GENERAL RELATIVITY HOMEWORK – WEEK 3

Exercise 1. *In this exercise, we play with matrices in 2d Euclidean space.*

1. *What is the formula for the transformation of a matrix A_{ij} under a rotation R_{ij}?*
2. *Write out the rotation matrix R_{ij} for the rotation angles $\pi/4$, $\pi/2$ and π.*
3. *Apply these rotations to the traceless symmetric matrix $A_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.*

Exercise 2. *Consider the particle worldline $x(t) = \sqrt{\rho^2 + t^2}$, where ρ is a constant. Rewrite it in the form $x^\mu(\tau)$, where τ is proper time. Find the 4-velocity $u^\mu(\tau)$ and 4-acceleration $\alpha^\mu(\tau)$. What are the (Lorentz-invariant) magnitudes of x^μ, u^μ and α^μ?*

Exercise 3. *The relativistic formula for momentum is $\mathbf{p} = \dfrac{m\mathbf{v}}{\sqrt{1-\mathbf{v}^2}}$. Find the force $\mathbf{F} = d\mathbf{p}/dt$ acting on a particle of mass m that moves along the worldline from Exercise 2.*