Flat space physics from AdS/CFT

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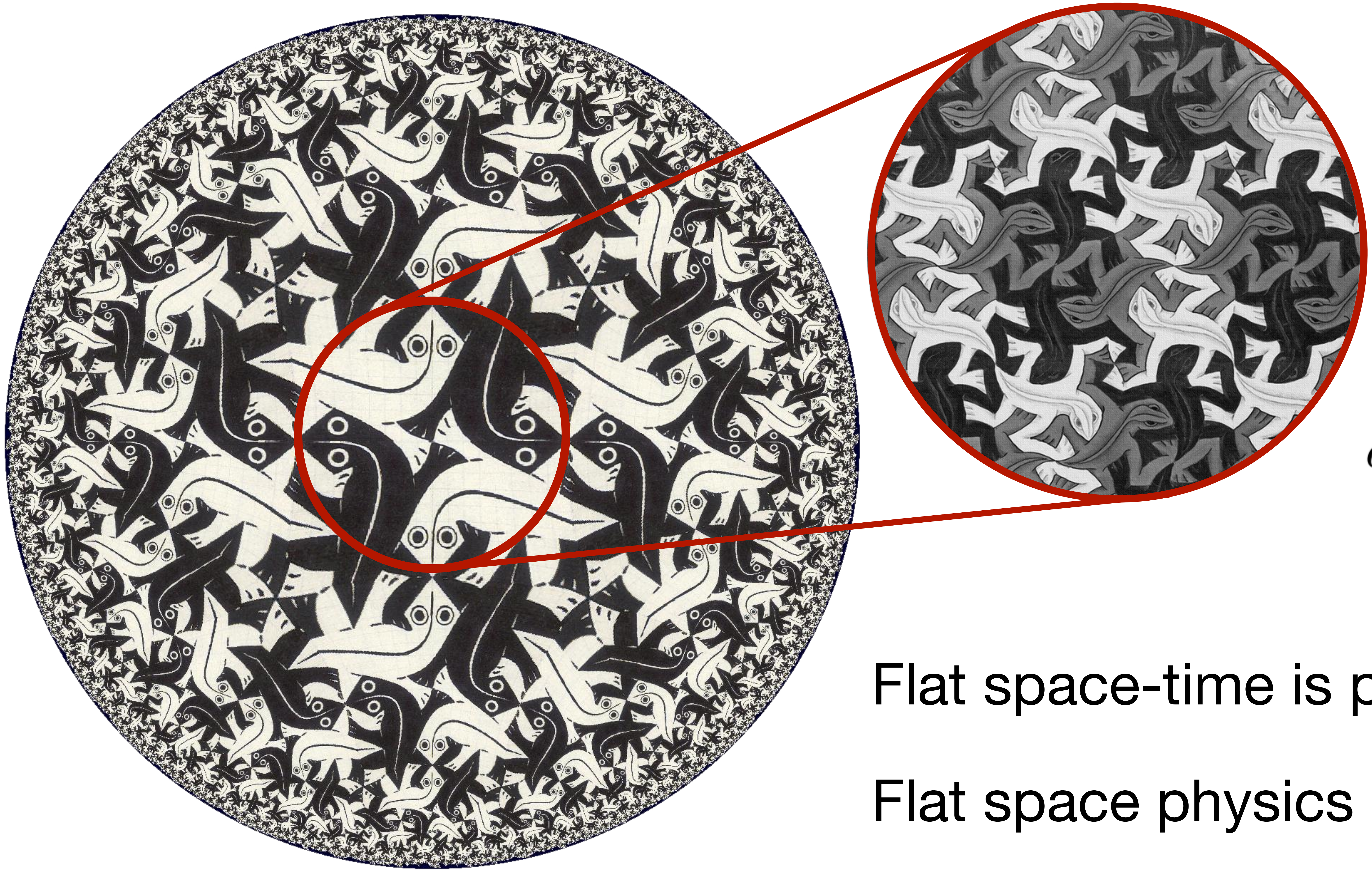
Based on: arxiv:1905.02729
arxiv:2005.03667
work with D. Neuenfeld

AdS/CFT
Conformal symmetry
Correlation functions
Mellin amplitudes

BMS symmetry
Flat space holography
Scattering amplitudes

Princeton gravity Initiative

Indirect holography



$$ds^2 \xrightarrow[L\to\infty]{} -dt^2 + dr^2 + r^2 d\Omega_{d-1}^2$$

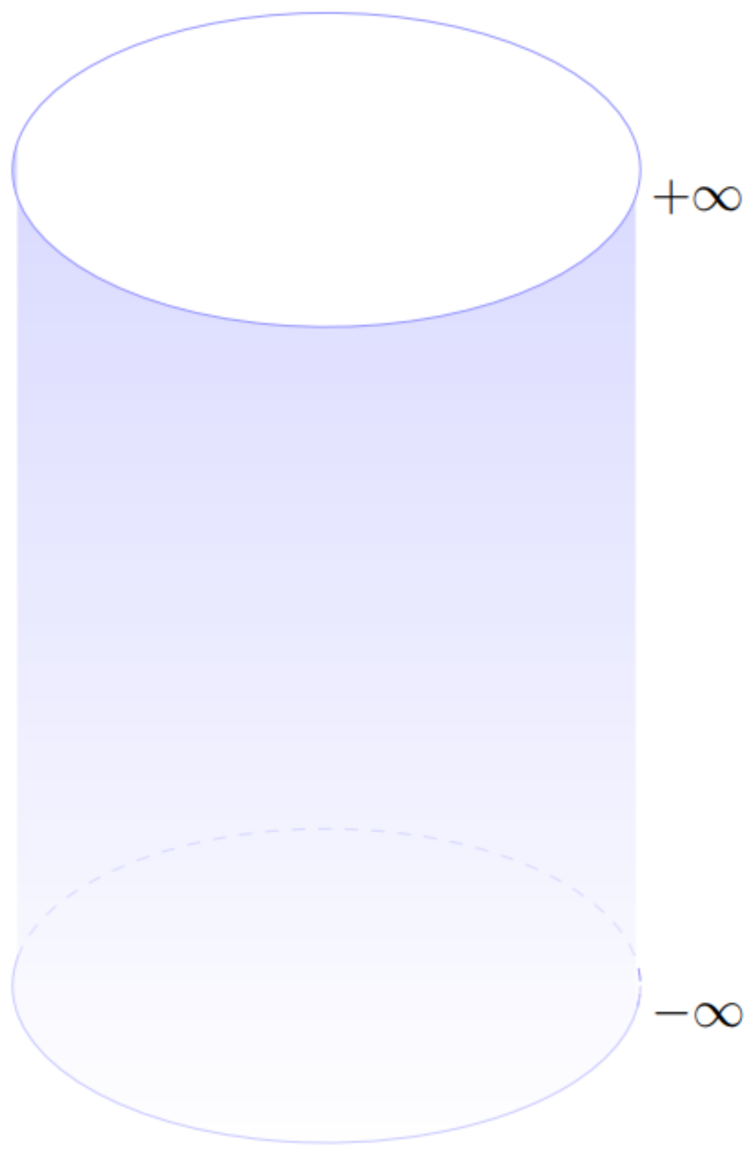
Flat space-time is part of AdS

Flat space physics are encoded into AdS/CFT

$$ds^2 = \frac{L^2}{\cos^2\rho}\left(-d\tau^2 + d\rho^2 + \sin^2\rho\, d\Omega_{d-1}^2\right)$$

Observables in AdS vs Flat space

In the presence of gravity, diffeomorphism invariance makes it impossible to define local correlation functions.



In AdS, gauge invariant observables can be defined at the boundary as conformal correlators

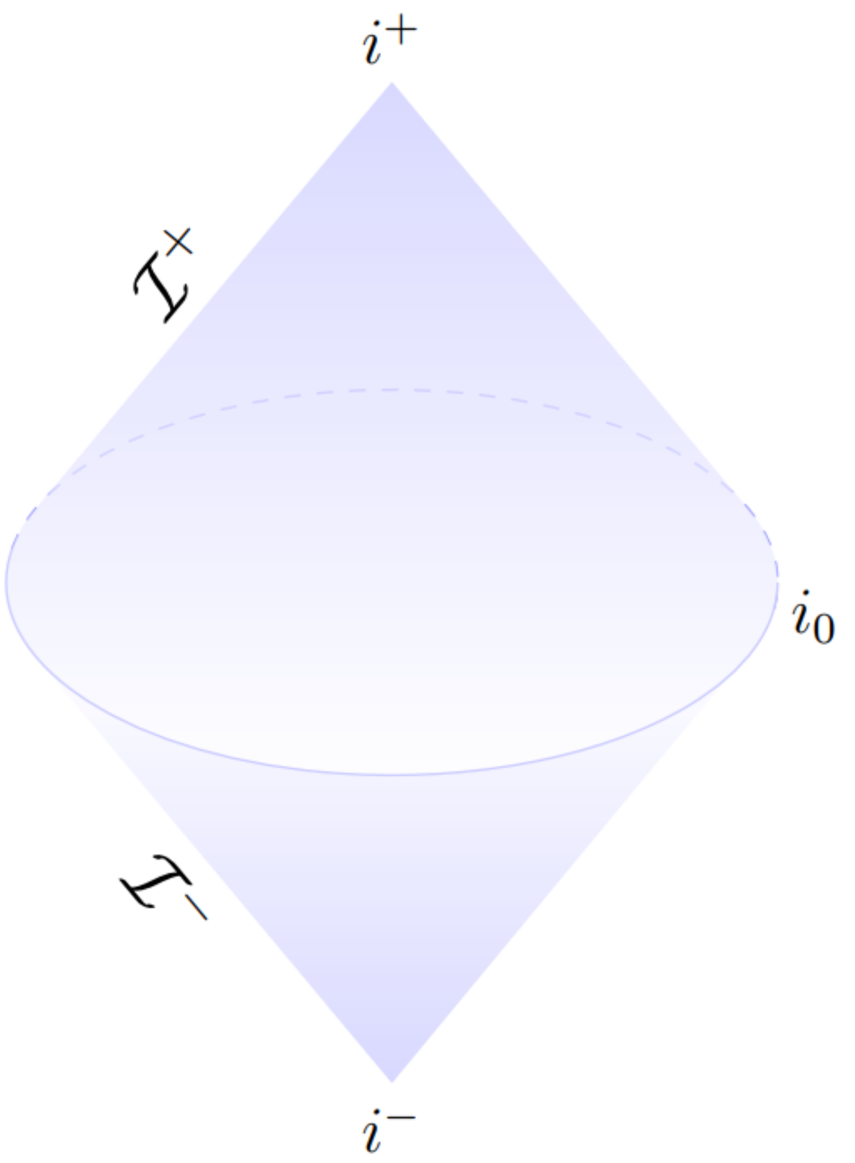
$$\langle \underbrace{\mathcal{O}_1(x_1)\mathcal{O}_2(x_2)}_{\text{operators}}\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle$$

Flat space has no boundary.

One can hope to define observables asymptotically

The only precise observable is the S-matrix

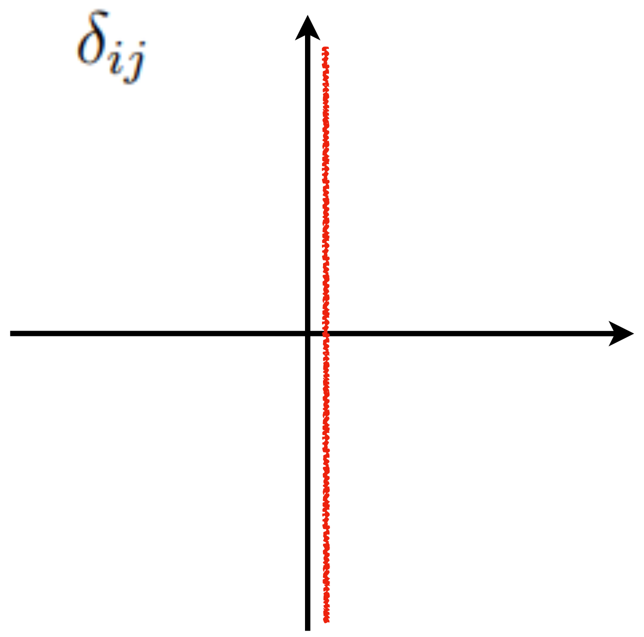
$$\underbrace{\langle \text{out}|\text{in}\rangle}_{\text{scattering states}}$$



A proposal involving Mellin amplitudes

[Penedones 2010]

$$\langle \mathcal{O}_1(x_1)\dots\mathcal{O}_n(x_n)\rangle = \frac{\mathcal{N}}{(2\pi i)^{n(n-3)/2}}\int d\delta_{ij}\, M(\delta_{ij}) \prod_{i<j}^{n}\Gamma(\delta_{ij})\left(x_{ij}^2\right)^{-\delta_{ij}} \qquad \sum_{j\neq i}^{n}\delta_{ij} = \Delta_i$$



Lorentzian vectors k_i

$$\delta_{ij} = k_i\cdot k_j = \frac{\Delta_i+\Delta_j - s_{ij}}{2} \qquad s_{ij} = -(k_i+k_j)^2$$

$M(s_{ij})$ {

- Crossing symmetric
- Meromorphic with simple poles: $s_{13} = \Delta_k - l_k + 2m$

M looks like a scattering amplitude!

$$M(s_{ij}) \approx \frac{R^{n(1-d)/2+d+1}}{\Gamma\left(\frac{1}{2}\sum_i \Delta_i - \frac{d}{2}\right)}\int_0^\infty d\beta\, \beta^{\frac{1}{2}\sum_i \Delta_i - \frac{d}{2}-1} e^{-\beta}\, T\left(S_{ij} = \frac{2\beta}{R^2}s_{ij}\right)$$

Flat space scattering amplitude

(MASSLESS case)

Proven using first principles

[Fitzpatrick Kaplan 2012]

MASSIVE case

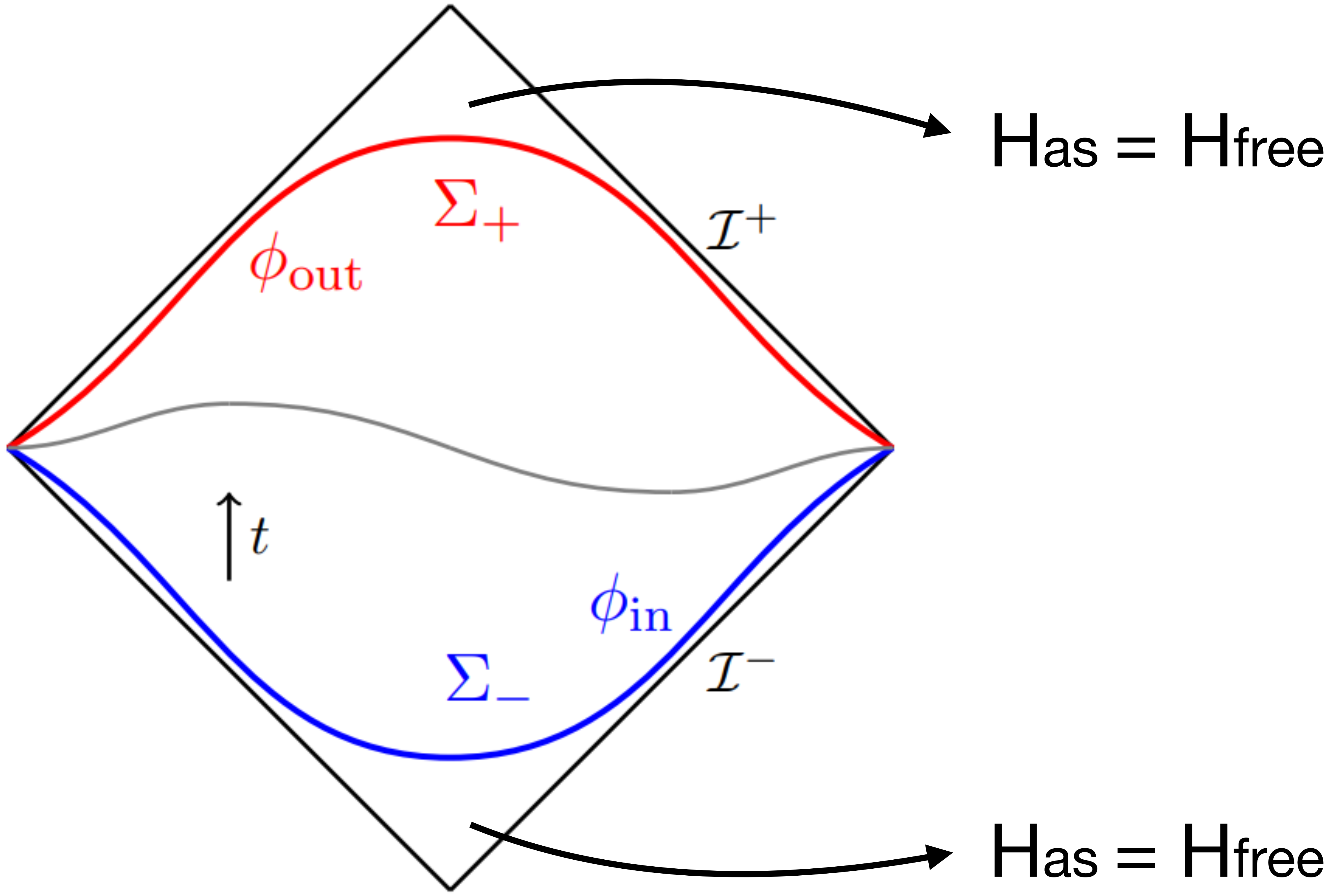
$$(m_1)^a\, T(k_i) = \lim_{\Delta_i\to\infty}\frac{(\Delta_1)^a}{\mathcal{N}} M\left(\gamma_{ij} = \frac{\Delta_i\Delta_j}{\Delta_1+\cdots+\Delta_n}\left(1+\frac{k_i\cdot k_j}{m_i m_j}\right)\right)$$

[Paulos, Penedones, Toledo, van Rees, Vieira 2010]

S-matrix

$$\langle \mathrm{out}|\mathrm{in}\rangle = \sum_{\phi_{\mathrm{a/b}}} \Psi^*_{\mathrm{out}}[\phi_{\mathrm{b}}] \int_{\phi|_{\Sigma_-}=\phi_{\mathrm{a}}}^{\phi|_{\Sigma_+}=\phi_{\mathrm{b}}} \mathcal{D}\phi\, e^{iS[\phi]}\Psi_{\mathrm{in}}[\phi_{\mathrm{a}}]\,,$$

Fock space wave-functionals



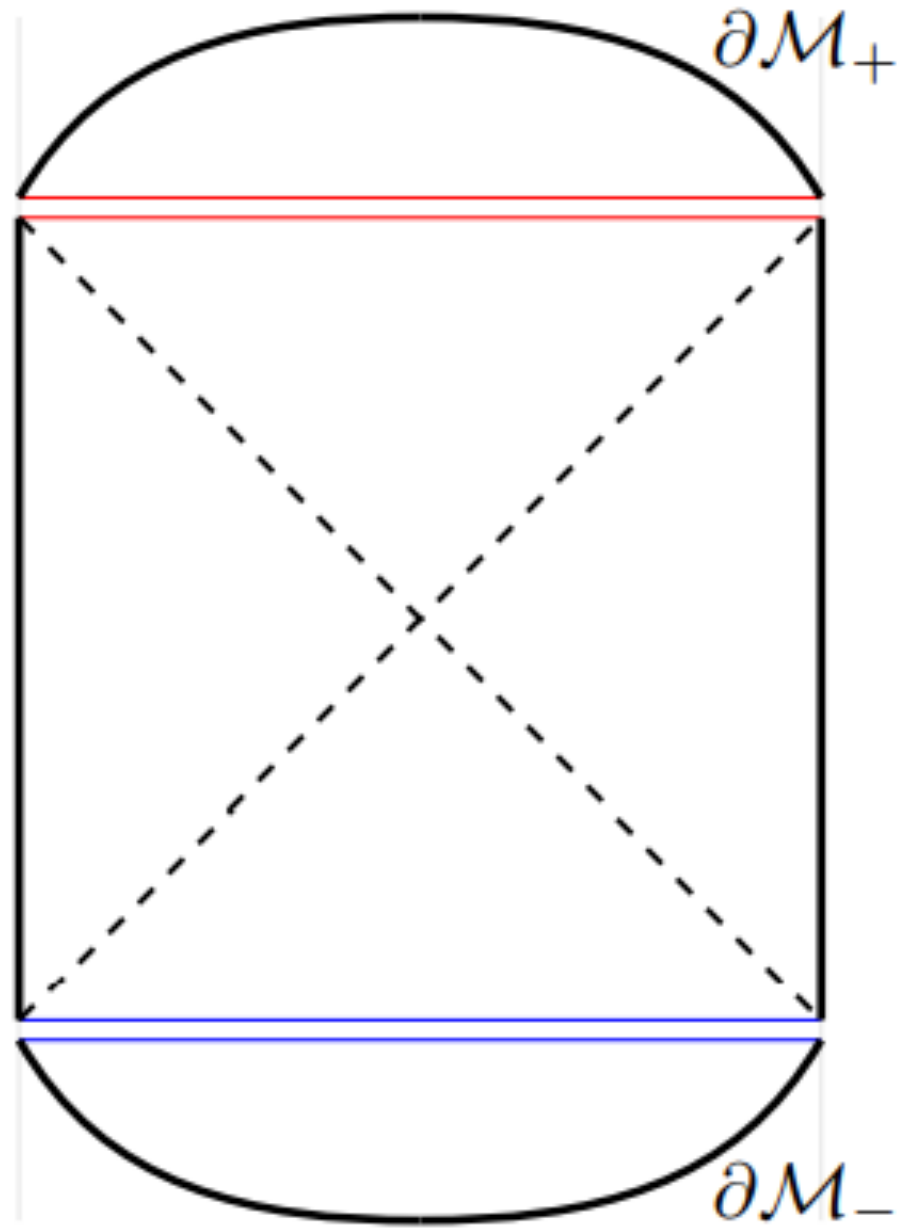
Objective of scattering theory: Compute overlap of two different scattering states of the **full** hamiltonian H

Such states are usually unavailable. Simpler scattering states must be used, by approximating the asymptotic Hamiltonian

Asymptotic decoupling: Asymptotic Hamiltonian is considered to be free.

Asymptotic states form a Fock space. How do we construct these in AdS/CFT?

Extrapolate dictionary in AdS/CFT



$$\langle \text{in}| e^{i\int_{\partial \text{AdS}} \phi_0(x)\mathcal{O}(x)} |\text{out}\rangle = \sum_{\phi_{\text{a/b}}} \Psi^*_{\text{out}}[\phi_{\text{b}}] \int_{\phi|_{\Sigma_-^{\text{AdS}}}=\phi_{\text{a}}}^{\phi|_{\Sigma_+^{\text{AdS}}}=\phi_{\text{b}}} [\mathcal{D}\Phi]_{\phi_0}\, e^{iS(\phi)} \Psi_{\text{in}}[\phi_{\text{a}}] \,.$$

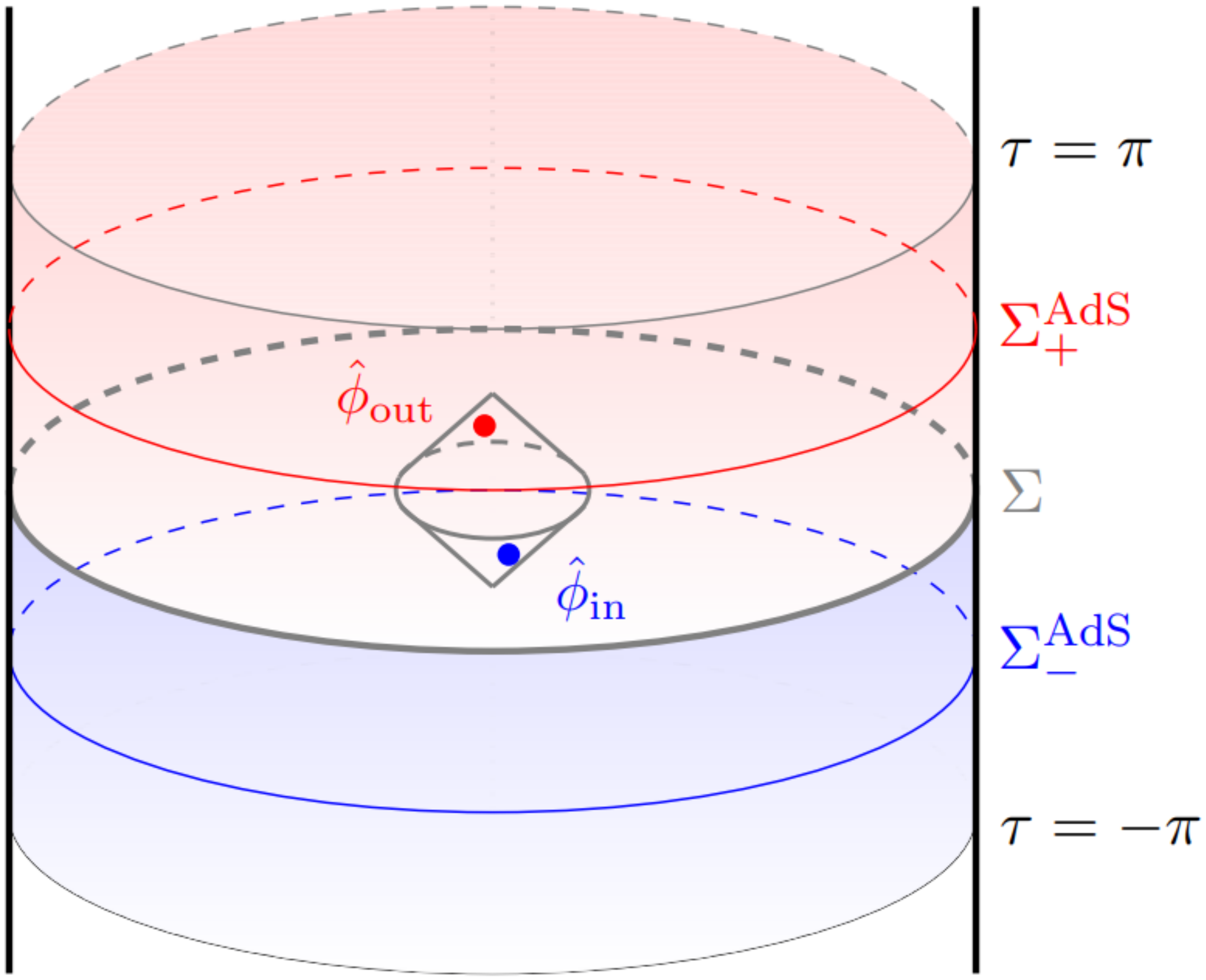
The in/out wave-functionals can be generated with Euclidean path integrals:

$$\Psi_{\text{in}}[\phi_{\text{a}}] \equiv \langle \phi_{\text{a}} | \Psi_{\text{in}} \rangle = \int_0^{\phi_{\text{a}}} [\mathcal{D}\Phi]_{\phi_i} e^{-S_{\mathcal{M}_-}[\Phi]} \,. \quad \Rightarrow \quad |\Psi_i\rangle = e^{-\int_{\partial\mathcal{M}_-} \mathcal{O}\phi_i} |0\rangle$$

It is not clear how to choose sources so that the state is a Fock space state in the flat limit!

Our approach will be different: $$|\Psi\rangle = \hat{\Psi}|0\rangle$$

Construct free local bulk operators in the CFT (HKLL) → Extract creation/annihilation operators (Fourier transform) → Take a flat limit (Large AdS radius)



Field can be reconstructed in the CFT using HKLL.

$$\hat\phi_{\rm in/out}(\rho,x) = \frac{1}{\pi}\int_{\mathcal{T}} d\tau' \int d^2\Omega\; \left[K_+(\rho,x;x')\mathcal{O}^+(x') + K_-(\rho,x;x')\mathcal{O}^-(x')\right]$$

Free field in Minkowski space at late/early times.

$$\hat\phi(x) = \int \frac{d^3\vec p}{(2\pi)^3}\frac{1}{\sqrt{2\omega_{\vec p}}}\left(\hat a_{\vec p}e^{ip\cdot x} + \hat a^\dagger_{\vec p}e^{-ip\cdot x}\right)\,, \quad \text{with} \quad x\in \text{Mink}_{3+1}\,.$$

Fock space states can be constructed by acting with creation and annihilation operators.

$$|\vec k, \text{in}\rangle = \sqrt{2\omega_{\vec k}}\,\hat a^\dagger_{\text{in},\vec k}|0\rangle\,, \quad \text{and} \quad \langle \vec p, \text{out}| = \langle 0|\sqrt{2\omega_{\vec p}}\,\hat a_{\text{out},\vec p}\,.$$

$$\hat a_{\text{in/out},\vec p} = \lim_{t\to\mp\infty}\frac{i}{\sqrt{2\omega_{\vec p}}}\int_\Sigma d^3\vec x\, e^{-ip\cdot x}\overleftrightarrow{\partial_0}\,\hat\phi(x)\,,$$

$$\hat a^\dagger_{\text{in/out},\vec p} = \lim_{t\to\mp\infty}\frac{-i}{\sqrt{2\omega_{\vec p}}}\int_\Sigma d^3\vec x\, e^{ip\cdot x}\overleftrightarrow{\partial_0}\,\hat\phi(x)\,.$$

Results

Massless fields

$$\sqrt{2\omega_{\vec{p}}}\, a_{\text{in},\vec{p}} = c_- \int_{-\pi}^{0} d\tau\, e^{i\omega_p L(\tau+\frac{\pi}{2})} \mathcal{O}^-(\tau, -\hat{p}) \;,$$

$$\sqrt{2\omega_{\vec{p}}}\, a^\dagger_{\text{in},\vec{p}} = c_+ \int_{-\pi}^{0} d\tau\, e^{-i\omega_p L(\tau+\frac{\pi}{2})} \mathcal{O}^+(\tau, -\hat{p}) \;,$$

$$\sqrt{2\omega_{\vec{p}}}\, a_{\text{out},\vec{p}} = c_+ \int_{0}^{\pi} d\tau\, e^{i\omega_p L(\tau-\frac{\pi}{2})} \mathcal{O}^-(\tau, \hat{p}) \;,$$

$$\sqrt{2\omega_{\vec{p}}}\, a^\dagger_{\text{out},\vec{p}} = c_- \int_{0}^{\pi} d\tau\, e^{-i\omega_p L(\tau-\frac{\pi}{2})} \mathcal{O}^+(\tau, \hat{p}) \;,$$

Integrals dominated by windows of size $\mathcal{O}(1/L)$ at $\tau = \pm\frac{\pi}{2}$

Massive fields

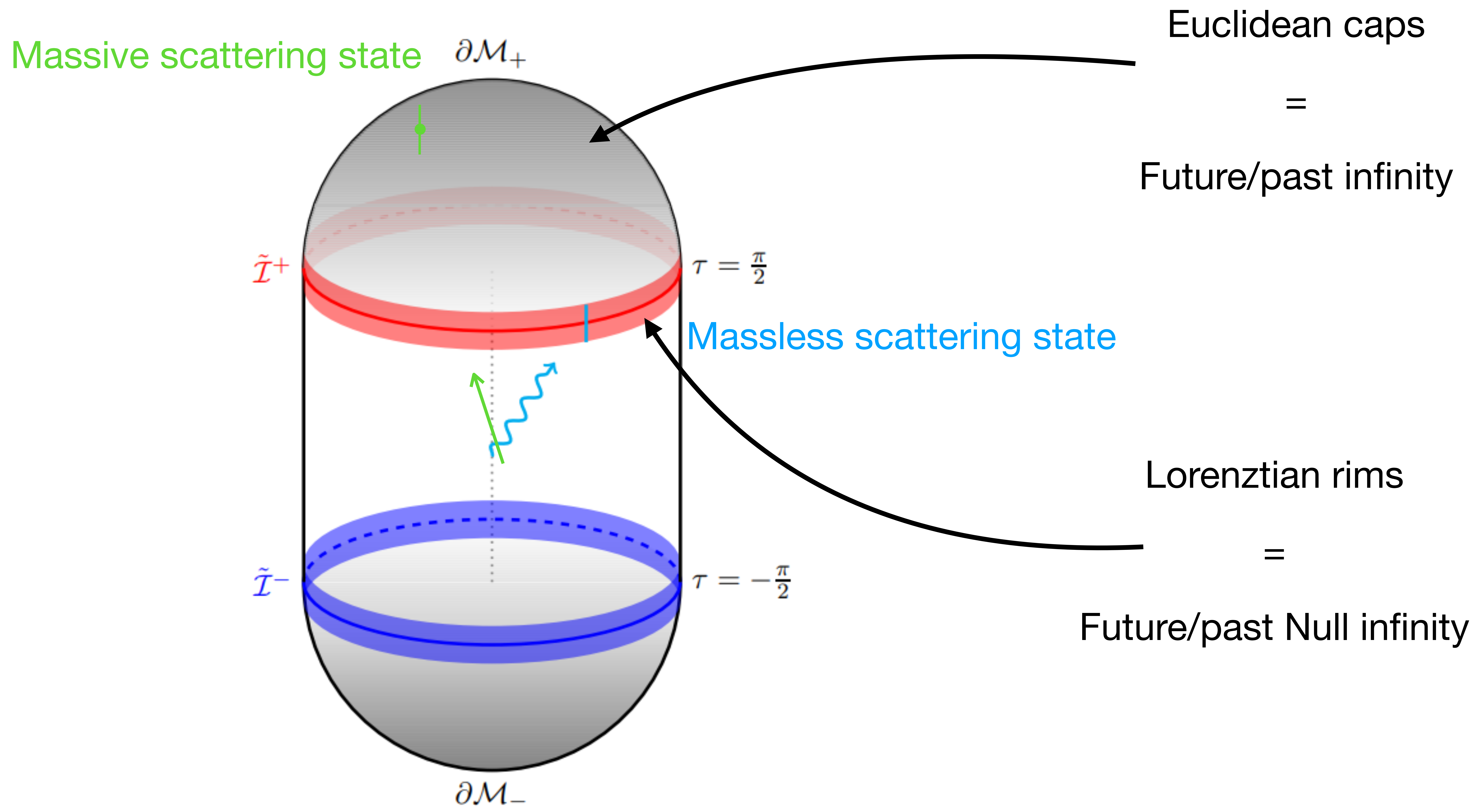
$$m^2L^2 = \Delta(\Delta - d)\,, \quad \Rightarrow \quad \Delta = \frac{d}{2} + mL + \mathcal{O}(L)^{-1}$$

Finite flat space mass involves large CFT operator dimensions

$$\sqrt{2\omega_{\vec{p}}}\,\hat{a}^\dagger_{\text{out},\vec{p}} = c_-(L, m, |\vec{p}|) \int_0^{\pi} d\tau' e^{i\omega_{\vec{p}}L\left[\frac{\pi}{2} + \frac{i}{2}\log\left(\frac{\omega_{\vec{p}}+m}{\omega_{\vec{p}}-m}\right) - \tau'\right]} \mathcal{O}^+(\tau', \hat{p})$$

Integrals dominated by windows of size $\mathcal{O}(1/L)$ at

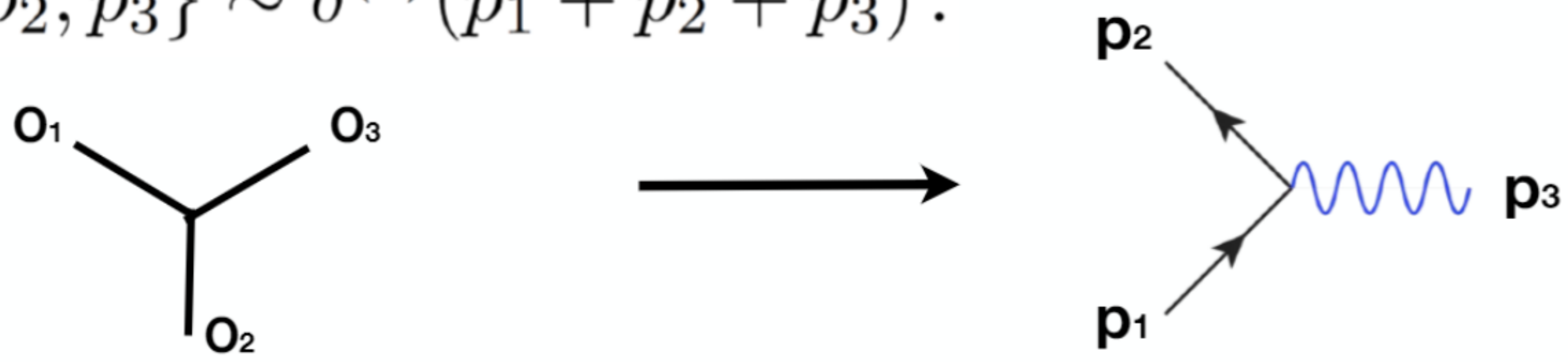
$$\mathrm{Re}(\tau') \sim \frac{\pi}{2} \qquad \mathrm{Im}\,(\tau') = \frac{1}{2}\log\left(\frac{\omega_{\vec{p}} + m}{\omega_{\vec{p}} - m}\right)$$


Massive scattering state
$\partial\mathcal{M}_+$
Euclidean caps
=
Future/past infinity
$\tilde{\mathcal{I}}^+$
$\tau = \frac{\pi}{2}$
Massless scattering state
Lorenztian rims
=
Future/past Null infinity
$\tilde{\mathcal{I}}^-$
$\tau = -\frac{\pi}{2}$
$\partial\mathcal{M}_-$

Some trivial examples in AdS_{d+1}

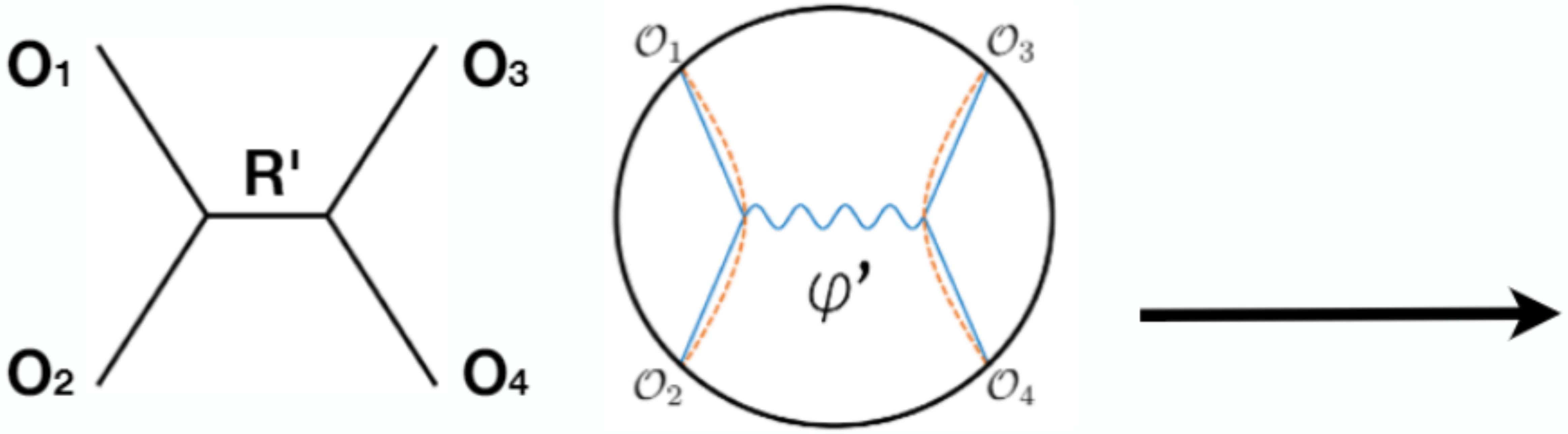
$$\mathcal{S}\{p_1, p_2\} \sim \delta^{(d)}(p_1 + p_2)\,,$$

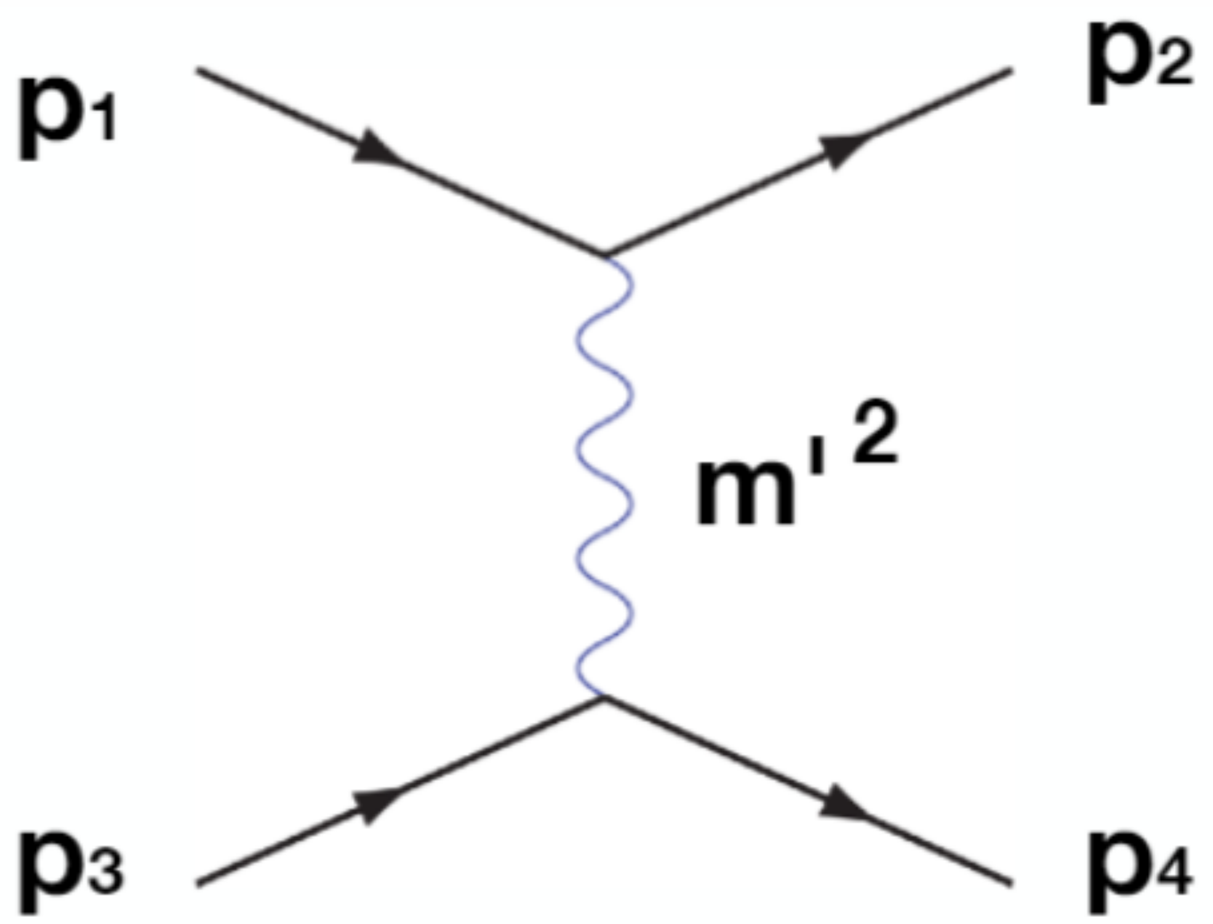
$$\mathcal{S}\{p_1, p_2, p_3\} \sim \delta^{(d)}(p_1 + p_2 + p_3)\,.$$



BMS_3 global block from CFT_2 global block

$$\mathcal{S}\{p_1, p_2, p_3, p_4\}_{p'^2=-m'^2} \sim \delta^{(3)}(p_1 + p_2 + p_3 + p_4)\delta\left(s + m'^2\right)$$

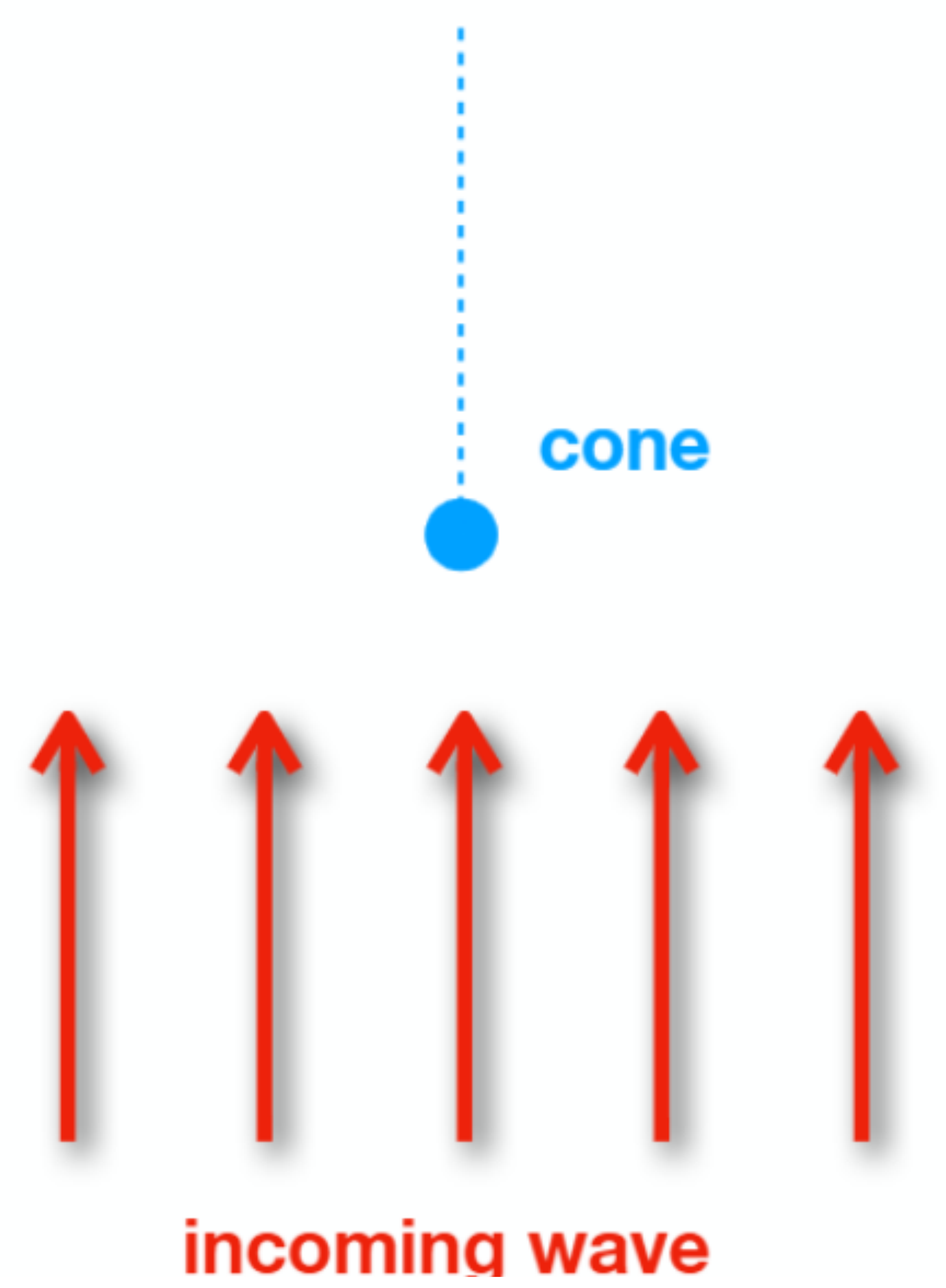


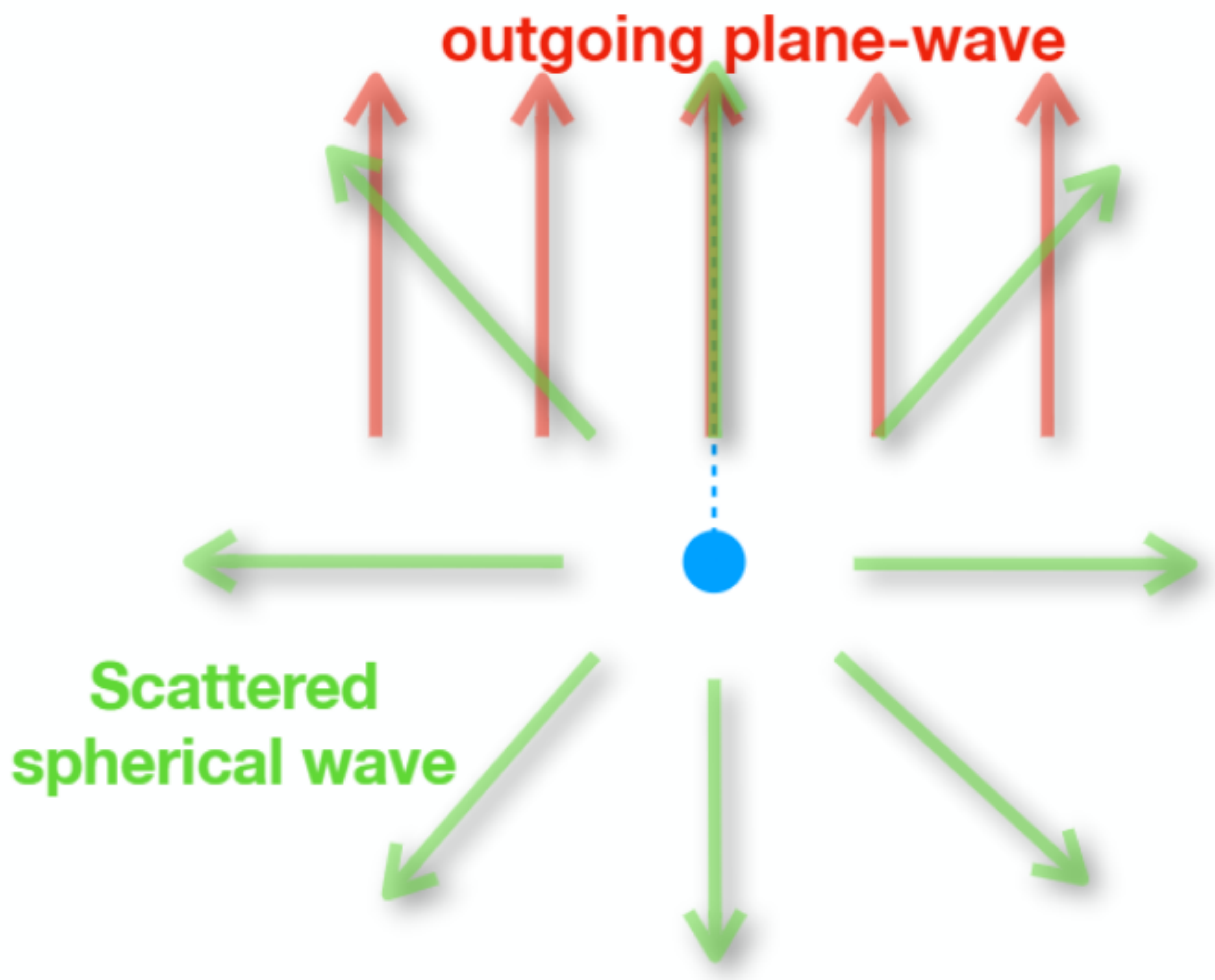


$$\mathcal{W}_{\Delta,0}(u,v) = \frac{\beta_{\Delta 12}}{2}\, u^{\frac{\Delta-\Delta_3-\Delta_4}{2}} \int_0^1 d\sigma\, \sigma^{\frac{\Delta+\Delta_{34}-2}{2}}(1-\sigma)^{\frac{\Delta-\Delta_{34}-2}{2}}(1-(1-v)\sigma)^{\frac{-\Delta+\Delta_{12}}{2}}$$
$$\times\, {}_2F_1\left(\frac{\Delta+\Delta_{12}}{2}, \frac{\Delta-\Delta_{12}}{2}; \Delta - \frac{d-2}{2}; \frac{u\sigma(1-\sigma)}{1-(1-v)\sigma}\right)\,.$$

Scattering against a cone (D=2+1)

[Deser and Jackiw '88, 't Hooft '88, Moreira '95]





$$\mathcal{S}\{p,p'\} = \delta^{(3)}(p+p') + \delta(\omega+\omega')\frac{\sin\frac{\pi}{\alpha}}{\cos\frac{\pi}{\alpha} - \cos(\phi-\phi')}$$

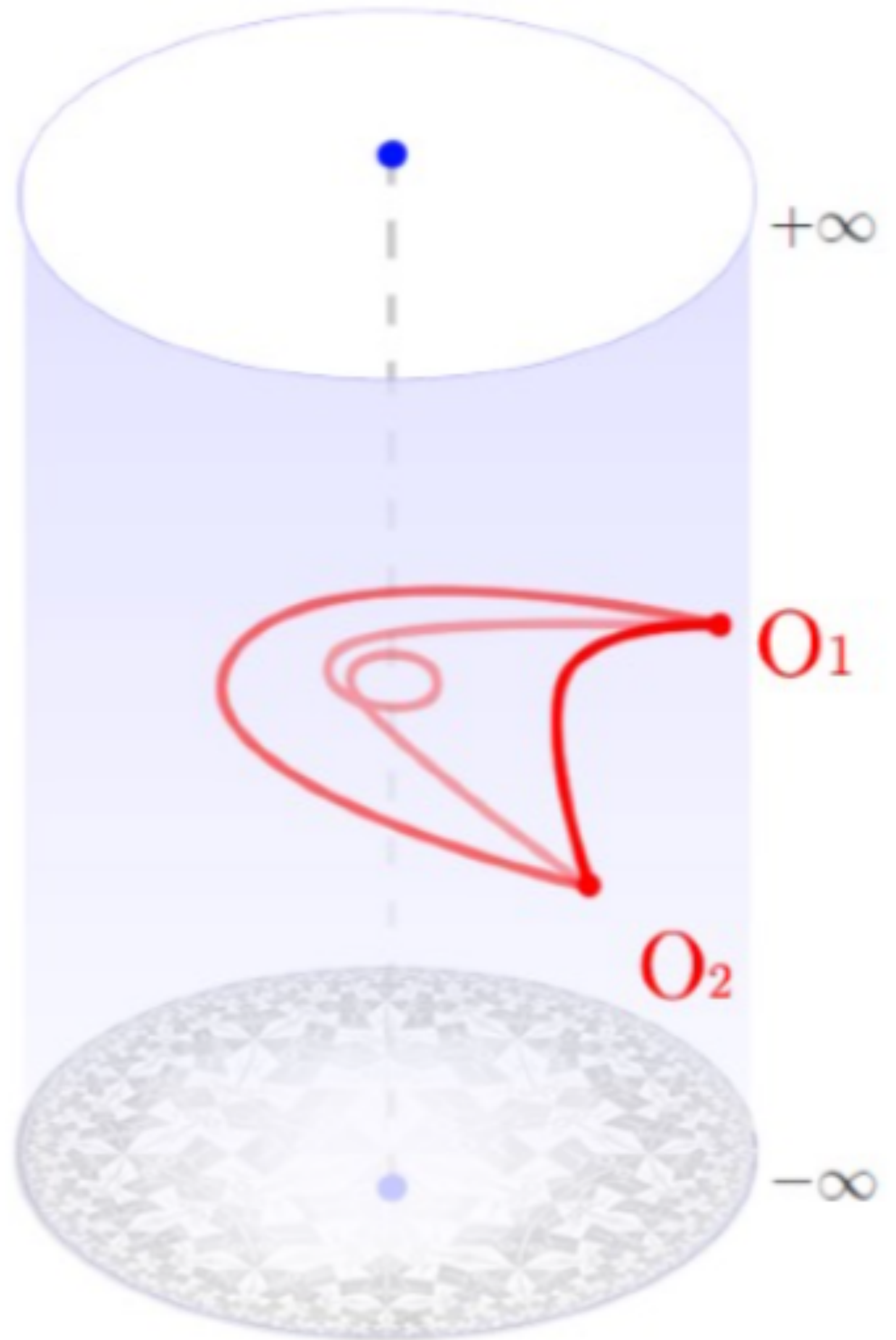
Same result from the flat limit of a CFT_2 correlator

CFT deficit state $$|\alpha\rangle_{\mathrm{CFT}_2} = \mathcal{O}_{\Delta=\frac{c}{12}(1-\alpha^2)}|0\rangle_{\mathrm{CFT}_2}$$

Dual to a conical deficit AdS geometry $$ds^2 = \frac{l^2}{\cos^2\rho}\left(d\rho^2 - \alpha^2 d\tau^2 + \alpha^2 \sin^2\rho^2 d\phi^2\right)$$

$$\mathcal{S}\{p_1, p_2\} = \lim_{l\to\infty} l^{\frac{d-3}{2}} \left[\prod_{i=1}^{2} C(p_i) \int dt_i\, e^{-i\omega_i t_i}\right] \langle\alpha\,|\mathcal{O}(\tau_1,\chi_1)\,\mathcal{O}(\tau_2,\chi_2)|\,\alpha\rangle$$

$$\mathcal{S}\{p, p'\} = \delta^{(3)}(p+p') + \delta(\omega+\omega')\frac{\sin\frac{\pi}{\alpha}}{\cos\frac{\pi}{\alpha} - \cos(\phi-\phi')}$$



Non-trivial CFT_2 correlators turn into non-trivial scattering events in asymptotically flat geometries.

Part II - Gauge fields

- What kind of CFT's are dual to U(1) gauge theories in AdS?
- Constructing photon creation/annihilation operators in Minkowski space using CFT
- Weinberg soft theorems as a consequence of CFT Ward identities

Scalar fields in AdS/CFT

Generally, a scalar field has two solutions next to the boundary of AdS

$$\phi(\rho, x) \xrightarrow[\rho \to \frac{\pi}{2}]{} (\cos\rho)^{\Delta_+}\alpha(x) + (\cos\rho)^{\Delta_-}\beta(x)$$

~~Usually β(x) is non-normalizable because Δ_- is not positive enough.~~

$$1 - \frac{d^2}{4} > m^2 L^2 > -\frac{d^2}{4}$$

In this range, both fall-offs are normalizable

Two possible quantizations:

❶ Fix β(x) at the boundary - β(x)=J'(x)
Quantize α(x) - Dual operator has $\Delta = \Delta_+$

❷ Fix α(x) at the boundary - α(x)=J(x)
Quantize β(x) - Dual operator has $\Delta = \Delta_-$

Related by RG flow

$$\delta S_{\mathrm{CFT}} = g \int d^d x \, O_-^2$$

relevant operator

Legendre transform

- Make α(x)=J(x) dynamical
- Couple to new background source J'(x)
- Resulting theory matches β(x)=J'(x) theory

U(1) gauge fields in AdS/CFT

Two possible fall-offs $$\mathcal{A}_\mu(\rho, x) \xrightarrow[\rho\to\frac{\pi}{2}]{} (\cos\rho)^1 \alpha_\mu(x) + (\cos\rho)^0 \beta_\mu(x)$$

Two possible quantizations:

❶ {
- Fix β(x) at the boundary - $\beta_\mu(x) = A_\mu(x)$ = non-dynamical background field in the CFT
- Quantize $\alpha_\mu(x)$ - Dual operator has Δ=d-1 and it is a conserved current $j_\mu(x)$

❷ {
- Fix α(x) at the boundary - $\alpha_\mu(x) = B_\mu(x)$
- Quantize β(x) - Dual operator has Δ=1, and it is a dynamical CFT gauge field.

Legendre transform

❶ Fixes the magnetic field F_{ab} at the boundary, while ❷ fixes the electric field $F_{\rho a}$

Electro-magnetic duality in the bulk ~ Legendre transform in the CFT

Ward identities

Q: What do these Ward identities teach us about flat space-time?

In CFT's of the form ❶ the dynamical current obeys a Ward identity

$$\partial_\mu \langle 0|T\{j^\mu(x)\mathcal{O}(x_1)\cdots\mathcal{O}(x_n)\bar{\mathcal{O}}(y_1)\cdots\bar{\mathcal{O}}(y_m)\}|0\rangle$$
$$= \left(\sum_{i=1}^{n} q_i \delta^{(3)}(x-x_i) - \sum_{j=1}^{m} q_j \delta^{(3)}(x-y_i)\right)\langle 0|T\{\mathcal{O}(x_1)\cdots\mathcal{O}(x_n)\bar{\mathcal{O}}(y_1)\cdots\bar{\mathcal{O}}(y_m)\}|0\rangle$$

In CFT's of the form ❷ The dynamical gauge field A_μ can be used to construct a topological current

$$f_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad \text{and} \quad (*f)^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho} f_{\nu\rho}$$

which obeys a Ward identity

$$\partial_\mu \langle 0|T\{(*f)^\mu(x)\mathcal{M}(x_1)\cdots\mathcal{M}(x_n)\bar{\mathcal{M}}(y_1)\cdots\bar{\mathcal{M}}(y_m)\}|0\rangle$$
$$= \left(\sum_{i=1}^{n} g_i \delta^{(3)}(x-x_i) - \sum_{j=1}^{m} g_j \delta^{(3)}(x-y_i)\right)\langle 0|T\{\mathcal{M}(x_1)\cdots\mathcal{M}(x_n)\bar{\mathcal{M}}(y_1)\cdots\bar{\mathcal{M}}(y_m)\}|0\rangle$$

Photon scattering states from a flat limit of AdS/CFT

Constructing flat space creation/annihilation operators using AdS/CFT

In Scattering theory, Fields are considered free asymptotically. We thus focus on free fields.

In Minkowski space, a free U(1) field reads $$\hat{A}_\mu(x) = \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{1}{\sqrt{2\omega_{\vec{q}}}} \sum_{\lambda=\pm} \left(\varepsilon_\mu^{(\lambda)} \hat{a}_{\vec{q}}^{(\lambda)} e^{iq\cdot x} + \varepsilon_\mu^{(\lambda)*} \hat{a}_{\vec{q}}^{(\lambda)\dagger} e^{-iq\cdot x} \right), \quad \text{with} \quad x \in \text{Mink}_{3+1}$$

Inverting:
$$\hat{a}_{\vec{q}}^{(\lambda)} = \lim_{t\to\pm\infty} \frac{i}{\sqrt{2\omega_{\vec{q}}}} \int d^3\vec{x}\, (\varepsilon^{(\lambda),\mu})^* e^{-iq\cdot x} \overleftrightarrow{\partial_0} \hat{A}_\mu(x),$$
$$\hat{a}_{\vec{q}}^{(\lambda)\dagger} = \lim_{t\to\pm\infty} \frac{-i}{\sqrt{2\omega_{\vec{q}}}} \int d^3\vec{x}\, \varepsilon^{(\lambda),\mu} e^{iq\cdot x} \overleftrightarrow{\partial_0} \hat{A}_\mu(x),$$

Free local bulk operators in AdS

❶

$$\mathcal{A}_\mu(\rho, x) \xrightarrow[\rho\to\frac{\pi}{2}]{} \cos\rho\, j_\mu(x)$$

❷

$$\mathcal{V}_\mu \xrightarrow[\rho\to\frac{\pi}{2}]{} A_\mu$$

$$\nabla_\mu F^{\mu\nu} = 0$$

$$\mathcal{A}_\mu(x) = \int d^3x' \left[K_\mu^V(\rho, x, x')\, \epsilon_{\tau'}^{ab} \nabla_a j_b^+(x') + K_\mu^S(\rho, x, x')\, \nabla^a j_a^+(x') \right] + \text{h.c.}$$

$$\mathcal{V}_\mu(x) = \int d^3x' \left[\tilde{K}_\mu^S(\rho, x; x')\, \epsilon_{\tau'}^{ab} \nabla_a A_b^+(x') + \tilde{K}_\mu^V(\rho, x; x')\, \nabla^a A_a^+(x') \right] + \text{h.c.}$$

Flat limit

$$\tau = \frac{t}{L}, \quad \text{and} \quad \rho = \frac{r}{L}, \quad \text{with} \quad L \to \infty.$$

Fourier transform

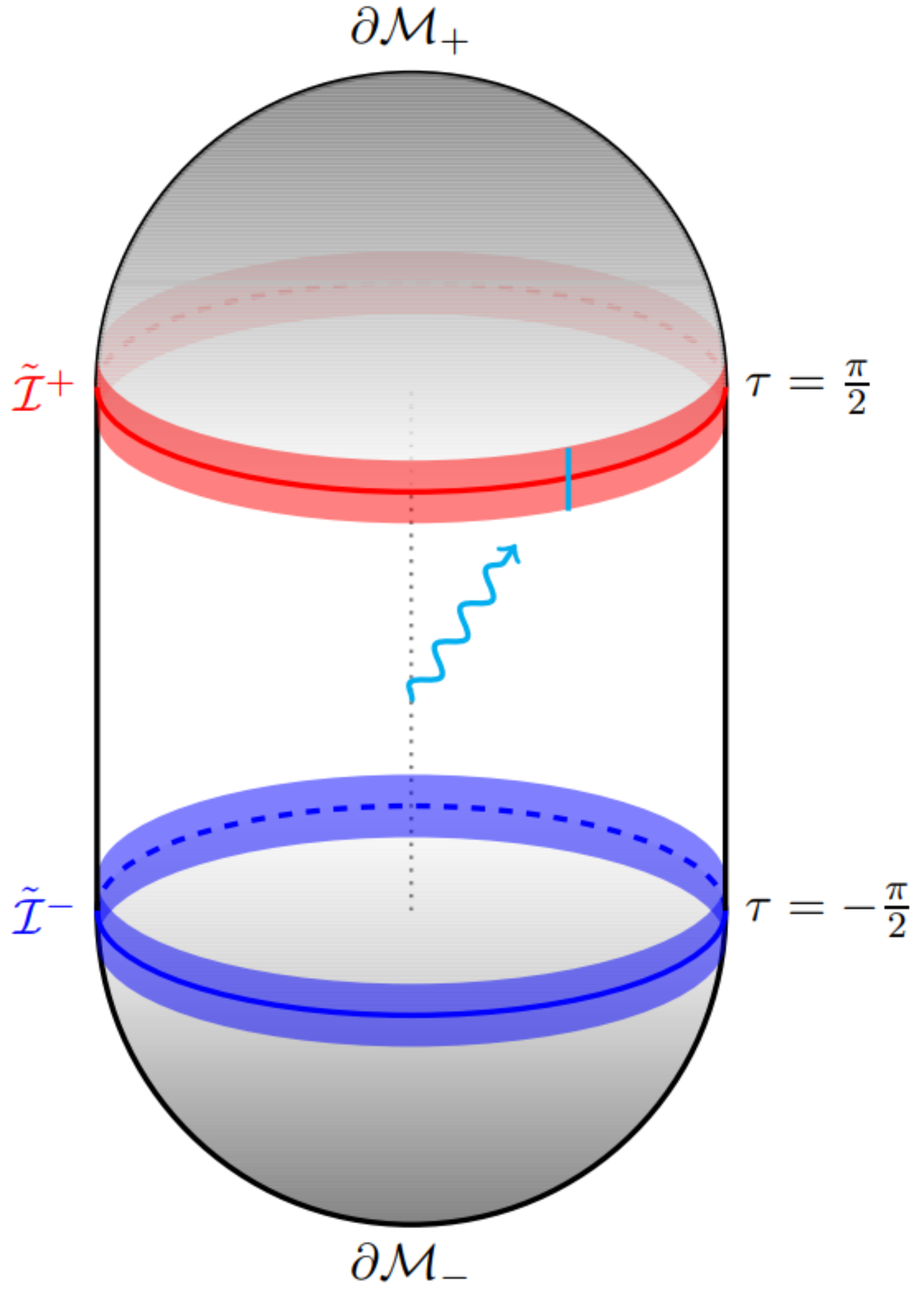
Result

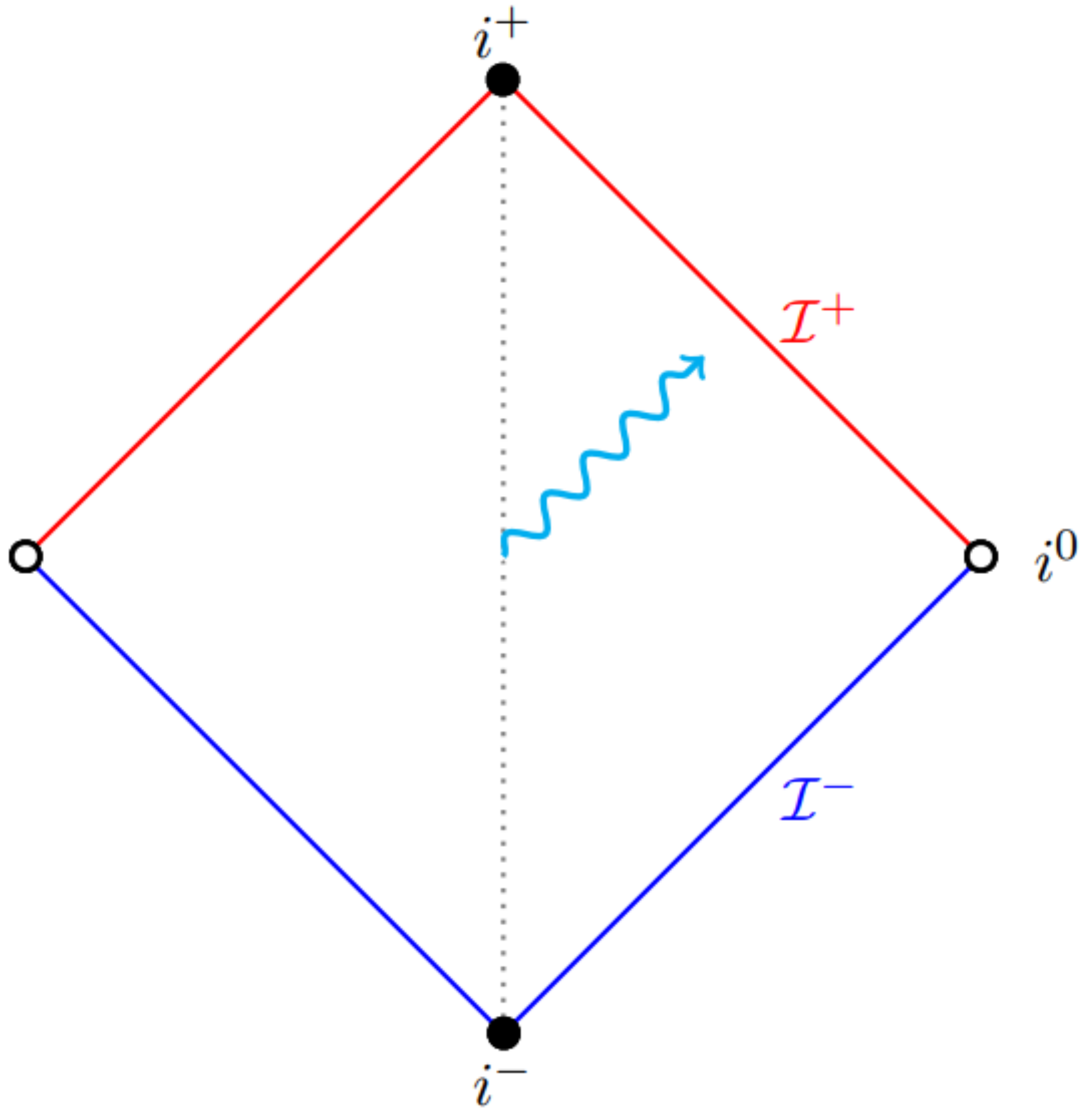
❶

$$\sqrt{2\omega_{\vec{q}}}\, a^{\dagger(-)}_{\vec{q}} = \frac{-1}{4\omega_q}\frac{1+z_q\bar{z}_q}{\sqrt{2}}\int d\tau'\, e^{i\omega_{\vec{q}}L\left(\frac{\pi}{2}-\tau'\right)}\int d^2z'\,\frac{1}{(z_q-z')^2}\,j^+_{\bar{z}'}(\tau',z',\bar{z}')\,,$$
$$\sqrt{2\omega_{\vec{q}}}\, a^{\dagger(+)}_{\vec{q}} = \frac{-1}{4\omega_q}\frac{1+z_q\bar{z}_q}{\sqrt{2}}\int d\tau'\, e^{i\omega_{\vec{q}}L\left(\frac{\pi}{2}-\tau'\right)}\int d^2z'\,\frac{1}{(\bar{z}_q-\bar{z}')^2}\,j^+_{z'}(\tau',z',\bar{z}')\,.$$

❷

$$\sqrt{2\omega_{\vec{q}}}\, v^{\dagger(-)}_{\vec{q}} = \frac{1}{4\omega_q}\frac{1+z_q\bar{z}_q}{\sqrt{2}}\int d\tau'\, e^{i\omega_{\vec{q}}L\left(\frac{\pi}{2}-\tau'\right)}\partial_{\tau'}A^+_{z'}(\tau',z'_q,\bar{z}'_q)\,,$$
$$\sqrt{2\omega_{\vec{q}}}\, v^{\dagger(+)}_{\vec{q}} = \frac{1}{4\omega_q}\frac{1+z_q\bar{z}_q}{\sqrt{2}}\int d\tau'\, e^{i\omega_{\vec{q}}L\left(\frac{\pi}{2}-\tau'\right)}\partial_{\tau'}A^+_{\bar{z}'}(\tau',z'_q,\bar{z}'_q)\,.$$





From CFT physics to Flat physics

Asymptotic symmetries of the S-matrix

The S-matrix is invariant under the action of an infinite set of symmetries

Maxwell equations

$$d*F=*j_E\,,\quad \text{and} \quad dF=*j_M$$

jE and jM are conserved

$$d*j_E=d^2*F=0\,,\quad \text{and}\quad d*j_M=d^2F=0$$

There are a family of conserved currents

$$*j_E^{\varepsilon}=d(\varepsilon*F)\,,\quad \text{and}\quad *j_M^{\varepsilon}=d(\varepsilon F)$$

These yield a family of conserved charges

$$Q_E^{\varepsilon}(\Sigma)=\int_\Sigma *j_E^{\varepsilon}=\int_\Sigma(d\varepsilon\wedge*F+\varepsilon*j_E)\,,$$

$$Q_M^{\varepsilon}(\Sigma)=\int_\Sigma *j_M^{\varepsilon}=\int_\Sigma(d\varepsilon\wedge F+\varepsilon*j_M)\,.$$

Conserved in the sense that $\langle\text{out}|Q^{\varepsilon}_{E/M}(\Sigma_+)\mathcal{S}-\mathcal{S}Q^{\varepsilon}_{E/M}(\Sigma_-)|\text{in}\rangle=0$

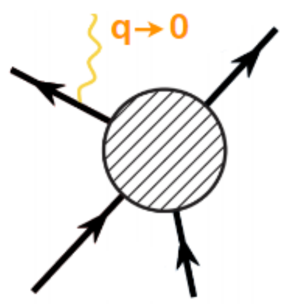
These charges have a 'soft'(radiative) part and a 'hard'(coulombic) part

$$Q_E^{\text{soft}}(\Sigma_+)=\frac{-1}{8\pi^2}\lim_{\omega_{\vec q}\to 0}\omega_{\vec q}\int d^2z\,\varepsilon(z,\bar z)\left[\partial_{\bar z}\left(\frac{\sqrt{2}}{1+z\bar z}\sqrt{2\omega_{\vec q}}\,\hat a^{(-)}_{\vec q}\right)+\partial_z\left(\frac{\sqrt{2}}{1+z\bar z}\sqrt{2\omega_{\vec q}}\,\hat a^{(+)}_{\vec q}\right)\right]$$

$$Q_E^{\text{hard}}(\Sigma_+)=\lim_{t\to\infty}\sum_i q_i\,\varepsilon\,(x)\,|_{r=\frac{|\vec p|}{p^0}t,\hat x=\hat p_i}$$

Conservation of the charges relates S-matrices with soft photons to S-matrices without them.

Choosing $\varepsilon(x)=\dfrac{1}{z-z'}\,,$ and $\varepsilon(x)=\dfrac{1}{\bar z-\bar z'}$ and assuming $F_{z\bar z}|_{\mathcal{I}^+_-}=F_{z\bar z}|_{\mathcal{I}^-_+}=0$ yields



$$=\langle\text{out}|\sqrt{2\omega_{\vec q}}\,a^{(\pm)}_{\vec q}\mathcal{S}|\text{in}\rangle\sim\left[\sum_i q_i\frac{p_i\cdot\epsilon^{\pm}}{p_i\cdot q}-\sum_j q_j\frac{p_j\cdot\epsilon^{\pm}}{p_j\cdot q}\right]\langle\text{out}|\mathcal{S}|\text{in}\rangle$$

<u>Objective:</u> Derive these formulas using CFT Ward identities and the map between scattering states and CFT operators we have constructed.

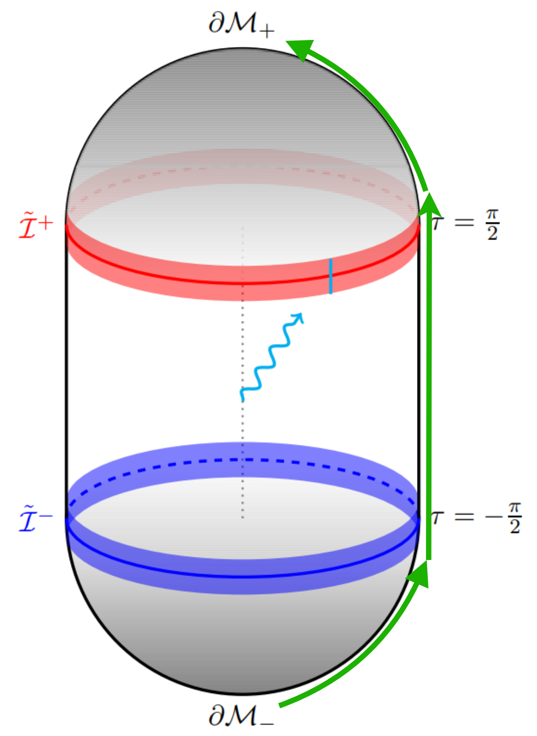
From CFT physics to Minkowski physics

Starting point: Integrated form of Ward identity

$$\underbrace{\int d^3x\, \alpha(x)\partial_\mu \langle 0|T\{j^\mu(x)X\}|0\rangle}_{\text{LHS}} = \underbrace{\left(\sum_{i=1}^{n} q_i\alpha(x_i) - \sum_{j=1}^{m} q_j\alpha(y_j)\right)\langle 0|T\{X\}|0\rangle}_{\text{RHS}}$$

$$X = \prod_i \mathcal{O}_i(x_i)\prod_j \bar{\mathcal{O}}_i(y_j)$$

Integration region:



1. Smear locations of operators such that they become creation/annihilation operators in flat space

2. Make a choice of parameter α(x) such that RHS becomes the hard part of Weinberg's theorem

$$\alpha(x) = \lim_{\rho\to\frac{\pi}{2}} \int d^2\hat{x}'\, \frac{1}{4\pi}\frac{\cos^2\rho - \cos^2\tau}{(\sin\tau - \sin\rho\,\hat{x}\cdot\hat{x}')^2}\varepsilon(\hat{x}')$$

$$\alpha(x)|_{\tilde{\mathcal{I}}^\pm} = \varepsilon(\hat{x})$$

yields RHS $= \langle \text{out}|Q_E^{\text{hard}}(\Sigma_+)\,\mathcal{S} - \mathcal{S}\,Q_E^{\text{hard}}(\Sigma_-)|\text{in}\rangle$

3. Integrate by parts the LHS

$$\int d^3x\, \partial_\mu\alpha(x)\langle 0|T\{j^\mu(x)X\}|0\rangle = \int_{\partial\mathcal{M}_\pm} d^3x\, \partial_\mu\alpha(x)\langle 0|T\{j^\mu(x)X\}|0\rangle$$

Can be argued away by computing Lienard-Wietchert potentials in AdS

$$-\int_{\tilde{\mathcal{I}}^\pm} d^3x\, \varepsilon(\hat{x})\left[D^z\langle 0|T\{j_z(x)X\}|0\rangle + D^{\bar{z}}\langle 0|T\{j_{\bar{z}}(x)X\}|0\rangle\right].$$

} Only radiative parts of the current contribute. Can be argued again by computing LW potentials explicitly.

$$= \int_{\tilde{\mathcal{I}}^\pm} d^3x\, \varepsilon(\hat{x})\left[D^z\langle 0|T\{j_z^{(\text{R})}(x)X\}|0\rangle + D^{\bar{z}}\langle 0|T\{j_{\bar{z}}^{(\text{R})}(x)X\}|0\rangle\right].$$

4. The operators appearing in LHS are soft photon operators according to our map!

$$\int_{\tilde{\mathcal{I}}^+} d^3x\, \varepsilon(\hat{x}) D^{\bar{z}} j^-_{\bar{z}} = \lim_{\omega_{\vec{q}}\to 0} \frac{2}{\pi}\omega_{\vec{q}} \int d^2z_q\, \varepsilon(\hat{q})\, \partial_{\bar{z}_q}\left(\frac{\sqrt{2}}{1+z\bar{z}}\sqrt{2\omega_{\vec{q}}}\, a^{(-)}_{\vec{q}}\right),$$

$$\int_{\tilde{\mathcal{I}}^+} d^3x\, \varepsilon(\hat{x}) D^{z} j^-_{z} = \lim_{\omega_{\vec{q}}\to 0} \frac{2}{\pi}\omega_{\vec{q}} \int d^2z_q\, \varepsilon(\hat{q})\, \partial_{z_q}\left(\frac{\sqrt{2}}{1+z\bar{z}}\sqrt{2\omega_{\vec{q}}}\, a^{(+)}_{\vec{q}}\right).$$

So LHS corresponds to the insertion of soft charge operators: LHS = $\langle \text{out}|Q^{\text{soft}}_E(\Sigma_+)\,\mathcal{S} - \mathcal{S}\,Q^{\text{soft}}_E(\Sigma_-)|\text{in}\rangle$

5. Putting all the pieces together yields

$$\langle \text{out}|Q^{\text{soft}}_E(\Sigma_+)\,\mathcal{S} - \mathcal{S}\,Q^{\text{soft}}_E(\Sigma_-)|\text{in}\rangle = \langle \text{out}|Q^{\text{hard}}_E(\Sigma_+)\,\mathcal{S} - \mathcal{S}\,Q^{\text{hard}}_E(\Sigma_-)|\text{in}\rangle$$

or simply

$$\boxed{\langle \text{out}|Q^{\varepsilon}_E(\Sigma_+)\,\mathcal{S} - \mathcal{S}\,Q^{\varepsilon}_E(\Sigma_-)|\text{in}\rangle = 0}$$

In order to show Weinberg soft theorems one proceeds as in flat space, but an extra assumption concerning the radiative modes must be made!

$$F_{z\bar{z}}|_{\mathcal{I}^+_-} = F_{z\bar{z}}|_{\mathcal{I}^-_+} = 0$$

Magnetic asymptotic symmetries from Ward identities

$$\partial_\mu \langle 0|T\{(*f)^\mu(x)X\}|0\rangle = \left(\sum_{i=1}^{n} g_i\delta^{(3)}(x-x_i) - \sum_{j=1}^{m} g_j\delta^{(3)}(x-y_j)\right)\langle 0|T\{X\}|0\rangle$$

Integrate by parts

$$2i\int_{\tilde{\mathcal{I}}^\pm} d\tau d^2z\,\varepsilon(\hat{x})\,[\partial_{\bar{z}}\langle 0|T\{\partial_\tau A_z(x)X\}|0\rangle - \partial_z\langle 0|T\{\partial_\tau A_{\bar{z}}(x)X\}|0\rangle]$$

Use map between CFT operators and scattering states

$$\langle \text{out}|Q_M^{\text{soft}}(\Sigma_+)\,\mathcal{S} - \mathcal{S}\,Q_M^{\text{soft}}(\Sigma_-)|\text{in}\rangle$$

choose

$$\alpha(x) = \lim_{\rho\to\frac{\pi}{2}}\int d^2\hat{x}'\,\frac{1}{4\pi}\,\frac{\cos^2\rho-\cos^2\tau}{(\sin\tau-\sin\rho\,\hat{x}\cdot\hat{x}')^2}\varepsilon(\hat{x}')$$

calculate

$$\langle \text{out}|Q_M^{\text{hard}}(\Sigma_+)\,\mathcal{S} - \mathcal{S}\,Q_M^{\text{hard}}(\Sigma_-)|\text{in}\rangle$$

$$\langle \text{out}|Q_M^{\varepsilon}(\Sigma_+)\,\mathcal{S} - \mathcal{S}\,Q_M^{\varepsilon}(\Sigma_-)|\text{in}\rangle = 0$$

Some Questions

- **Gravity:** Scattering states involving gravitons from the CFT perspective.

$$a_{\vec{q}}^{(\pm)} \sim \int d\tau\, e^{i\omega_{\vec{q}}L\left(\frac{\pi}{2}-\tau\right)}\, \epsilon_{(\pm)}^{CD} \int d^2z\, I_{CD}{}^{AB} T_{AB}^{\mathrm{CFT}}$$

Soft theorems from Stress tensor Ward identities

BMS4 algebra from a flat limit of AdS/CFT

- **IR finite S-matrix:**
Relaxing the condition for asymptotic decoupling involves including interactions in our definition of scattering states. This should yield Faddeev-Kulish asymptotic states and an IR finite S-matrix in the flat limit.

Thanks!