On Broken Supersymmetry In String Theory

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(Two recent reviews with Mourad : 2107.04064, 1711.11494)

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Alexander von Humboldt Stiftung/Foundation

Broken SUSY in 10D
with
NO Tachyons

Vacuum Energy in Field Theory

- BOSE (FERMI) OSCILLATOR:

$$H = \frac{p^2}{2m} + \frac{1}{2} m \, \omega^2 \, q^2 \longrightarrow \mathcal{E}_0 = (-) \frac{\hbar \, \omega}{2}$$

- QUANTUM FIELD THEORY:

$$\frac{\mathcal{E}_0}{V} = \sum_i (-1)^{F_i} \int \frac{d^d \mathbf{k}}{(2\pi)^d} \, \frac{\hbar \, c}{2} \, \sqrt{\mathbf{k}^2 + m_i^2}$$

- THE COSMOLOGICAL CONSTANT ISSUE:

$$\frac{\mathcal{E}_0}{V} \sim \frac{\mathcal{E}_{Pl}}{V_{Pl}} \sum_i (-1)^{F_i} + \ldots$$

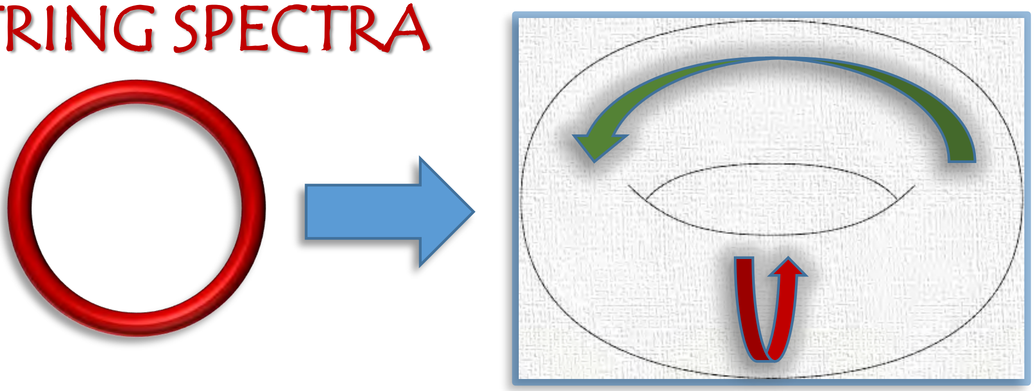
(Zeldovich, 1968)

- (Exact, GLOBAL) SUPERSYMMETRY removes problem

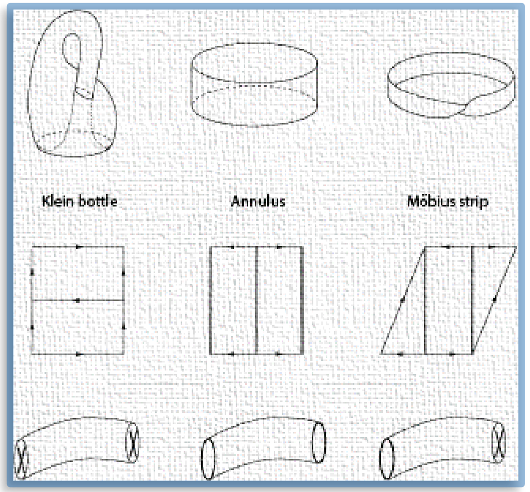
(Zumino, 1975)

Vacuum Energy and String Theory

VACUUM ENERGY: DETERMINES CONSISTENT STRING SPECTRA



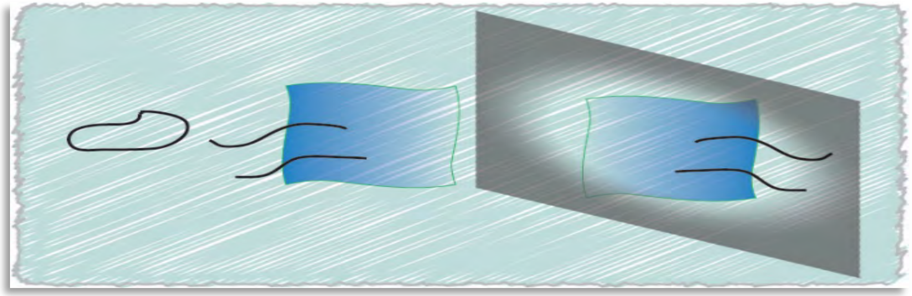
- VACUUM ENERGY & "CLOSED" STRINGS:
  - ❖ MODULAR INVARIANCE !



- ORIENTIFOLDS: MORE SUBTLE, OPEN AND CLOSED STRINGS

(AS, 1987)
[+Bianchi, Pradisi, 1988-96]
[+Stanev, 1994-96]

- VACUUM HOSTS (EXTENDED) SOLITONS



(Polchinski, 1995)

- OK WITH SUPERSYMMETRY (NO BACK-REACTION) !

The (SUSY) 10D-11D Zoo

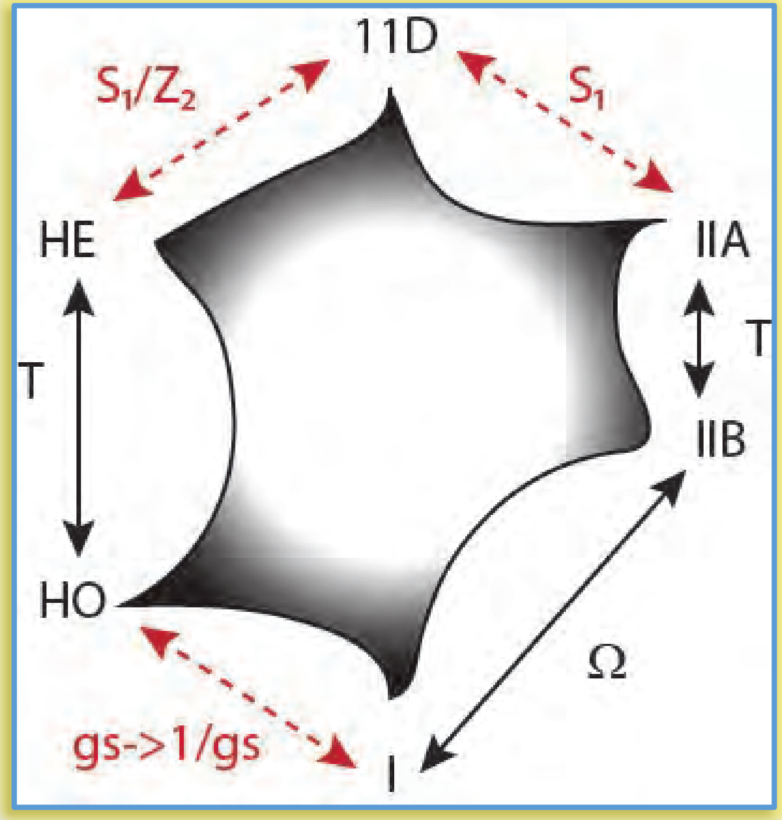
- **Highest point** of (SUSY) String Theory

BUT:

- Exhibits **dramatically our limitarions**
- perturbative → **Solid arrows**
- [10&11D supergravity → **Dashed arrows**]

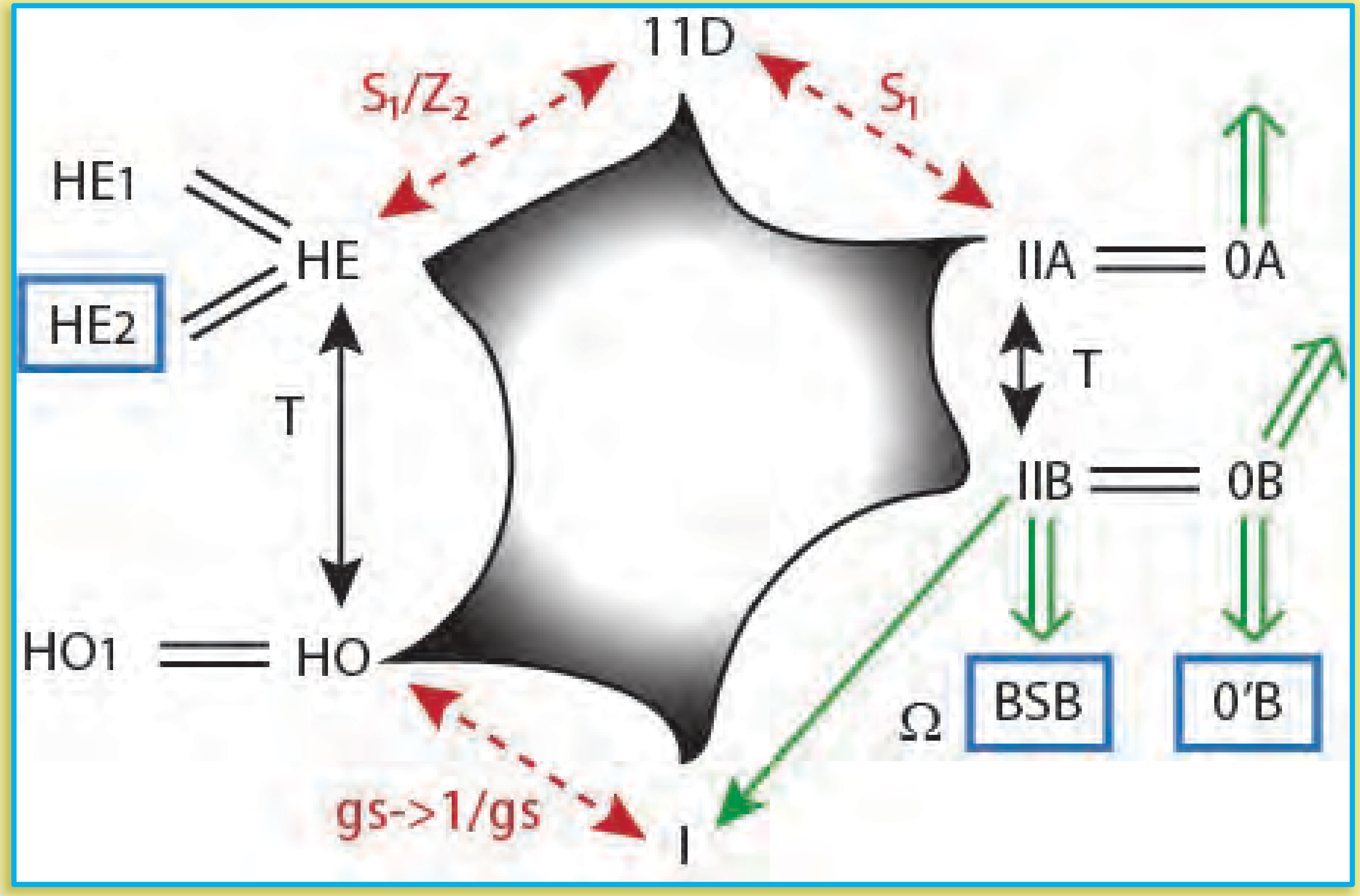
(Witten, 1995)

- **SUSY**: **stabilizes** these 10D Minkowski vacua



BROKEN SUSY ?

The 10D-11D Zoo



- 3 D=10 **non-SUSY non-tachyonic** strings
- SO(16)xSO(16) *(Dixon, Harvey, 1987) (Alvarez-Gaumé, Ginsparg, Moore, Vafa, 1987)*
- O'B U(32) *(AS, 1995)*
- [BSB Usp(32)] *(Sugimoto, 1999, Antoniadis, Dudas, AS, 1999)*

- String consistency rules OK
- BUT: vacuum modified (Tadpole potential)

- QUESTIONS:
  - Compactifications? Stability?
  - NON-PERTURBATIVE LINKS?

$$\mathcal{S} = \frac{1}{2\,k_{10}^2}\int d^{10}x\,\sqrt{-G}\left\{e^{-2\phi}\left[-R+4(\partial\phi)^2\right]-\frac{1}{12}\,\mathcal{H}_3^2-\frac{1}{4}\,e^{-\phi}\,\mathrm{tr}\,\mathcal{F}^2 - T\,e^{-\phi}+\ldots\right\}$$

The Non-Tachyonic 10D String Models

$$O_{2n} = \frac{\theta^n\left[{0 \atop 0}\right](0|\tau) + \theta^n\left[{0 \atop 1/2}\right](0|\tau)}{2\,\eta^n(\tau)}\,, \qquad S_{2n} = \frac{\theta^n\left[{1/2 \atop 0}\right](0|\tau) + i^{-n}\,\theta^n\left[{1/2 \atop 1/2}\right](0|\tau)}{2\,\eta^n(\tau)}\,,$$

$$V_{2n} = \frac{\theta^n\left[{0 \atop 0}\right](0|\tau) - \theta^n\left[{0 \atop 1/2}\right](0|\tau)}{2\,\eta^n(\tau)}\,, \qquad C_{2n} = \frac{\theta^n\left[{1/2 \atop 0}\right](0|\tau) - i^{-n}\,\theta^n\left[{1/2 \atop 1/2}\right](0|\tau)}{2\,\eta^n(\tau)}\,,$$

$$\eta(\tau) = q^{\frac{1}{24}} \prod_{n=1}^{\infty}(1 - q^n)\,, \qquad q = e^{2\pi i\tau}\,,$$

$$\theta\left[{\alpha \atop \beta}\right](z|\tau) = \sum_{n\in Z} q^{\frac{1}{2}(n+\alpha)^2}\, e^{i2\pi(n+\alpha)(z-\beta)}$$

SO(16)xSO(16):

$$\mathcal{T}_{SO(16)\times SO(16)} = \int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2}\,\frac{1}{(Im\tau)^4\,\eta^8\,\bar\eta^8}\,\left[O_8(\bar V_{16}\,\bar C_{16} + \bar C_{16}\,\bar V_{16}) + V_8(\bar O_{16}\,\bar O_{16} + \bar S_{16}\,\bar S_{16}) - S_8(\bar O_{16}\,\bar S_{16} + \bar S_{16}\,\bar O_{16}) - C_8(\bar V_{16}\,\bar V_{16} + \bar C_{16}\,\bar C_{16})\right]$$

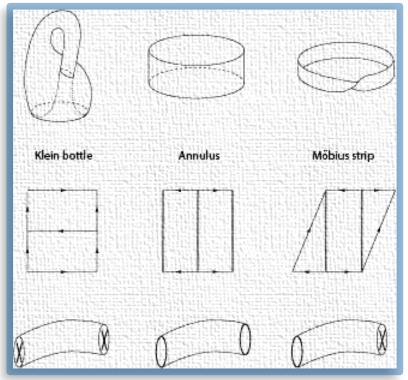
(Dixon, Harvey, 1987)
(Alvarez-Gaumé, Ginsparg, Moore, Vafa, 1987)

U(32):

$$\frac{1}{2}(\mathcal{T} + \mathcal{K}) = \int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2}\,\frac{|O_8|^2 + |V_8|^2 + |S_8|^2 + |C_8|^2}{(Im\tau)^4\,\eta^8\,\bar\eta^8} + \frac{1}{2}\int_0^{\infty}\frac{d\tau_2}{(\tau_2)^2}\,\frac{(-)\,O_8 + V_8 + S_8 - C_8}{(\tau_2)^4\,\eta^8}\,[2i\tau_2]$$

$$\frac{1}{2}(\mathcal{A} + \mathcal{M}) = \int_0^{\infty}\frac{d\tau_2}{(\tau_2)^2}\,\frac{\mathcal{N}\bar{\mathcal{N}}\,V_8 - \frac{1}{2}(\mathcal{N}^2 + \bar{\mathcal{N}}^2)\,C_8}{(\tau_2)^4\,\eta^8}\,[i\tau_2/2] - \frac{\mathcal{N} + \bar{\mathcal{N}}}{2}\int_0^{\infty}\frac{d\tau_2}{(\tau_2)^2}\,\frac{\hat C_8}{(\tau_2)^4\,\hat\eta^8}\,[i\tau_2/2 + 1/2]$$

(AS, 1995)



USp(32):

$$\mathcal{T}_{IIB} = \int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2}\,\frac{|V_8 - S_8|^2}{(Im\tau)^4\,\eta^8\,\bar\eta^8} \longrightarrow \frac{1}{2}(\mathcal{T} + \mathcal{K}) = \frac{1}{2}\int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2}\,\frac{|V_8 - S_8|^2}{(Im\tau)^4\,\eta^8\,\bar\eta^8} + \frac{1}{2}\int_0^{\infty}\frac{d\tau_2}{(\tau_2)^2}\,\frac{V_8 - S_8}{(\tau_2)^4\,\eta^8}\,[2i\tau_2]$$

$$\mathcal{A} + \mathcal{M} = \frac{1}{2}\,\mathcal{N}^2\int_0^{\infty}\frac{d\tau_2}{(\tau_2)^2}\,\frac{V_8 - S_8}{(\tau_2)^4\,\eta^8}\,[i\tau_2/2] - \frac{1}{2}\,\mathcal{N}\int_0^{\infty}\frac{d\tau_2}{(\tau_2)^2}\,\frac{(-)\hat V_8 - \hat S_8}{(\tau_2)^4\,\eta^8}\,[i\tau_2/2 + 1/2]$$

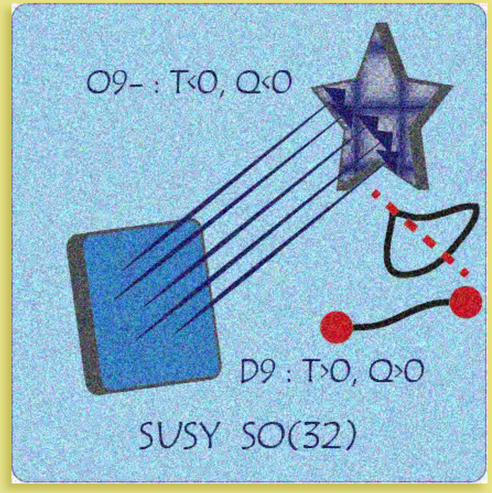
(Sugimoto, 1999, Antoniadis, Dudas, AS, 1999)

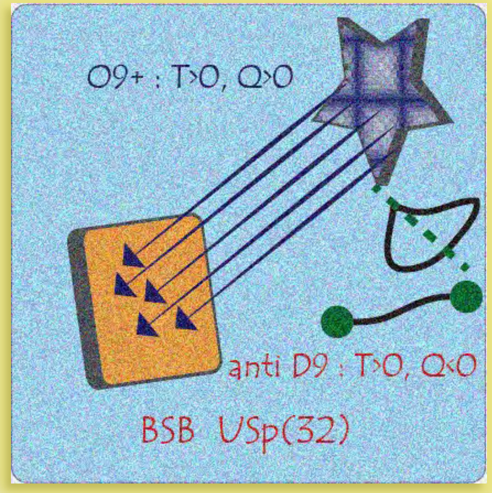
Brane SUSY Breaking

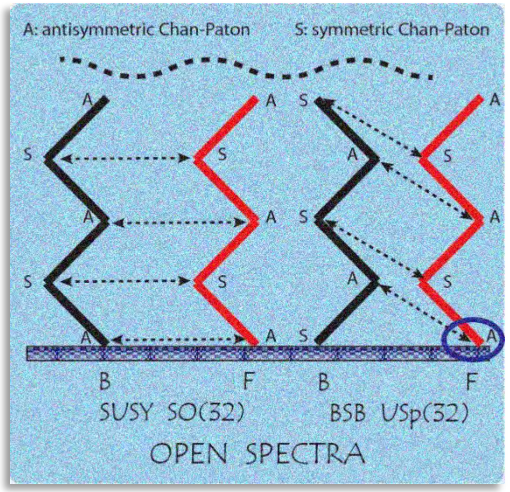
(Sugimoto, 1999)
(Antoniadis, Dudas, AS, 1999)
(Angelantonj, 1999)
(Aldazabal, Uranga, 1999)

- Non-linear SUSY: ∃ goldstino!
- NO TACHYONS

(Dudas, Mourad, 2000)
(Pradisi, Riccioni, 2001)







$$\mathcal{S} = \frac{1}{2\,k_{10}^2}\int d^{10}x\,\sqrt{-G}\left\{e^{-2\phi}\left[-R+4(\partial\phi)^2\right]-\frac{1}{12}\,\mathcal{H}_3^2-\frac{1}{4}\,e^{-\phi}\,\mathrm{tr}\,\mathcal{F}^2 - T\,e^{-\phi}+\ldots\right\}$$

VACUUM ENERGY → POTENTIAL

NOTE:
- Expansion in powers of $\alpha' R$
- Expansion in powers of $g_s = e^{\phi}$

SUMMARIZING

NO SUSY → Typically tachyonic modes

- **BUT:** 3 D=10 non-SUSY non-tachyonic strings

- SO(16)xSO(16) heterotic — *(Dixon, Harvey, 1987) (Alvarez-Gaumé, Ginsparg, Moore, Vafa, 1987)*
- 0'B U(32) orientifold (no SUSY) — *(AS, 1995)*
- Usp(32) orientifold (non-linear SUSY) — *(Sugimoto, 1999, Antoniadis, Dudas, AS, 1999)*

Vacuum modified (Tadpole potential)

$$\mathcal{S} = \frac{1}{2\,k_{10}^2} \int d^{10}x \sqrt{-G} \left\{ e^{-2\phi} \left[-R + 4(\partial\phi)^2 \right] - \frac{1}{12}\,\mathcal{H}_3^2 - \frac{1}{4}\, e^{-\phi}\,\mathrm{tr}\,\mathcal{F}^2 - T\, e^{-\phi} + \ldots \right\}$$

The "Climbing" Scalar

A. Sagnotti – OIST
October 6, 2021 (online)

Cosmological Potentials

What potentials lead to slow-roll, and where ?

$$ds^2 = -dt^2 + e^{2A(t)}\, d\mathbf{x}\cdot d\mathbf{x}$$

$$\ddot\phi + 3\dot\phi\sqrt{\frac{1}{3}\dot\phi^2 + \frac{2}{3}V(\phi)} + V' = 0$$

Driving force from V′ *vs* friction from V

- **If V does not vanish :** convenient gauge "makes the damping term neater"

$$ds^2 = e^{2\mathcal{B}(t)}\, dt^2 - e^{\frac{2\mathcal{A}(t)}{d-1}}\, d\mathbf{x}\cdot d\mathbf{x}$$
$$V e^{2\mathcal{B}} = V_0$$
$$\tau = t\sqrt{\frac{d-1}{d-2}}, \qquad \varphi = \phi\sqrt{\frac{d-1}{d-2}}$$

$$\dot{\mathcal{A}}^2 - \dot\varphi^2 = 1$$
$$\ddot\varphi + \dot\varphi\sqrt{1+\dot\varphi^2} + \frac{V_\varphi}{2V}\left(1+\dot\varphi^2\right) = 0$$

$$m\ddot{x} + b\dot{x} = f$$

- Now driving from logV *vs* O(1) damping

$$V = \varphi^n \longrightarrow \frac{V'}{2V} = \frac{n}{2\varphi}$$

❖ **Quadratic potential?** Far away from origin *(Linde, 1983)*

❖ **Exponential potential?** YES or NO

$$V(\varphi) = V_0\, e^{2\gamma\varphi} \longrightarrow \frac{V'}{2V} = \gamma$$

$V = e^{3\gamma\varphi/2}$: Climbing & Descending Scalars

(Halliwell, 1987;…, Dudas and Mourad, 1999; Russo, 2004; Dudas, Kitazawa, AS, 2010)

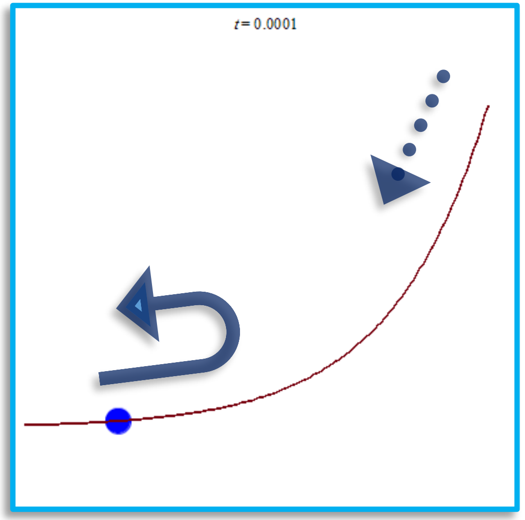
- $\gamma < 1$? Both signs of speed

a. "Climbing" solution (φ climbs, then descends):

$$\dot{\varphi} = \frac{1}{2}\left[\sqrt{\frac{1-\gamma}{1+\gamma}}\ \coth\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right) - \sqrt{\frac{1+\gamma}{1-\gamma}}\ \tanh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]$$

b. "Descending" solution (φ only descends):

$$\dot{\varphi} = \frac{1}{2}\left[\sqrt{\frac{1-\gamma}{1+\gamma}}\ \tanh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right) - \sqrt{\frac{1+\gamma}{1-\gamma}}\ \coth\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]$$



Limiting τ- speed (LM attractor):

$$v_{lim} = -\frac{\gamma}{\sqrt{1-\gamma^2}}$$

(Lucchin and Matarrese, 1985)

$\gamma = 1$ is "critical": LM attractor & descending solution disappear there and beyond

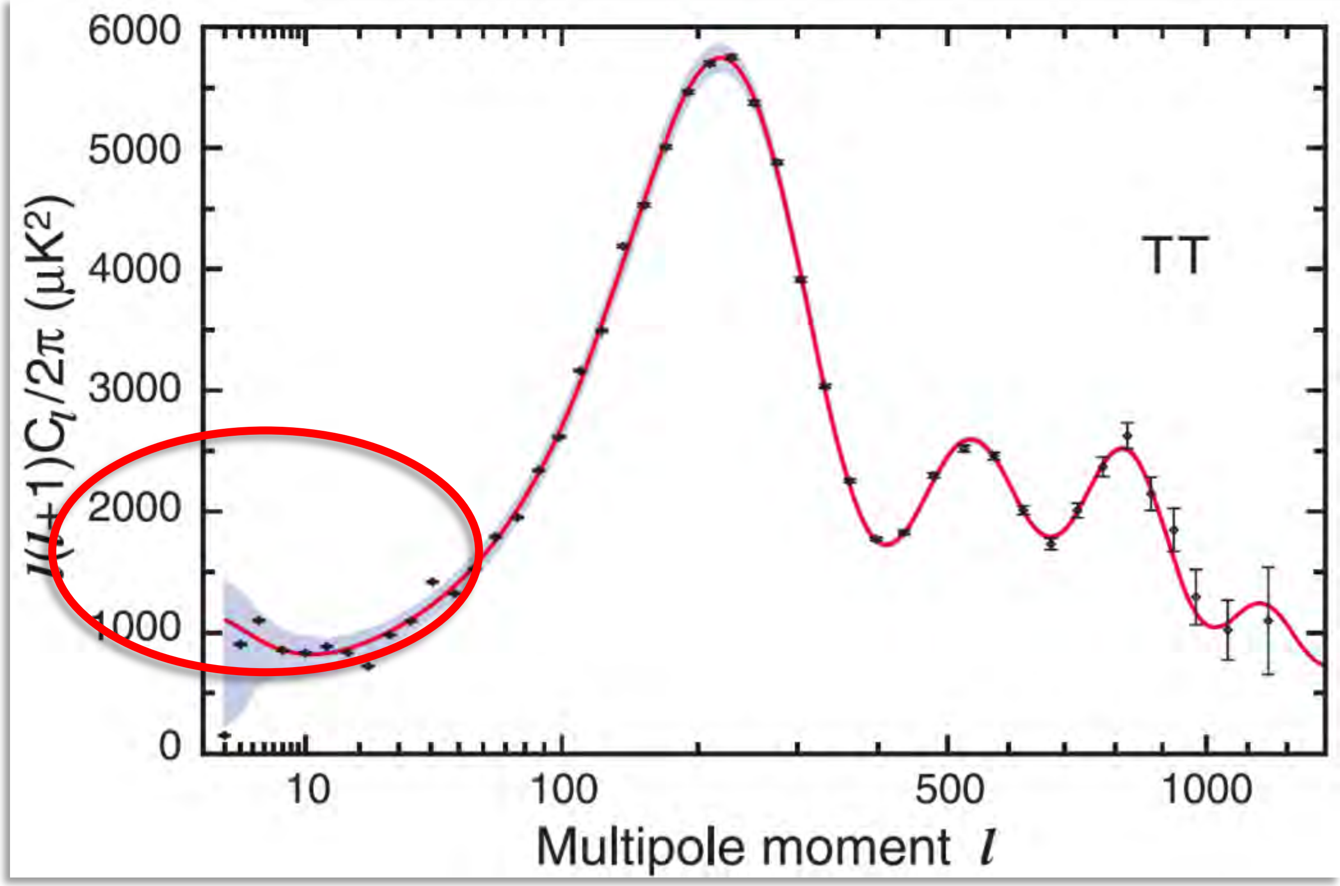
CLIMBING : in ALL asymptotically exponential potentials with $\gamma \geq 1$!

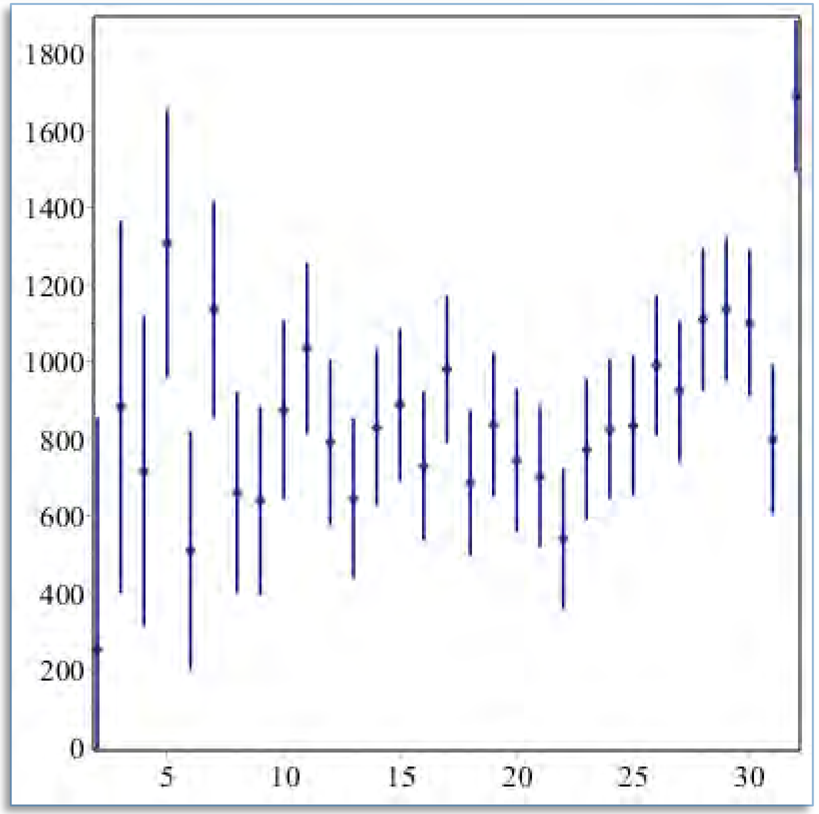
10D STRING THEORY HAS PRECISELY $\gamma = 1$ (bounded g_s !)

- $\gamma = 1$:

$$\varphi(\tau) = \varphi_0 + \frac{1}{2}\left[\log|\tau-\tau_0| - \frac{1}{2}(\tau-\tau_0)^2\right]$$
$$\mathcal{A}(\tau) = \mathcal{A}_0 + \frac{1}{2}\left[\log|\tau-\tau_0| + \frac{1}{2}(\tau-\tau_0)^2\right]$$

Are we seeing signs of the onset of inflation ?





PLANCK 015

$+$: $$A_\ell \sim \ell(\ell+1)\int \frac{dk}{k}\, P_R(k)\, j_\ell(k\Delta\eta)^2 \sim P_R\left(k=\frac{\ell}{\Delta\eta}\right)$$

$-$: Cosmic Variance

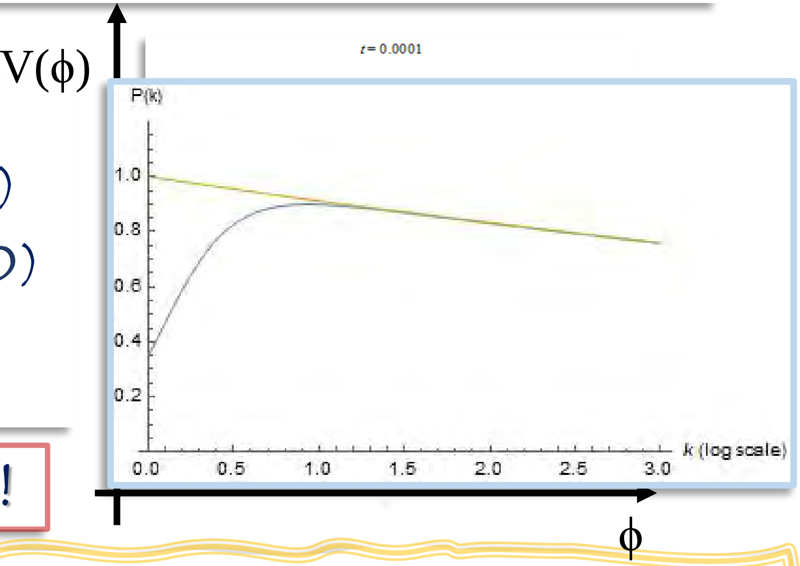
Cosmology with Tadpoles: The Climbing Scalar

(Halliwell, 1987;…, Dudas and Mourad, 1999; Russo, 2004)
(Dudas, Kitazawa, AS, 2010)

$V = T(e^{\gamma\phi} [+ e^{\gamma'\phi}])$ (Einstein frame, [$\gamma' < 3/2$])

For $\gamma < 3/2$ [canonical: beware of different notation in earlier work]:

- "Climbing" solution (ϕ climbs, then descends) → $g_s = e^{\phi}$ BOUNDED)
- "Descending" solution (ϕ only descends) → $g_s = e^{\phi}$ UNBOUNDED)
- Limiting τ- speed (LM attractor)   (Lucchin and Matarrese, 1985)



LM attractor & descending solution disappear for $\gamma \geq 3/2$!

CLIMBING : BSB (Usp(32)) and U(32) HAVE $\gamma = 3/2$! [SO(16) x SO(16) has $\gamma = 5/2$] !

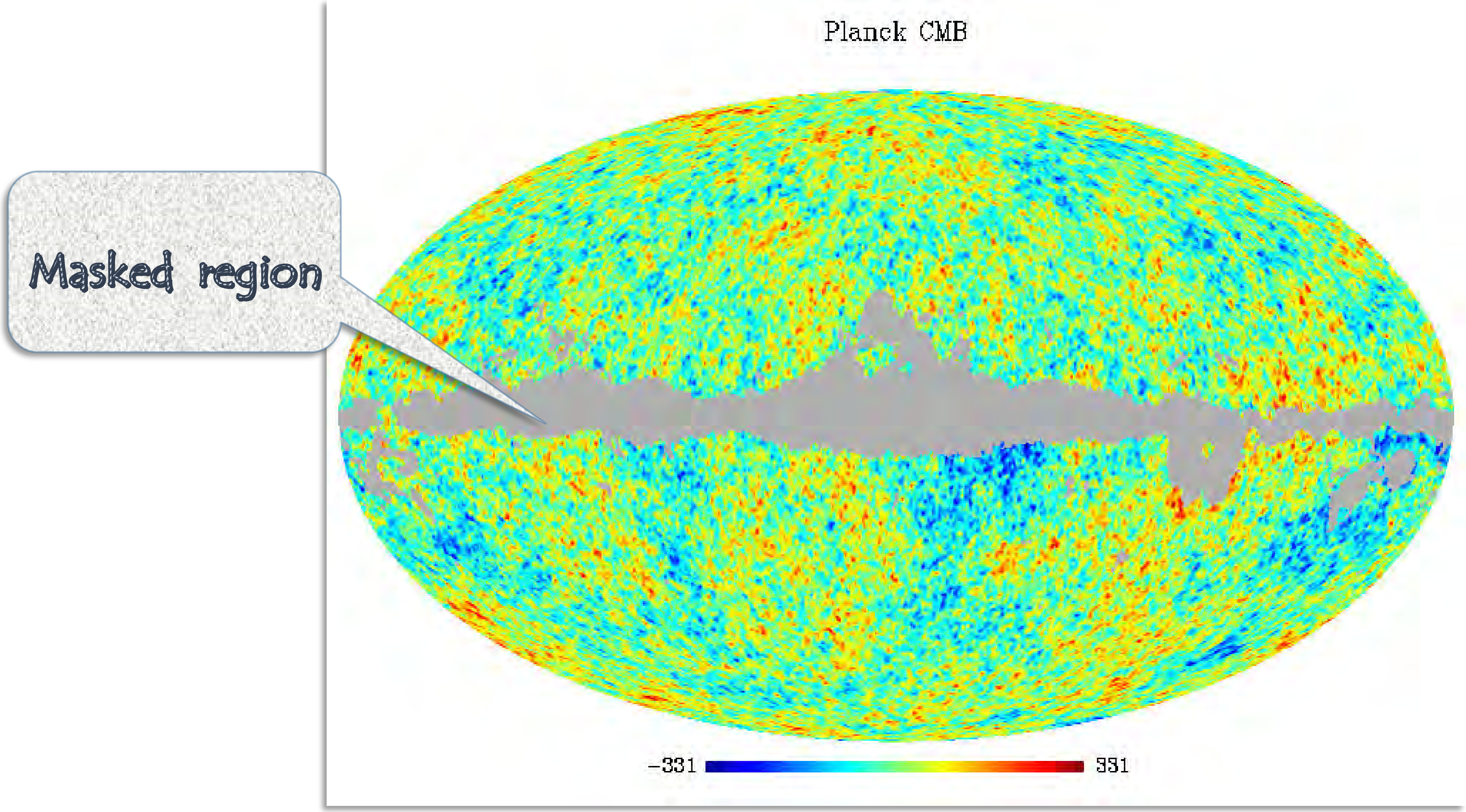
CLUES for the onset of INFLATION?

$$P(k) = A k^{n_s - 1} \rightarrow \frac{k^3}{[k^2 + \Delta^2]^{2 - \frac{n_s}{2}}}$$

Tensor-to-scalar ratio & (squeezed) non-gaussianity : enhanced at transition

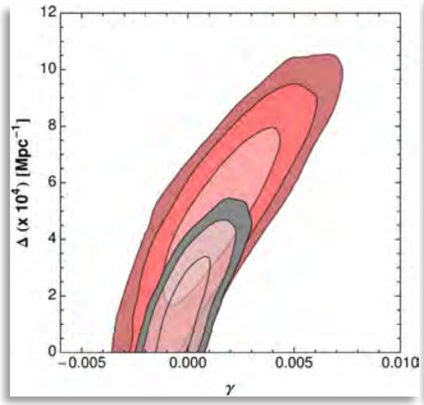
(Bartolo, Matarrese, AS, Vanzan, work in progress)

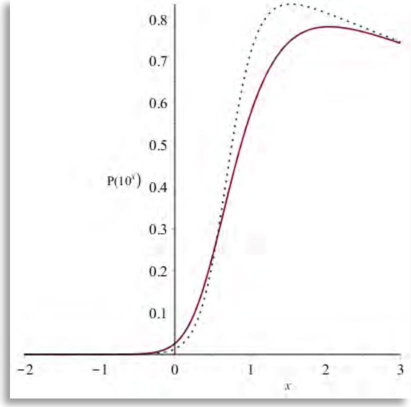
Pre-Inflationary Relics in the CMB?



Pre-Inflationary Relics in the CMB?

(Gruppuso, Kitazawa, Mandolesi, Natoli, AS, 2015)



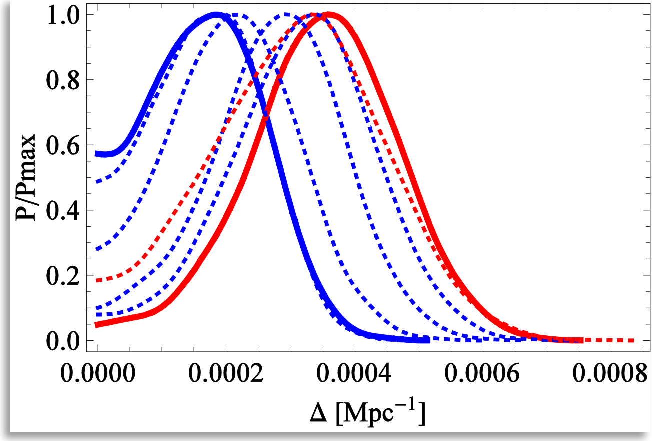


- Extend ΛCDM to allow for low-ℓ suppression:

(Kitazawa, AS, 2014)

$$\mathcal{P}(k) \;=\; A\;(k/k_0)^{n_s-1} \;\rightarrow\; \frac{A\;(k/k_0)^3}{\left[(k/k_0)^2+(\Delta/k_0)^2\right]^{\nu}}$$

- ❖ NO effects on standard ΛCDM parameters (6+16 nuisance)
- ❖ A new scale Δ. Preferred value? Depends on Galactic masking.



$$\Delta = (0.351 \pm 0.114) \times 10^{-3}\,\mathrm{Mpc}^{-1}$$

RED : + 30−degree extended mask

$> 99\%$ confidence level

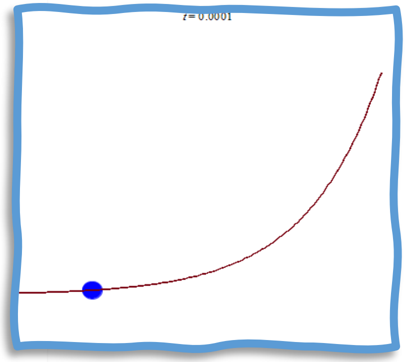
- What is the corresponding energy scale at onset of inflation?

$$\Delta^{Infl} \;\sim\; 2.4 \times 10^{12}\; e^{N-60}\;\mathrm{GeV} \;\sim\; 10^{12} - 10^{14}\mathrm{GeV} \;\;\text{for}\;\; \mathrm{N} \;\sim\; 60-65$$

SUMMARIZING

String Theory PREDICTS the exponent of $V = T e^{\gamma \phi}$

- Einstein frame: $\gamma=3/2$ (U(32) and Usp(32) orientifolds); $\gamma=5/2$ (SO(16)xSO(16)) $: \geq \gamma_c$
- For $\gamma \geq \gamma_c$: ϕ can only emerge from the initial singularity CLIMBING UP!
- g_s BOUNDED [NOT R !], & MODEL of FAST-SLOW TRANSITION

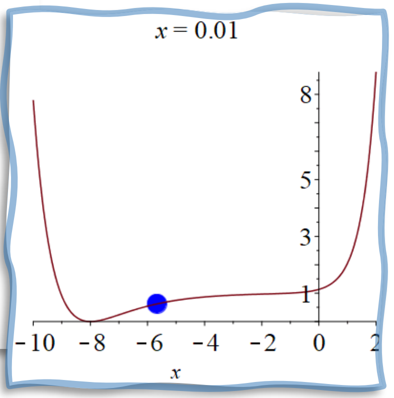


- ONSET OF INFLATION ?

$$P(k) = A k^{n_s - 1} \rightarrow \frac{k^3}{[k^2 + \Delta^2]^{2 - \frac{n_s}{2}}}$$

✓ Primordial power spectrum depressed [POSTDICTION]

❖ Tensor-to-scalar ratio & non-gaussianity: enhanced at transition [PREDICTIONS]



CLIMBING: can occur in generic inflationary potentials

Compactifications

9D Dudas-Mourad Vacua

(Dudas, and Mourad, 2000, 2001)

$$\mathcal{S} = \frac{1}{2\,k_{10}^2}\int d^{10}x\,\sqrt{-G}\left\{e^{-2\phi}\left[-R + 4(\partial\phi)^2\right] - \frac{1}{12}\,\mathcal{H}_3^2 - \frac{1}{4}\,e^{-\phi}\,\mathrm{tr}\,\mathcal{F}^2 - T\,e^{-\phi} + \ldots\right\}$$

9D solutions → T DRIVES the compactification

[For both Usp(32) and U(32), & similar but more complicated for SO(16) x SO(16)]

- SPONTANEOUS COMPACTIFICATIONS: intervals of FINITE length $\sim \frac{1}{\sqrt{T}}$
- FINITE 9D Planck mass and gauge coupling

- g_s diverges at one end & curvature at the other
- Internal interval & 9D flat space (with warping)
- QUESTIONS:
  - Fermions?
  - String corrections: are large values of g_s NEEDED for these types of compactification ?
  - Stability ?

$$e^{\phi} = e^{u+\phi_0}\,u^{\frac{1}{3}}$$

$$ds^2 = e^{-\frac{u}{6}}\,u^{\frac{1}{18}}\,dx^2 + \frac{2}{3\,T\,u^{\frac{3}{2}}}\,e^{-\frac{3}{2}(u+\phi_0)}\,du^2$$

AdS x S Flux Vacua with Tadpoles

(Gubser, Mitra, 2002)
(Mourad, AS, 2016)

$$S = \frac{1}{2\,k_{10}^2} \int d^{10}x\,\sqrt{-G}\left\{e^{-2\phi}\left[-R+4(\partial\phi)^2\right] - \frac{1}{12}\,\mathcal{H}_3^2 - \frac{1}{4}\,e^{-\phi}\,\mathrm{tr}\,\mathcal{F}^2 - T\,e^{-\phi} + \ldots\right\}$$

- Dilaton Eq: constraint from positivity of T (orientifolds NEED H_3 fluxes, SO(16)xSO(16) H_7 fluxes)
- Eqs. determine AdS (in Poincaré coordinates or in other slicings)
- $AdS_3 \times S^7$ (orientifolds) ; $AdS_7 \times S^3$ (SO(16)xSO(16))

❖ WIDE REGIONS where the two couplings $\alpha' R$ and $g_s = e^{\phi}$ are SMALL

❖ (H_3 or H_7) FLUXES SUPPORT THESE SYMMETRIC COMPACTIFICATIONS

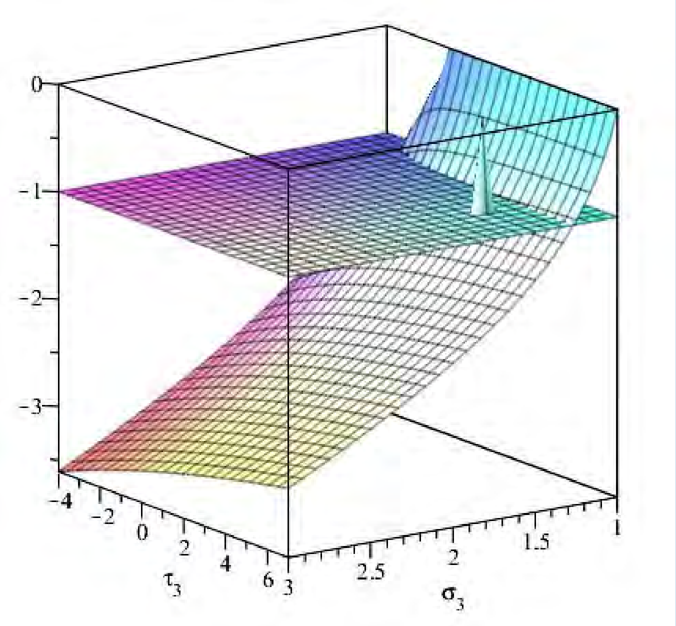
Stability ?

$AdS_3 \times S^7$ ($\&$ $AdS_7 \times S^3$) Vacua

(Gubser, Mitra, 2001)
(Mourad, AS, 2016)
(Basile, Mourad, AS, 2018)

❖ Orientifold & SO(16)xSO(16) vacua: WEAK coupling but UNSTABLE

- VIOLATIONS of Breitenlohner-Freedman bounds for modes with INTERNAL EXCITATIONS (mixings)
- Wide NEARBY regions of stability. Quantum corrections?
- At least in SO(16)xSO(16): perturbative instabilities can be removed by internal projections on S^3



Dudas-Mourad Vacua

(Basile, Mourad, AS, 2018)

❖ Dudas-Mourad vacua: STRONG COUPLING but STABLE !

- E.g.: Scalar perturbations:

$$ds^2 = e^{2\Omega(z)}\left[(1+A)\,dx^\mu\,dx_\mu + (1-7A)\,dz^2\right] ,$$

$$A'' + A'\left(24\,\Omega' - \frac{2}{\phi'}\,e^{2\Omega}\,V_\phi\right) + A\left(m^2 - \frac{7}{4}\,e^{2\Omega}\,V - 14\,e^{2\Omega}\,\Omega'\,\frac{V_\phi}{\phi'}\right) = 0$$

❖ Schrödinger-like form

$$m^2\,\Psi = \left(b + \mathcal{A}^\dagger\,\mathcal{A}\right)\Psi$$

$$\mathcal{A} = \frac{d}{dr} - \alpha(r)\,, \qquad \mathcal{A}^\dagger = -\frac{d}{dr} - \alpha(r)\,, \qquad b = \frac{7}{2}\,e^{2\Omega}\,V\,\frac{1}{1+\frac{9}{4}\,\alpha_O\,y^2} > 0$$

NO tachyons in 9D : PERTUBATIVE STABILITY

The Climbing Scalar

(Basile, Mourad, AS, 2018)

- COSMOLOGY : the issue is the time evolution of perturbations
- INITIALLY (large η) V is negligible: tensor perturbations evolve as

$$
\begin{aligned}
h''_{ij} \; &+ \; \frac{1}{\eta}\, h'_{ij} \; + \; \mathbf{k}^2\, h_{ij} \; = \; 0 \\
h_{ij} \; &\sim \; A_{ij}\, J_0(k\eta) \; + \; B_{ij}\, Y_0(k\eta) \qquad (\mathbf{k} \neq 0) \\
h_{ij} \; &\sim \; A_{ij} \; + \; B_{ij}\, \log\left(\frac{\eta}{\eta_0}\right) \qquad (\mathbf{k} = 0)
\end{aligned}
$$

- NOTICE: logarithmic growth for k=0 (instability of isotropy) !!

- RESONATES with (Kim, Nishimura, Tsuchiya, 2018)
(Anagnostopoulos, Auma, Ito, Nishimura, Papadoulis, 2018)

Dynamical origin of compactification ?

SUMMARIZING

Perturbative Stability of Vacua

1. Dudas-Mourad vacuum: $m^2 \geq 0$ for modes → STABLE

2. Compactifications to AdS: $\left(m^2 \geq -\frac{c}{R^2}\right)$ *(Breitenlohner, Freedman, 1982)*

   [AdS x S non-SUSY vacua → NOT STABLE]

Cosmology

Question: do perturbations decay in time ?

[INSTABILITY OF ISOTROPY for climbing phase → clue for 10→4?]

Stability with a Bounded g_s

More General Setup

(Mourad, AS, 2021)

$$ds^2 = e^{2A(r)}dx^2 + e^{2B(r)}dr^2 + e^{2C(r)}dy^2 \quad : \qquad B = (p+1)A + (D-p-2)C$$

Harmonic gauge

General H-T System:

$$A'' = -\frac{T}{8}\, e^{2\left[(p+1)A+(8-p)C+\frac{\gamma}{2}\,\phi\right]} + \frac{(7-p)}{16}\, e^{2[\beta_p\,\phi+(p+1)\,A]}\, H^2_{p+2}\,,$$
$$C'' = -\frac{T}{8}\, e^{2\left[(p+1)A+(8-p)C+\frac{\gamma}{2}\,\phi\right]} - \frac{(p+1)}{16}\, e^{2[\beta_p\,\phi+(p+1)\,A]}\, H^2_{p+2}\,,$$
$$\phi'' = T\,\gamma\, e^{2\left[(p+1)A+(8-p)C+\frac{\gamma}{2}\,\phi\right]} + \beta_p\, e^{2[\beta_p\,\phi+(p+1)\,A]}\, H^2_{p+2}$$

H vs h: Symmetry:

$$[A, C,\, p] \longleftrightarrow [C, A,\, D-p-3]$$
$$\left[H^2_{p+2}, \beta_p; h^2_{p+1}, \beta_{p-1}\right] \longleftrightarrow \left[-h^2_{p+1}, -\beta_{p-1}; -H^2_{p+2}, -\beta_p\right]$$

New Variables:

$$U = \phi + \{[(p+1)\gamma - 2\,\beta_p]\,A + [(7-p)\gamma + 2\,\beta_p]\,C\}\,,$$
$$X = (p+1)A + \frac{\gamma}{2}\,\phi + (8-p)C\,,$$
$$Z = (p+1)A + \beta_p\,\phi$$

Triangular for $\gamma = \gamma_c$:

$$U'' = 0$$
$$X'' = \frac{T}{2}\,(\gamma^2 - \gamma_c^2)\, e^{2X} + \frac{H^2_{p+2}}{16}\, b\, e^{2Z}$$
$$Z'' = \frac{T}{8}\,(8\,\beta_p\,\gamma - p - 1)\, e^{2X} + \frac{H^2_{p+2}}{8}\left[8\,\beta_p^2 + \frac{(7-p)(p+1)}{2}\right] e^{2Z}$$

With curvature terms: setup for (spherical) black holes or branes

D=4 with Fluxes on $T^5 \times I$

(Mourad, AS, to appear)

- Five-form flux in IIB → ϕ CONSTANT, SPATIAL INTERVAL of length ℓ

$$
\begin{aligned}
ds^2 &= \Lambda^2 \frac{\eta_{\mu\nu}\, dx^\mu\, dx^\nu}{[\sinh(r)]^{\frac{1}{2}}} + \ell^2\, [\sinh(r)]^{\frac{1}{2}}\, e^{-\frac{5r}{\sqrt{10}}}\, dr^2 + \left(\sqrt{2}\,\Phi\,\ell\right)^{\frac{2}{5}} [\sinh(r)]^{\frac{1}{2}}\, e^{-\frac{r}{\sqrt{10}}}\, d\vec{y}^{\,2} \\
\mathcal{H}_5^{(0)} &= \frac{\Lambda^4}{\sqrt{2}}\, \frac{dx^0 \wedge \ldots \wedge dr}{[\sinh(r)]^2} + \Phi\, dy^1 \wedge \ldots \wedge dy^5 \\
q_3\, \Phi &= N \qquad\qquad q_3 = \sqrt{\pi}\, m_{Pl(10)}^4
\end{aligned}
$$

$$\Lambda = i\,\gamma^0\,\gamma^1\,\gamma^2\,\gamma^3\,\sigma_2$$

- FINITE gs , BUT STILL CURVATURE SINGULARITY]

Used extensively: (Bergshoeff, Kallosh, Ortin, Roest, Van Proeyen, 2001)

- Split perturbations according to SO(1,3)xSO(5) [or SO(4) for internal excitations]
- SUSY BREAKING ~ $1/\ell$
- Tensor eqs:
- (+ Einstein eqs.)

$$
\begin{aligned}
\partial_{[\mu}\, b^{(2)}{}_{\nu]}{}^{lm} + \frac{1}{2}\,\epsilon^{lmnpq}\,\partial_n\, b_{\mu\nu pq} &= -\frac{e^{-4A-4C}}{2}\,\epsilon_{\mu\nu\rho\sigma}\,\partial_r\, b^{\rho\sigma lm}\,, \\
\partial_r\, b^{(2)}{}_\mu{}^{lm} &= e^{2A+6C}\left(\partial^{[l}\, b_\mu{}^{m]} + \frac{1}{2}\,\epsilon^{\alpha\beta\gamma}{}_\mu\,\partial_\alpha\, b_{\beta\gamma}{}^{lm}\right), \\
\partial_\mu\, b^m - \partial_n\, b^{(2)}{}_\mu{}^{mn} &= e^{-2C}\left[\frac{H_5}{2}\, h_\mu{}^m - e^{-6A}\,\partial_r\, b_\mu{}^m\right], \\
\partial_r\, b^m &= e^{-2C}\left[\frac{H_5}{2}\, h_r{}^m - e^{10C}\left(\partial^m\, b - \partial_\mu\, b^{\mu m}\right)\right], \\
\partial_p\, b^p &= \frac{H_5}{4}\left(-\,e^{-2A}\, h_\alpha{}^\alpha - e^{-2B}\, h_{rr} + e^{-2C}\, h_i{}^i\right) + e^{-8A}\,\partial_r\, b\,.
\end{aligned}
$$

Stability with Fluxes on T^5 x I

(Mourad, AS, to appear)

$$ds^2 = e^{2A(r)} dx^2 + e^{2B(r)} dr^2 + e^{2C(r)} dy^2$$

- NO instabilities for **k=0, BUT mixings** induce them for **k≠0**
- E.g:

$$M\,Z = m^2\,Z$$

$$\begin{pmatrix} \mathcal{K}^2 + (-\partial + \alpha)_z (\partial + \alpha)_z & \frac{kH_5}{\sqrt{2}}\, e^{2(A-3C)} \\ \frac{kH_5}{\sqrt{2}}\, e^{2(A-3C)} & \mathcal{K}^2 + (-\partial + \beta)_z (\partial + \beta)_z \end{pmatrix} \qquad \alpha = \frac{C-A}{2}\,, \qquad \beta = -\frac{5A+3C}{2}$$

- PERTURBATIONS → Schrödinger-like systems

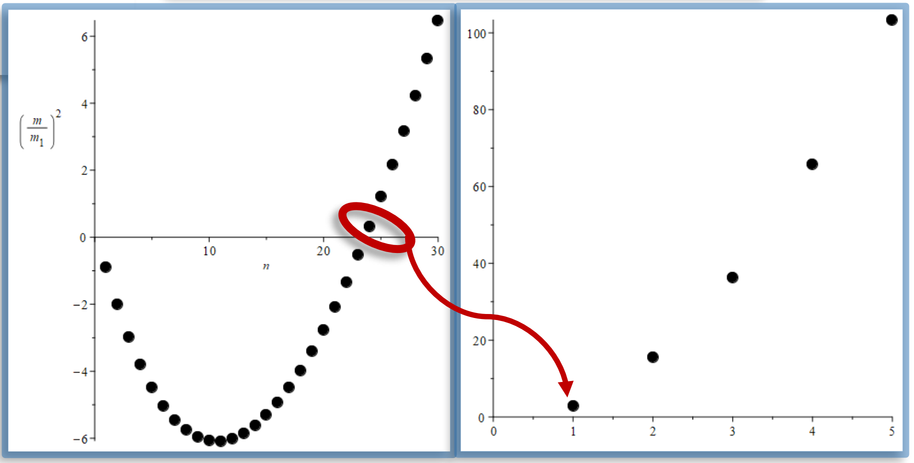
$$m^2\,\Psi = \left(b + \mathcal{A}^\dagger \mathcal{A}\right)\Psi$$
$$\mathcal{A} = \frac{d}{dr} - \alpha(r)$$

- Variational method:

$$m_\Psi^2 = \frac{\langle \Psi | \widetilde{M} | \Psi \rangle}{\langle \Psi | \Psi \rangle} \geq m_0^2$$

- Mixings → unstable KK excitations
- BUT: NO link between 4D & internal scale

$$m_{Pl(10)}\,\ell > \mathcal{O}\left(10^2\right) N^{\frac{1}{4}}$$



Scalar Perturbations with Fluxes on T^5 x I

(Mourad, AS, to appear)

- Here a Schrödinger-like system, but with a **non-symmetric potential**
- [NO NATURAL measure for self-dual form] → from Schrödinger-like system ?
- → The **eigenvalues** for **m²** may be **complex**
- BASIC TOOL: (extension of) variational principle

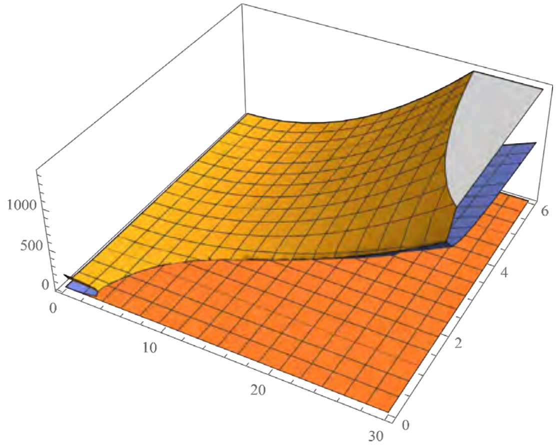
$$m^2_\Psi \;=\; \frac{1}{2}\,\frac{\langle\Psi|\left(\widetilde{M}+\widetilde{M}^\dagger\right)|\Psi\rangle}{\langle\Psi|\Psi\rangle} \;\geq\; m_0^2$$

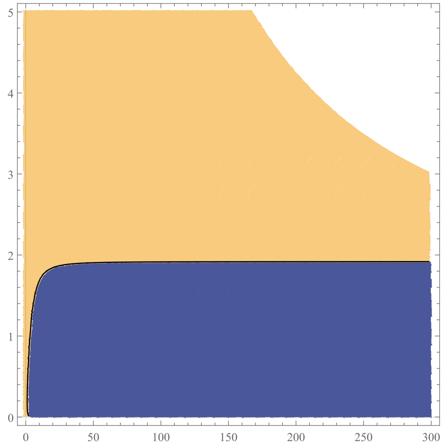
$$m^2 \;=\; Inf\left\{\frac{\int dz\left[\partial_z Y^\dagger H^{-1}\partial_z Y + \frac{\epsilon(z)}{2}Y^\dagger\left(N-N^T\right)\partial_z Y + Y^\dagger H^{-1} U\, Y\right]}{\int dz\, Y^\dagger H^{-1} Y}\right\}$$

$$H^{-1} \;=\; 1-\frac{F(z)}{2}\left(N+N^T\right)\;+\;\frac{F(z)^2}{4}N^T N$$

$$F \;=\; -\frac{1}{\sqrt{2}\,h}\int_{\frac{r}{\rho}}^{\infty} dx\,\frac{e^{\frac{x}{2\sqrt{10}}}}{[\sinh(x)]^{\frac{3}{2}}}$$

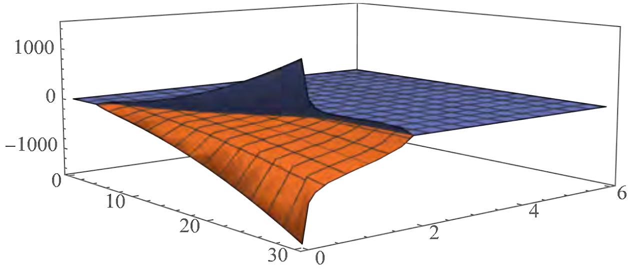
$$N \;=\; \begin{pmatrix} 0 & 1 & 1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$$





SOME EVIDENCE THAT:

NO Instabilities beyond a critical value H_{min}
for the five-form field strength H_5



Summarizing

$$ds^2 = e^{2A(r)} dx^2 + e^{2B(r)} dr^2 + e^{2C(r)} dy^2$$

- More general (warped) compactifications to Minkowski space with fluxes

  - ❖ IIB to Minkowski$_4$ with Flux: g_s can be BOUNDED EVERYWHERE (not R, however!)
  - ❖ MODE ANALYSIS: complicated by modes of the self-dual H_5, NO STANDARD ACTION!
  - ❖ BASIC TOOL: reduce to Schrödinger-like systems → VARIATIONAL METHOD for m^2 !
  - ❖ STABILITY: possible since internal scales leave no sign in Minkowski$_4$!
  - ❖ [ONLY ONE peculiar sector with non-Hermitian Hamiltonian]

There seems to be room for stable 4D Minkowski vacua

ALSO: useful BENCHMARKS to explore brane deformations by dilaton tadpoles

NEXT TIME …

Thank You