]$ and resum to get $\zeta(10)$

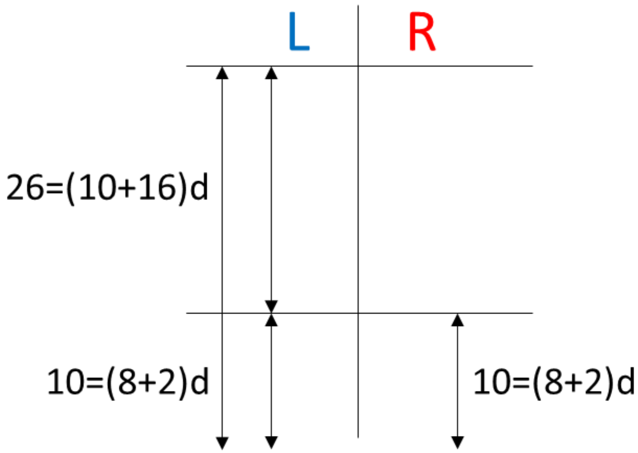
  $\Rightarrow$ the first (dominant) term up to exp. accuracy

- contribution from $\tau_2 < 1$, exp. suppressed.

Contents

For presentation, mostly the SO(32) case only, omitting the E_8xE_8 case

II) ● Idea of Heterotic strings



adopt the lightcone coordinates

Right mover: 10d superstring $\bar{X}^i_R(\tau-\sigma),\ \bar{\psi}^i(\tau-\sigma)$

Left mover: 26d bosonic string out of which

internal 16d realize rank 16 current algebra

$X^i_L(\tau+\sigma),\ X^I_L(\tau+\sigma)$ (or fermions)

● State counting & characters

- $\mathrm{Tr} q^{L_0}\bar{q}^{\bar{L}_0}$ counts #(states) at level m as coeff. in $q(\bar{q})$ expansion
- It takes the form of $\sum_{i,j}\bar{\chi}_i^{\mathrm{Vir}}(\bar{q})X_{ij}\chi_j^{\mathrm{Vir}}(q)$ and involves spacetime & internal SO(2n), n=4, 8 characters $\mathrm{ch(rep)} = O_{2n}, V_{2n}, S_{2n}, C_{2n}$ expressible in terms of the four theta constants and the Dedekind eta fn

$$\eta(\tau) = q^{1/24}\prod_{n=1}^{\infty}(1-q^n)$$

- SO(32) hetero $\quad Z_B^{(8)}\left(\bar{V}_8-\bar{S}_8\right)\left(O_{16}O_{16}+V_{16}V_{16}+S_{16}S_{16}+C_{16}C_{16}\right)$

  E_8 x E_8 hetero $\quad Z_B^{(8)}(\bar{V}_8-\bar{S}_8)(O_{16}+S_{16})(O_{16}+S_{16})$

Boost and enhanced gauge symmetry

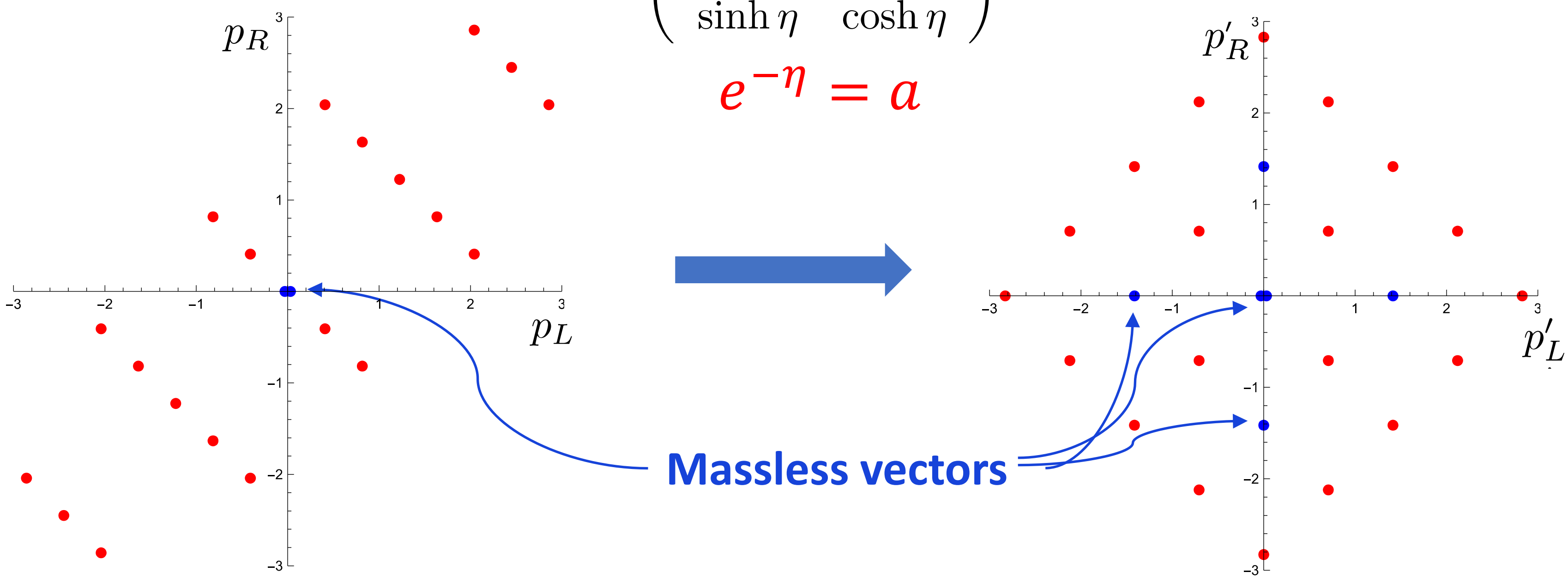
- Simplest example: bosonic strings on S^1

Mass formula: $M^2 = 4(N-1) + 2p_L^2 = 4(\tilde{N}-1) + 2p_R^2$

(※) $\begin{pmatrix} p_L \\ p_R \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} na + w/a \\ na - w/a \end{pmatrix}$ $\xrightarrow{\text{boost}}$ $\begin{pmatrix} p'_L \\ p'_R \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} n + w \\ n - w \end{pmatrix}$

$$\begin{pmatrix} \cosh\eta & \sinh\eta \\ \sinh\eta & \cosh\eta \end{pmatrix}$$

$$e^{-\eta} = a$$



$U(1)^2$ $\xrightarrow{\text{gauge sym. is enhanced}}$ $SU(2) \times SU(2)$

(※) forms SO(1,1) Lorentzian lattice

Narain moduli space of d-dim toroidal compactification

- the comp. ⇒ (16+d,d) even self-dual Lorentzian lattice
  - The space of marginal deformations (the moduli space) is the coset

(※) $$\frac{SO(16+d,d)}{SO(16+d)\times SO(d)}$$

these boosts are generated by the constant background fields whose worldsheet action is

$$A_{Ii}\int d^2z\partial X_L^I\bar{\partial}X_R^i + C_{ji}\int d^2z\partial X_L^j\bar{\partial}X_R^i,$$

$$\left[\, I=1,\cdots,16,\ i,j=10-d,\cdots,9 \,\right]$$

Narain, Sarmadi, Witten, (1986)

(※) the case d=1 is our first concern

Idea of compactification on a twisted circle

- choose $\mathcal{T}$: the translation by a half period

$$\mathcal{T}: \ X^9 \to X^9 + \pi R$$

- choose Q : the Z_2 action on the "internal" part that defines the model M_2

- Actually $Q = Q_L \bar{Q}_R$ and $\bar{Q}_R = (-)^F$, namely the sign flip by the spacetime fermion number

- adopt $\mathcal{T}Q$ as our Z_2 action (no fixed point) and project onto $\mathcal{T}Q = 1$ e.v.,

  namely $\dfrac{1 + \mathcal{T}Q}{2}$

- restore modular inv. by adding the twisted sectors

- need to prepare

$$\Lambda_{\alpha,\beta} \equiv (\eta\bar{\eta})^{-1} \sum_{n \in 2(\mathbf{Z}+\alpha), \ w \in \mathbf{Z}+\beta} q^{\frac{\alpha'}{2} p_L^2} \bar{q}^{\frac{\alpha'}{2} p_R^2}$$

α and β are 0 or 1/2, and $\alpha = 0 \ (1/2)$ and $\beta = 0 \ (1/2)$

III) ● Construction

Dixon-Harvey '86
Blum-Dienes '97

start over
- $Z_+^{(9)+} = \left(\Lambda_{0,0} + \Lambda_{1/2,0}\right) Z_B^{(7)} Z_+^+,$
- $\mathcal{T}Q: \; Z_+^{(9)+} \to Z_-^{(9)+} = \left(\Lambda_{0,0} - \Lambda_{1/2,0}\right) Z_B^{(7)} Z_-^+,$

Z_-^+ is the Q -action of Z_+^+.

- $\boldsymbol{S}: \; Z_-^{(9)+} \to Z_+^{(9)-} = \left(\Lambda_{0,1/2} + \Lambda_{1/2,1/2}\right) Z_B^{(7)} Z_+^-,$

$Z_-^+(-1/\tau) \equiv Z_+^-(\tau).$

- $\mathcal{T}Q: \; Z_+^{(9)-} \to Z_-^{(9)-} = \left(\Lambda_{0,1/2} - \Lambda_{1/2,1/2}\right) Z_B^{(7)} Z_-^-,$

Z_-^- is the Q -action of Z_+^-.

$$\begin{aligned} \therefore \quad Z_{\rm int}^{(9)} &= \frac{1}{2}\left(Z_+^{(9)+} + Z_-^{(9)+} + Z_+^{(9)-} + Z_-^{(9)-}\right) \\ &= \frac{1}{2} Z_B^{(7)} \left\{\Lambda_{0,0}\left(Z_+^+ + Z_-^+\right) + \Lambda_{1/2,0}\left(Z_+^+ - Z_-^+\right)\right. \\ &\quad \left. + \Lambda_{0,1/2}\left(Z_+^- + Z_-^-\right) + \Lambda_{1/2,1/2}\left(Z_+^- - Z_-^-\right)\right\}. \end{aligned}$$

In $a \to \infty$ limit, $Z_{\rm int}^{(9)}$ produces model M_2 :

$$Z_{M_2} = Z_B^{(8)}\left(Z_+^+ + Z_-^+ + Z_+^- + Z_-^-\right).$$

$SO(16) \times SO(16) \leftrightarrow$ SUSY $SO(32)$

- The partition function

$$Z_{\text{int}}^{(9)} = Z_B^{(7)} \{\Lambda_{0,0} \left[\bar{V}_8 \left(O_{16}O_{16} + S_{16}S_{16}\right) - \bar{S}_8 \left(V_{16}V_{16} + C_{16}C_{16}\right)\right]$$
$$+\Lambda_{1/2,0}\left[\bar{V}_8 \left(V_{16}V_{16} + C_{16}C_{16}\right) - \bar{S}_8 \left(O_{16}O_{16} + S_{16}S_{16}\right)\right]$$
$$+\Lambda_{0,1/2} \left[\bar{O}_8 \left(V_{16}C_{16} + C_{16}V_{16}\right) - \bar{C}_8 \left(O_{16}S_{16} + S_{16}O_{16}\right)\right]$$
$$+\Lambda_{1/2,1/2} \left[\bar{O}_8 \left(O_{16}S_{16} + S_{16}O_{16}\right) - \bar{C}_8 \left(V_{16}C_{16} + C_{16}V_{16}\right)\right]\}$$

$l_L^I = m^I$ is the momentum for X_L^I.

Massless vectors with

- $n = w = m^I = 0$ × 16

$$\begin{cases} n = w = 0 \\ m^I = \left(\underline{\pm, \pm, (0)^6}; (0)^8\right), \ \left((0)^8; \underline{\pm, \pm, (0)^6}\right) \end{cases}$$

$SO(16) \times SO(16)$ adjoint

Massless spinors with

$$\begin{cases} n = w = 0 \\ m^I = \left(\underline{\pm, (0)^7}; \underline{\pm, (0)^7}\right) \end{cases}$$

$(\mathbf{16}, \mathbf{16})$ of $SO(16) \times SO(16)$

$SO(16) \times SO(16) \leftrightarrow E_8 \times E_8$

- The partition function

$$Z_{\text{int}}^{(9)} = Z_B^{(7)} \{\Lambda_{0,0} [\bar{V}_8 (O_{16}O_{16} + S_{16}S_{16}) - \bar{S}_8 (O_{16}S_{16} + S_{16}O_{16})]$$
$$+\Lambda_{1/2,0} [\bar{V}_8 (O_{16}S_{16} + S_{16}O_{16}) - \bar{S}_8 (O_{16}O_{16} + S_{16}S_{16})]$$
$$+\Lambda_{0,1/2} [\bar{O}_8 (V_{16}C_{16} + C_{16}V_{16}) - \bar{C}_8 (V_{16}V_{16} + C_{16}C_{16})]$$
$$+\Lambda_{1/2,1/2} [\bar{O}_8 (V_{16}V_{16} + C_{16}C_{16}) - \bar{C}_8 (V_{16}C_{16} + C_{16}V_{16})] \}$$

$l_L^I = m^I$ is the momentum for X_L^I.

Massless vectors with

- $n = w = m^I = 0$ × 16

$$\begin{cases} n = w = 0 \\ m^I = \left(\underline{\pm, \pm, (0)^6}; (0)^8\right), \ \left((0)^8; \underline{\pm, \pm, (0)^6}\right) \end{cases}$$

$SO(16) \times SO(16)$ adjoint

Massless spinors with

$$\begin{cases} n = w = 0 \\ m^I = \begin{cases} \frac{1}{2}\left(\underline{\pm, \pm, \pm, \pm, \pm, \pm, \pm, \pm}_+; (0)^8\right), \\ \frac{1}{2}\left((0)^8; \underline{\pm, \pm, \pm, \pm, \pm, \pm, \pm, \pm}_+\right) \end{cases} \end{cases}$$

$(\mathbf{128}, \mathbf{0}) \oplus (\mathbf{0}, \mathbf{128})$ of $SO(16) \times SO(16)$

IV) ● boosting the momentum lattice

The momenta of $X^9_{L,R}, X^I_L$ change as

by A_I : Wilson lines

(world-sheet action)

+ $A_I \int d^2 z \partial X^I_L \bar{\partial} X^9_R$

$$\begin{cases} l^I_L = m^I , \\ p_L = \frac{1}{\sqrt{2}} (an + w/a) , \\ p_R = \frac{1}{\sqrt{2}} (an - w/a) , \end{cases}$$

→ Boost $SO(17,1)$

$$\begin{cases} {l'}^I_L = \frac{1}{\sqrt{2}} \left(\sqrt{2} m^I - 2 A^I a_0^{-1} w \right) , \\ p'_L = \frac{1}{\sqrt{2}} \left(\sqrt{2} A \cdot m + a_0 n - (1 - |A|^2) a_0^{-1} w \right) , \\ p'_R = \frac{1}{\sqrt{2}} \left(\sqrt{2} A \cdot m + a_0 n - (1 + |A|^2) a_0^{-1} w \right) , \end{cases}$$

$\left(a_0 = \sqrt{1 + |A|^2} a \right)$

In the partition function,

Boost $SO(17,1)$

$$\Lambda_{\alpha,\beta}(a) X_{16} Y_{16} = (\eta\bar{\eta})^{-1} \eta^{-16} \sum_{n,w} q^{p_L^2} \bar{q}^{p_R^2/2} \sum_{m^I} q^{|l_L|^2/2}$$

→

$$\chi^{(\alpha,\beta)}_{XY}(a, A^I)$$

$$(\eta\bar{\eta})^{-1} \eta^{-16} \sum_{n,w,m^I} q^{(p'^2_L + |l'_L|^2)/2} \bar{q}^{p_R^2/2}$$

$\left[X_{16}, Y_{16} = (O_{16}, V_{16}, S_{16}, C_{16}) \right]$

The sum of m^I depends on X, Y.

Moduli space and shift symmetry

- Moduli space of 9D interpolating models is 17-dimensional: a, A^I
- Defining t_1^I and t_2 as

$$t_1^I = \frac{1}{\sqrt{2}}\frac{A^I}{a_0} = \frac{1}{\sqrt{2}}\frac{A^I}{\sqrt{1+|A|^2a}}, \quad t_2 = \frac{1}{\sqrt{2}}\frac{1}{a_0} = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1+|A|^2a}},$$

  we can find the shift symmetry:

$$\chi_{XY}^{(\alpha,\beta)}\left(t_1^I, t_2\right) = \chi_{XY}^{(\alpha,\beta)}\left(t_1^I + 2, t_2\right)$$

- The fundamental region of moduli space is

$$-1 < t_1^I \leq 1, \quad 0 \leq t_2.$$

- Moduli t_1^I, t_2 are the parameters of the boost on the momentum lattice.

● The case of one WL

INkjm 1

Sorry, here, $\chi^{(\alpha,\beta)}_{XY} = X^{(\alpha,\beta)}Y$

- **SUSY SO(32) —SO(16)×SO(16) interpolating model with WL**

Massless spectrum, at generic *R, A,* comes from ***n=w=m=0* part**

$$Z^{(9)}_{\rm int} = Z^{(7)}_{\rm boson}\left\{\bar{V}_8\left(O^{(0,0)}_{16}O_{16}+S^{(0,0)}_{16}S_{16}\right)-\bar{S}_8\left(V^{(0,0)}_{16}V_{16}+C^{(0,0)}_{16}C_{16}\right)\right.$$
$$+\bar{O}_8\left(V^{(0,1/2)}_{16}C_{16}+C^{(0,1/2)}_{16}V_{16}\right)-\bar{C}_8\left(O^{(0,1/2)}_{16}S_{16}+S^{(0,1/2)}_{16}O_{16}\right)$$
$$+\bar{V}_8\left(V^{(1/2,0)}_{16}V_{16}+C^{(1/2,0)}_{16}C_{16}\right)-\bar{S}_8\left(O^{(1/2,0)}_{16}O_{16}+S^{(1/2,0)}_{16}S_{16}\right)$$
$$\left.+\bar{O}_8\left(O^{(1/2,1/2)}_{16}S_{16}+S^{(1/2,1/2)}_{16}O_{16}\right)-\bar{C}_8\left(V^{(1/2,1/2)}_{16}C_{16}+C^{(1/2,1/2)}_{16}V_{16}\right)\right\}$$

massless states at generic *A, R*

Massless bosons:

- $g_{\mu\nu},\ \ B_{\mu\nu},\ \ \phi$
- gauge bosons in adjoint rep of $\mathrm{SO}(16)\times\mathrm{SO}(14)\times\mathrm{U}(1)\times\mathrm{U}(1)^2$

Massless fermions:

- $\mathbf{8}_S\otimes(\mathbf{16},\mathbf{14})$

$$\longrightarrow\qquad n^0_F - n^0_B = 32$$

- **SUSY SO(32) —SO(16)×SO(16) interpolating model with WL**

Massless spectrum ∃**a few conditions under which the gauge group gets enhanced**

$$Z_{\rm int}^{(9)} = Z_{\rm boson}^{(7)} \Big\{ \bar{V}_8 \left(O_{16}^{(0,0)} O_{16} + S_{16}^{(0,0)} S_{16} \right) - \bar{S}_8 \left(V_{16}^{(0,0)} V_{16} + C_{16}^{(0,0)} C_{16} \right)$$
$$+ \bar{O}_8 \left(V_{16}^{(0,1/2)} C_{16} + C_{16}^{(0,1/2)} V_{16} \right) - \bar{C}_8 \left(O_{16}^{(0,1/2)} S_{16} + S_{16}^{(0,1/2)} O_{16} \right)$$
$$+ \bar{V}_8 \left(V_{16}^{(1/2,0)} V_{16} + C_{16}^{(1/2,0)} C_{16} \right) - \bar{S}_8 \left(O_{16}^{(1/2,0)} O_{16} + S_{16}^{(1/2,0)} S_{16} \right)$$
$$+ \bar{O}_8 \left(O_{16}^{(1/2,1/2)} S_{16} + S_{16}^{(1/2,1/2)} O_{16} \right) - \bar{C}_8 \left(V_{16}^{(1/2,1/2)} C_{16} + C_{16}^{(1/2,1/2)} V_{16} \right) \Big\}$$

condition ① $\sqrt{2}A + \sqrt{1+A^2}\, a n_1 = 0, \quad n_1 \in 2\mathbf{Z}$

new massless state： two $\mathbf{8}_V \otimes (\mathbf{1}, \mathbf{14})$ • two $\mathbf{8}_S \otimes (\mathbf{16}, \mathbf{1})$

⇒
- **SO(16)×SO(14)×U(1)** ⟶ **SO(16)×SO(16)**
- $\mathbf{8}_S \otimes (\mathbf{16}, \mathbf{14})$ ⟶ $\mathbf{8}_S \otimes (\mathbf{16}, \mathbf{16})$

⟶ $n_F^0 - n_B^0 = 64$

- **SUSY SO(32) —SO(16)×SO(16) interpolating model with WL**

Massless spectrum ∃ **a few conditions under which the gauge group gets enhanced**

$$Z_{\rm int}^{(9)}=Z_{\rm boson}^{(7)}\Big\{\bar{V}_8\left(O_{16}^{(0,0)}O_{16}+S_{16}^{(0,0)}S_{16}\right)-\bar{S}_8\left(V_{16}^{(0,0)}V_{16}+C_{16}^{(0,0)}C_{16}\right)$$
$$+\bar{O}_8\left(V_{16}^{(0,1/2)}C_{16}+C_{16}^{(0,1/2)}V_{16}\right)-\bar{C}_8\left(O_{16}^{(0,1/2)}S_{16}+S_{16}^{(0,1/2)}O_{16}\right)$$
$$+\bar{V}_8\left(V_{16}^{(1/2,0)}V_{16}+C_{16}^{(1/2,0)}C_{16}\right)-\bar{S}_8\left(O_{16}^{(1/2,0)}O_{16}+S_{16}^{(1/2,0)}S_{16}\right)$$
$$+\bar{O}_8\left(O_{16}^{(1/2,1/2)}S_{16}+S_{16}^{(1/2,1/2)}O_{16}\right)-\bar{C}_8\left(V_{16}^{(1/2,1/2)}C_{16}+C_{16}^{(1/2,1/2)}V_{16}\right)\Big\}$$

condition ② $\sqrt{2}A+\sqrt{1+A^2}an_2=0,\quad n_2\in 2\boldsymbol{Z}+1$

new massless state : **two** $\mathbf{8}_V\otimes(\mathbf{16},\mathbf{1})$ • **two** $\mathbf{8}_S\otimes(\mathbf{1},\mathbf{14})$

➡ **SO(16)×SO(14)×U(1)** ⟶ **SO(18)×SO(14)**

$\mathbf{8}_S\otimes(\mathbf{16},\mathbf{14})$ ⟶ $\mathbf{8}_S\otimes(\mathbf{18},\mathbf{14})$

⟶ $n_F^0-n_B^0=0$

Summary of INkjm 1

SO(32) case

Conditions	$\tilde{\tau}_1 = n_1/\sqrt{2} \quad (n_1 \in \boldsymbol{Z})$	$\tilde{\tau}_1 = n_2/\sqrt{2} \quad (n_2 \in \boldsymbol{Z} + 1/2)$
Gauge group	$SO(16) \times SO(16)$	$SO(14) \times SO(18)$
$N_F - N_B$	positive	zero

$\tilde{\tau}_1 = \sqrt{2} t_1^1$

E_8 x E_8 case

Conditions	$\tilde{\tau}_1 = n_1/\sqrt{2} \quad (n_1 \in 2\boldsymbol{Z})$	$\tilde{\tau}_1 = n_1/\sqrt{2} \quad (n_1 \in 2\boldsymbol{Z} + 1)$	$\tilde{\tau}_1 = n_2/\sqrt{2} \quad (n_2 \in \boldsymbol{Z} + 1/2)$
Gauge group	$SO(16) \times SO(16)$	$SO(16) \times E_8$	$SO(16) \times SO(14) \times U(1)$
$N_F - N_B$	positive	negative	negative

- We found out $\cos(2\pi t_1)$ potential, these points giving the extrema
- need to investigate potential instability w. r. t. turning on the other Wilson lines

still in **IV)** ● **Back to the full set of WL**

$SO(16) \times SO(16) \leftrightarrow$ SUSY $SO(32)$

- The partition function

case only for today

$$Z^{(9)}(t_1^I, t_2) = Z_B^{(7)} \left\{ \bar{V}_8 \left(\chi_{OO}^{(0,0)} + \chi_{SS}^{(0,0)} \right) - \bar{S}_8 \left(\chi_{VV}^{(0,0)} + \chi_{CC}^{(0,0)} \right) + \bar{V}_8 \left(\chi_{VV}^{(1/2,0)} + \chi_{CC}^{(1/2,0)} \right) - \bar{S}_8 \left(\chi_{OO}^{(1/2,0)} + \chi_{SS}^{(1/2,0)} \right) + \bar{O}_8 \left(\chi_{VC}^{(0,1/2)} + \chi_{CV}^{(0,1/2)} \right) - \bar{C}_8 \left(\chi_{OS}^{(0,1/2)} + \chi_{SO}^{(0,1/2)} \right) + \bar{O}_8 \left(\chi_{OS}^{(1/2,1/2)} + \chi_{SO}^{(1/2,1/2)} \right) - \bar{C}_8 \left(\chi_{VC}^{(1/2,1/2)} + \chi_{CV}^{(1/2,1/2)} \right) \right\}$$

Massless vectors with $n = w = m^I = 0$ × 16

The gauge symmetry is $U(1)^{16}$ and there are no massless fermions at generic points of moduli space.

There are special points in moduli space, where the additional massless states appear.

- On the plane in moduli space satisfying

$$t_1^{a_1} = t_1^{a_2} = \cdots = t_1^{a_p} = x, \quad 1 \leq a_i \leq 8,\ 2 \leq p \leq 8$$

The gauge symmetry is enhanced:

$$U(1)^{p-1} \subset U(1)^{16} \longrightarrow SU(p)$$

- On the plane in moduli space satisfying

$$t_1^{a_1} = t_1^{a_2} = \cdots = t_1^{a_p} = x, \quad 1 \le a_i \le 8,\ 2 \le p \le 8$$

The gauge symmetry is enhanced:

$$U(1)^{p-1} \subset U(1)^{16} \longrightarrow SU(p)$$

- On the plane in moduli space satisfying

$$\begin{cases} t_1^{a_1} = t_1^{a_2} = \cdots = t_1^{a_p} = x, \quad 1 \le a_i \le 8,\ 2 \le p \le 8 \\ t_1^{b_1} = t_1^{b_2} = \cdots = t_1^{b_q} = y, \quad 1 \le b_j \le 8 \text{ or } 9 \le b_j \le 16, 2 \le q \le 8 \end{cases}$$

The gauge symmetry is enhanced:

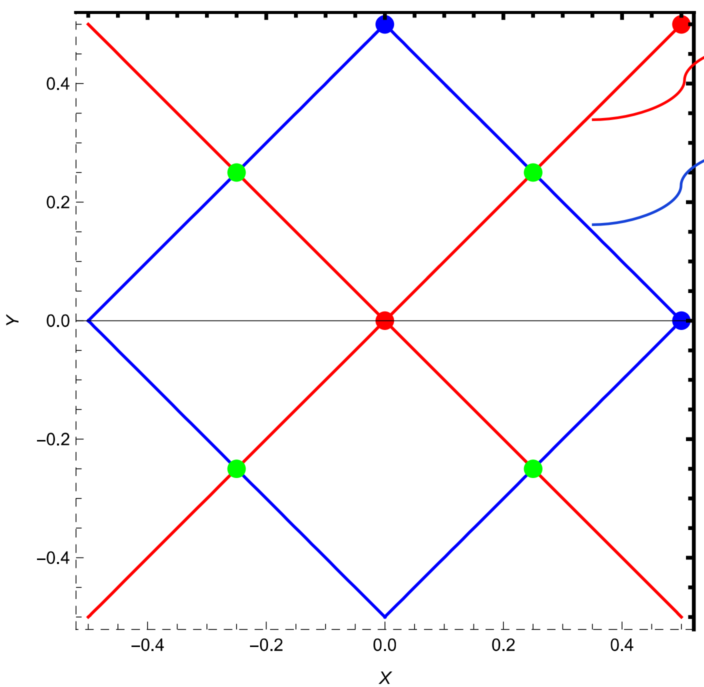
$$U(1)^{p+q-2} \subset U(1)^{16} \longrightarrow SU(p) \times SU(q)$$

case 1

$$\begin{cases} t_1^{a_1} = t_1^{a_2} = \cdots = t_1^{a_p} = x, \quad 1 \le a_i \le 8,\ 2 \le p \le 8 \\ t_1^{b_1} = t_1^{b_2} = \cdots = t_1^{b_q} = y, \quad \underline{1 \le b_j \le 8,}\ 2 \le q \le 8 \end{cases}$$

➡ Massless vectors in $SU(p) \times SU(q)$ adj rep

- There are the special lines and points in x-y plane:



Red line: vectors in $SU(p+q)$ adj rep

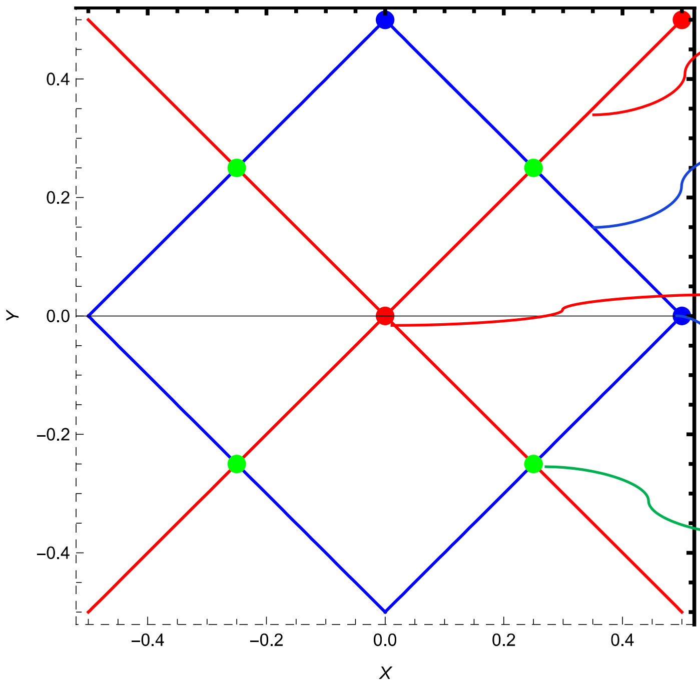
Blue line: spinors in two bi-fund reps of $SU(p) \times SU(q)$

case 1

$$\begin{cases} t_1^{a_1} = t_1^{a_2} = \cdots = t_1^{a_p} = x, & 1 \le a_i \le 8,\ 2 \le p \le 8 \\ t_1^{b_1} = t_1^{b_2} = \cdots = t_1^{b_q} = y, & \underline{1 \le b_j \le 8,}\ 2 \le q \le 8 \end{cases}$$

➡ Massless vectors in $SU(p) \times SU(q)$ adj rep

- There are the special lines and points in x-y plane:



Red line: vectors in $SU(p+q)$ adj rep

Blue line: spinors in two bi-fund reps of $SU(p) \times SU(q)$

Red points: vectors in $SO(2p+2q)$ adj rep

Blue points: vectors in $SO(2p) \times SO(2q)$ adj rep, spinors in bi-fund rep of $SO(2p) \times SO(2q)$

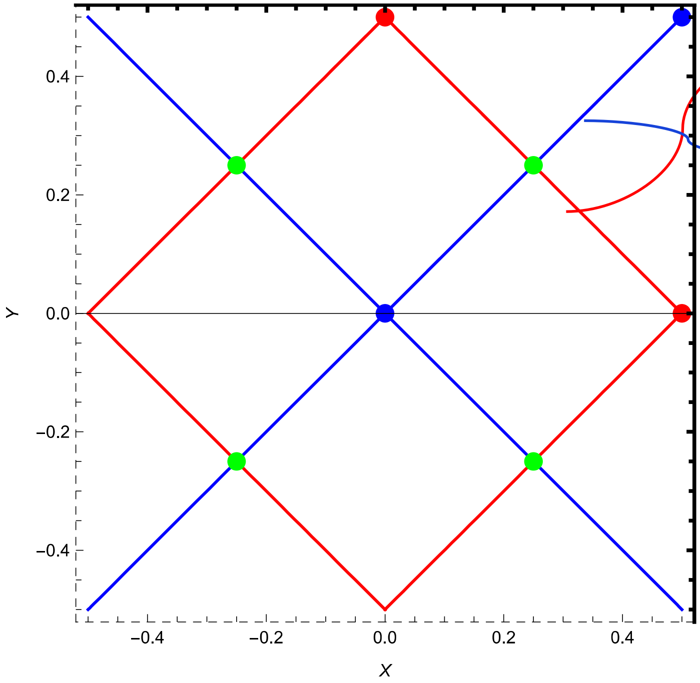
Green points: vectors in $SU(p+q)$ adj rep, spinors in anti-sym rep of $SU(p+q)$

case 2

$$\begin{cases} t_1^{a_1} = t_1^{a_2} = \cdots = t_1^{a_p} = x, & 1 \le a_i \le 8,\ 2 \le p \le 8 \\ t_1^{b_1} = t_1^{b_2} = \cdots = t_1^{b_q} = y, & \underline{9 \le b_j \le 16,}\ 2 \le q \le 8 \end{cases}$$

➡ Massless vectors in $SU(p) \times SU(q)$ adj rep

➢ There are the special lines and points in x-y plane:



Red line: vectors in $SU(p+q)$ adj rep

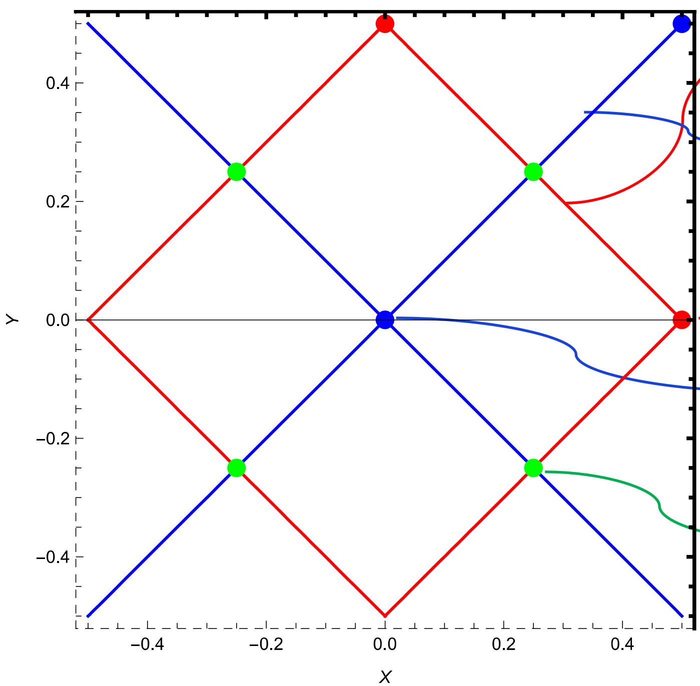
Blue line: spinors in two bi-fund reps of $SU(p) \times SU(q)$

case 2

$$\begin{cases} t_1^{a_1} = t_1^{a_2} = \cdots = t_1^{a_p} = x, & 1 \le a_i \le 8,\ 2 \le p \le 8 \\ t_1^{b_1} = t_1^{b_2} = \cdots = t_1^{b_q} = y, & \underline{9 \le b_j \le 16,}\ 2 \le q \le 8 \end{cases}$$

➡ Massless vectors in $SU(p) \times SU(q)$ adj rep

➢ There are the special lines and points in x-y plane:



Red line: vectors in $SU(p+q)$ adj rep

Blue line: spinors in two bi-fund reps of $SU(p) \times SU(q)$

Red points: vectors in $SO(2p+2q)$ adj rep

Blue points: vectors in $SO(2p) \times SO(2q)$ adj rep, spinors in bi-fund rep of $SO(2p) \times SO(2q)$

Green points: vectors in $SU(p+q)$ adj rep, spinors in anti-sym rep of $SU(p+q)$

➢ these special points of intersection are written as

$$t_1^A = \left((0)^p, \left(\frac{1}{2}\right)^q, \left(\frac{1}{4}\right)^r, \left(-\frac{1}{4}\right)^s\right), \quad t_1^{A'} = \left((0)^{p'}, \left(\frac{1}{2}\right)^{q'}, \left(\frac{1}{4}\right)^{r'}, \left(-\frac{1}{4}\right)^{s'}\right),$$

$$\left[\ p+q+r+s = p'+q'+r'+s' = 8, \quad I = (A, A')\ \right]$$

the massless spectrum is

- The gauge bosons of $SO(2P) \times SO(2Q) \times (SU(R) \times U(1))$;
- The spinors in $(\mathbf{2P}, \mathbf{2Q}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \frac{\boldsymbol{R(R-1)}}{\mathbf{2}}) \oplus \left(\mathbf{1}, \mathbf{1}, \overline{\frac{\boldsymbol{R(R-1)}}{\mathbf{2}}}\right)$ of $SO(2P) \times SO(2Q) \times SU(R)$

$$\left[\ P = p + q', \quad Q = q + p', \quad R = r + s + r' + s'\ \right]$$

$n_F = n_B$ cases

$(P, Q) = (9, 7)$ or $(7, 9)$

$(P, Q) = (6, 6)$

The gauge symmetry is maximally enhanced at the points with $p = q' = 8$ or $q = p' = 8$, where the gauge symmetry is $SO(32)$ and there are no massless fermions.

- The cosmological constant is calculated up to exponentially suppressed terms:

$$\Lambda^{(9)}_{ModelI}(t_1^I, t_2) \simeq \frac{48}{\pi^{14}}\left(\frac{a_0}{\sqrt{\alpha'}}\right)^9 8\left\{-24 + 4\sum_{A=1}^{8}\sum_{A'=9}^{16}\cos\left(2\pi t_1^A\right)\cos\left(2\pi t_1^{A'}\right) - 4\sum_{\substack{A,B=1\\A>B}}^{8}\cos\left(2\pi t_1^A\right)\cos\left(2\pi t_1^B\right) - 4\sum_{\substack{A',B'=9\\A'>B'}}^{16}\cos\left(2\pi t_1^{A'}\right)\cos\left(2\pi t_1^{B'}\right)\right\}$$

- the stable points in moduli space:

$$\frac{\partial \Lambda^{(9)}}{\partial t_1^I} = 0, \quad \frac{\partial^2 \Lambda^{(9)}}{\partial t_1^I \partial t_1^J} \geq 0 \quad \xrightarrow{\text{solve}} \quad t_1^I = \left((0)^8; \left(\frac{1}{2}\right)^8\right), \left(\left(\frac{1}{2}\right)^8; (0)^8\right)$$

➡ stabilized when the gauge symmetry is maximally enhanced.

the $n_F = n_B$ cases are only extremal

V)

- completed the analysis in the case of $d = 1$ in susy restoring region
- a few $n_F = n_B$ models found
- the minimum is SO(32)/$E_8 \times E_8$ gauge sym., massless bosons only
- $\frac{\partial}{\partial \alpha'} \Lambda_{\text{string}} =$ dilaton tadpole is small to this order & will be made harmless

VI) ● Simplest d dim. generalization: sketch & results

INkjm3

- **assumptions**
  - still in the susy restoring region
  - only the X^9 direction is twisted. Otherwise just d. dim toroidal comp.
- **construction**
  - prepare the following (16+d,d) momentum lattice

$$\Lambda\left[\Gamma;\alpha,\beta\right]\equiv\left(\eta\bar{\eta}\right)^{-D}\eta^{-16}\sum_{m^I\in\Gamma}\sum_{w^9\in\boldsymbol{Z}+\alpha}\sum_{n_9\in 2(\boldsymbol{Z}+\beta)}\sum_{w^{i\neq 9},n_{i\neq 9}\in\boldsymbol{Z}}q^{\frac{1}{2}\left(|\ell_L|^2+p_L^2\right)}\bar{q}^{\frac{1}{2}p_R^2},$$

  where Γ is a 16-dimensional Euclidean lattice.

  - Γ_{16} : 16 dim. even self-dual lattice
  - The $\boldsymbol{Z}_2$ action is $(-1)^F Q_L \mathcal{T}^{(9)}$
  - By using a shift vector $\delta^I\in\frac{1}{2}\Gamma_{16}$, Q_L can be represented by $\exp\left(2\pi i m\cdot\delta\right)$ for $m^I\in\Gamma_{16}$. We split Γ_{16} into

$$\Gamma_{16}^{+}=\left\{m^I\in\Gamma_{16}\,\middle|\,\delta\cdot m\in\boldsymbol{Z}\right\},\qquad\Gamma_{16}^{-}=\left\{m^I\in\Gamma_{16}\,\middle|\,\delta\cdot m\in\boldsymbol{Z}+\frac{1}{2}\right\}.$$

- **output:**

$$\begin{aligned}Z_{int}^{(10-D)}=Z_B^{(8-D)}\{&\bar{V}_8\left(\Lambda\left[\Gamma_{16}^{+};0,0\right]+\Lambda\left[\Gamma_{16}^{-};0,1/2\right]\right)\\&-\bar{S}_8\left(\Lambda\left[\Gamma_{16}^{+};0,1/2\right]+\Lambda\left[\Gamma_{16}^{-};0,0\right]\right)\\&+\bar{O}_8\left(\Lambda\left[\Gamma_{16}^{+}+\delta;1/2,0\right]+\Lambda\left[\Gamma_{16}^{-}+\delta;1/2,1/2\right]\right)\\&-\bar{C}_8\left(\Lambda\left[\Gamma_{16}^{+}+\delta;1/2,1/2\right]+\Lambda\left[\Gamma_{16}^{-}+\delta;1/2,0\right]\right)\}.\end{aligned}$$

- **results**
  - gauge symmetry enhancement pattern is the same as before
  - also in 1:1 correspondence with the corresponding toroidal comp.

    in M_1 superstring.

VII) Recall

- $$Z_{\mathrm{SO(32)susy}} = (\bar{V}_8 - \bar{S}_8)(O_{16}O_{16} + V_{16}V_{16} + \text{massive only})$$
$$\approx 0 \qquad \Uparrow \qquad \Uparrow$$
heterotic gauged 10d sugra   gravity(ino) adj.   bifund.

in evaluation $\bar{V}_8 \approx \bar{S}_8 \equiv (\overline{VS})_{\mathrm{eval}}$

- $$Z_{\mathrm{SO(16)\times SO(16)nosusy}} = \bar{V}_8 O_{16}O_{16} - \bar{S}_8 V_{16}V_{16} + \text{massive only}$$
$$\approx (\overline{VS})_{\mathrm{eval}}(O_{16}O_{16} - V_{16}V_{16}) + \cdots$$
Ⓕ gravitino & gaugino removed   Ⓑ bifund. vector removed

- $$Z_{\mathrm{IT}} = \Lambda_{00}\bar{V}_8 O_{16}O_{16} - \Lambda_{00}\bar{S}_8 V_{16}V_{16} + \Lambda_{1/2,0}\bar{V}_8 V_{16}V_{16} - \Lambda_{1/2,0}\bar{S}_8 O_{16}O_{16} + \cdots$$
$$\approx (\Lambda_{00} - \Lambda_{1/2,0})(\overline{VS})_{\mathrm{eval}}(O_{16}O_{16} - V_{16}V_{16}) + \text{massive}$$
They came back!! as 1st KK excitations

- Both $\Lambda^{\text{1-loop}}_{\text{cosmo}}$ & gauge sym. enhancement can be understood in QFT of SO(16) × SO(16) heterotic gauged supergravity coupled with bifund. supermultiplet where SUSY broken by the twisted circle.

Gauge symmetry enhancement in EFT

- starting point : 10D $SO(32)$ SYM + SUGRA

twisted circle comp. :

$$A_M = \sum_{I<J} A_M^{(IJ)} T^{(IJ)} \qquad I,J = 1,\cdots,32$$

$T^{(IJ)}$: $SO(32)$ generator

$$A_M(x^\mu, y+2\pi R) = e^{2\pi i\omega} A_M(x^\mu, y) \;\Rightarrow\; A_M(x^\mu, y) = \sum_N \sum_{I<J} A_{M(N)}^{(IJ)}(x^\mu)\, e^{i(N+\omega_{IJ})y/R}\, T^{(IJ)}$$

$\omega_{IJ} = 0$ or $1/2$

Turn on one WL :

- VEV of A_9 ⟷ Wilson line $\mathcal{A}$: $A_9 = \mathcal{A}\, T^{15,16}$ ($T^{15,16}$: Cartan of $SO(32)$)
- mass formula of $A_{\mu\,(N)}^{(IJ)}$: $m_{N(I,J)}^2 = \left[\frac{1}{R}(N+\omega_{IJ}) - \mathcal{A}\right]^2$
- condition: $n = AR$ (1) $n \in \mathbf{Z}$ (2) $n \in \mathbf{Z} + 1/2$

➡ gauge symmetry is enhanced : $SO(16)\times SO(14)\times U(1) \rightarrow$
- (1) $SO(16)\times SO(16)$
- (2) $SO(18)\times SO(14)$

Turn on the full set of WL : $A_9 = \sum_{m=1}^{16} \mathcal{A}^{(m)} T^{(2m-1,2m)}$

Cartan of $SO(32)$

- Define: $A_{\mu(N)}^{(m,n)} \equiv \left(A_{\mu(N)}^{(2m-1,2n-1)} \pm i A_{\mu(N)}^{(2m-1,2n)}\right) \pm i \left(A_{\mu(N)}^{(2m,2n-1)} \pm i A_{\mu(N)}^{(2m,2n)}\right)$ $\quad 1 \leq m < n \leq 16$
- mass formula of $A_{\mu\,(N)}^{(m,n)}$: $m_{N(m,n)}^2 = \left[-\frac{1}{R}(N + \omega_{m,n}) \pm \mathcal{A}^{(m)} \pm \mathcal{A}^{(n)}\right]^2$
- condition of WLs : $\mathcal{A}^{(a_1)} = \cdots = \mathcal{A}^{(a_p)} \quad (1 \leq a_i \leq 8,\ \ 2 \leq p \leq 8)$

➡ gauge symmetry is enhanced : $U(1)^{p-1} \subset U(1)^{16} \to SU(p)$

Other enhancement patterns in Interpolating model also appear

➡ Gauge symmetry enhancement can be understood in EFT