SPECIAL RELATIVITY HOMEWORK – WEEK 5

Exercise 1. *Consider a particle with mass m and charge q in an electromagnetic field. Recall that the Lorentz force law $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ is relativistically correct, under the true definition of force $\mathbf{F} = d\mathbf{p}/dt$, with $\mathbf{p}$ the spatial component of $p^\mu = mu^\mu$.*

Find the particle's trajectory $x^\mu(\tau)$, 4-velocity $u^\mu(\tau)$, 4-acceleration $\alpha^\mu(\tau)$ and its magnitude $\alpha_\mu \alpha^\mu$, in the following cases (τ is proper time):

1. *Constant electric field $\mathbf{E} = (E, 0, 0)$, $\mathbf{B} = 0$, initial velocity $\mathbf{v}(0) = 0$.*
2. *Constant magnetic field $\mathbf{B} = (B, 0, 0)$, $\mathbf{E} = 0$, initial velocity $\mathbf{v}(0) = (v_\parallel, v_\perp, 0)$.*

Exercise 2. *A proton is the ground state of two "up" quarks and one "down" quark. Its mass is $M_p = 939 MeV$. The lowest excited state of the same quarks is the baryon called Δ^+, whose mass is $M_\Delta = 1232 MeV$.*

- *A proton at rest gets hit by a photon, and turns into a Δ^+. What is the minimal energy of the photon?*
- *Now, consider the same process in a different reference frame, where the energy of the photon is $\varepsilon = 10^{-3} eV$ – typical of the Cosmic Microwave Background. What is the minimal energy of the proton at which the Δ^+ can be formed? This is the GZK bound on the energy of cosmic rays.*