Celestial CFT and extended SuperBMS algebra

Angelos Fotopoulos
Northeastern University

OIST

Okinawa Institute of Science and Technology

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based on:

Wei. Fan, A.F., T.R. Taylor

Soft Limits of Yang-Mills Amplitudes and Conformal Correlators

arXiv:1903.01676, JHEP 05 (2019) 121

A.F. S.Stieberger., T.R. Taylor, Bin Zhu:

BMS Algebra from Soft and Collinear Limits

arXiv:1912.10973, JHEP 03 (2020) 130

Extended Super BMS Algebra of Celestial CFT

arXiv:2007.03785 JHEP 09 (2020) 198

Wei Fan, A.F., S. Stieberger T.R. Taylor:

On Sugawara construction on Celestial Sphere

arXiv:2005.10666 JHEP 09 (2020) 139

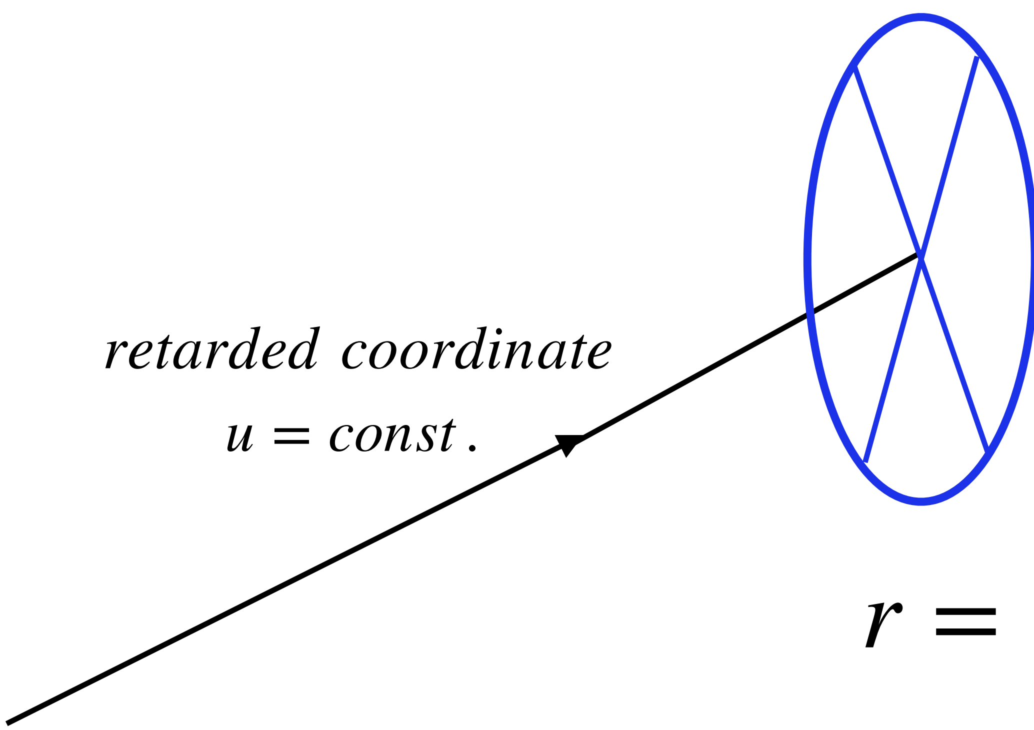
$$ds^2 = -dt^2 + d\vec{x}^2$$

Flat Minkowski metric in <u>retarded (or Bondi) coordinates</u> $(u, r, z, \bar{z})$

$$\begin{cases} x^0 = u + r \\ x^1 = \dfrac{r\,(z+\bar{z})}{1+|z|^2} \\ x^2 = -i\dfrac{r\,(z-\bar{z})}{1+|z|^2} \\ x^3 = \dfrac{r(1-|z|^2)}{1+|z|^2} \end{cases}$$

$$r^2 = \vec{x}^2$$

$$ds^2 = -du^2 - 2\,dudr + \underbrace{\frac{4r^2}{(1+|z|^2)^2}}_{S^2}\,dzd\bar{z}$$



Null infinity $\mathscr{I}^+$

$S^2 \times \mathbf{R}$

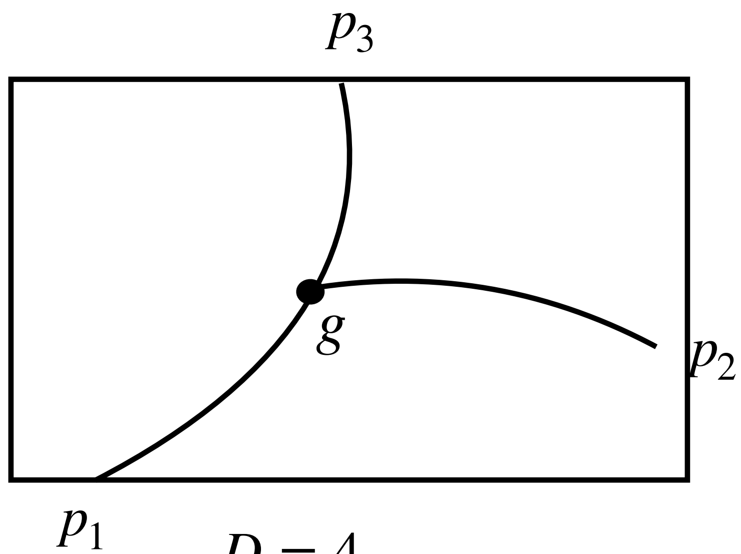
$(u, z, \bar{z})$

natural arena
for in and out states

Basic Idea

Amplitudes = conformal correlators of primary fields on celestial sphere

traditional amplitudes describe <u>transitions</u> between <u>momentum</u> <u>eigenstates</u>

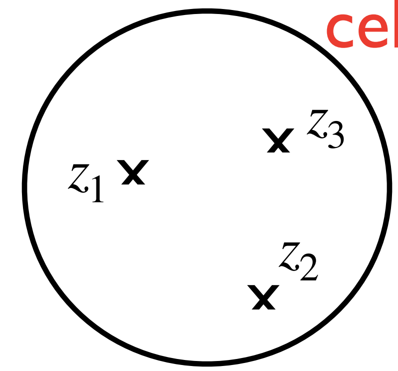


$D = 4$

$$z_k = \frac{p_k^1 + i p_k^2}{p_k^0 + p_k^3}$$

=

celestial sphere



$D = 2$

D=2 Euclidean CFT

D=4 space-time QFT

$$\mathscr{A}(\{p_i, \epsilon_j\}) = i(2\pi)^4 \, \delta^{(4)}\left(p_1 + p_2 + p_3\right) \, A(\{p_i, \epsilon_j\})$$

$$\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \rangle \sim \frac{g}{|z_1 - z_2|^{h_1+h_2-h_3} \, |z_2 - z_3|^{h_2+h_3-h_1} \, |z_1 - z_3|^{h_1+h_3-h_2}}$$

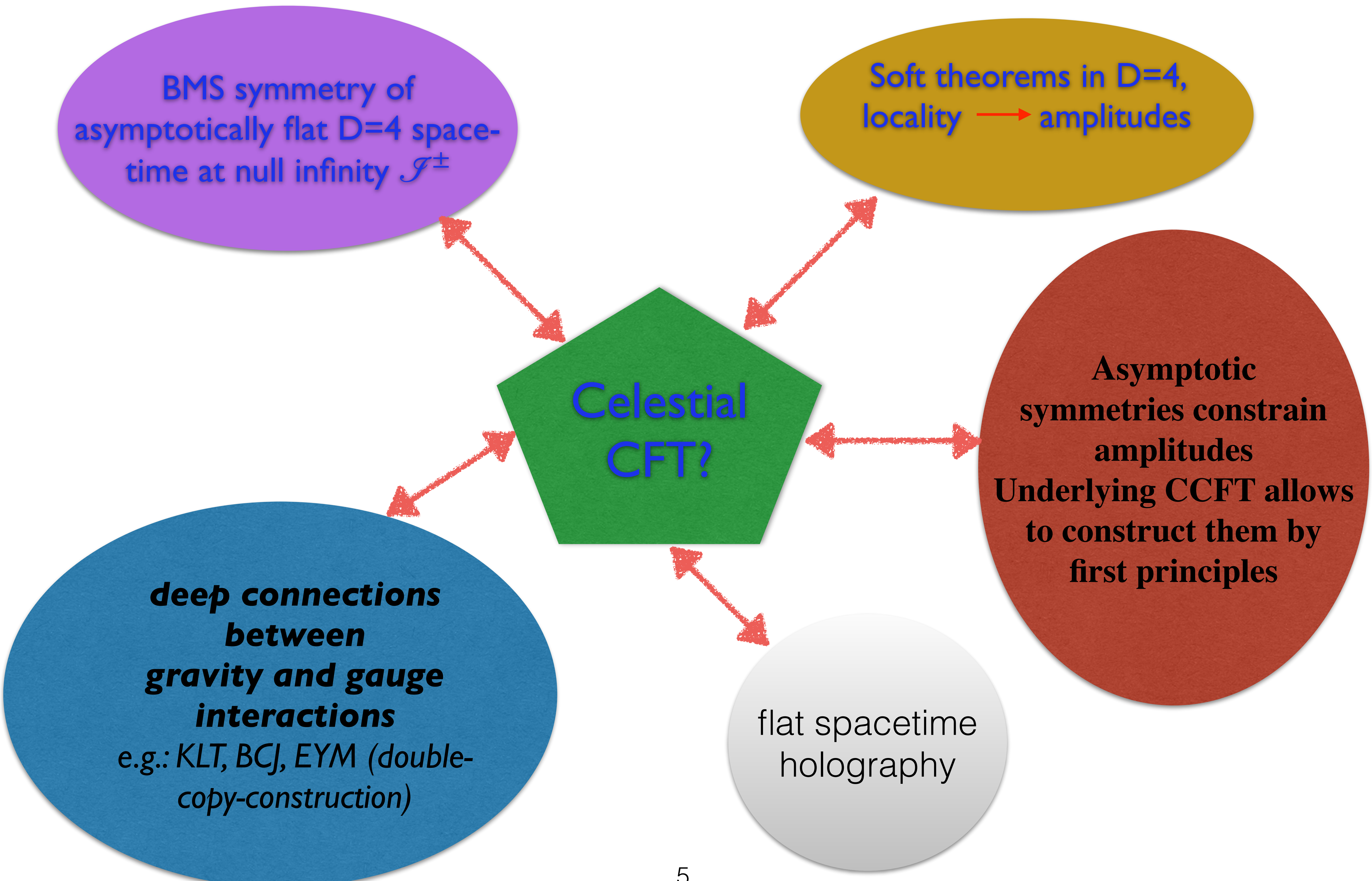
Lorentz symmetry

$SO(1,3) \simeq SL(2,\mathbf{C})$

$$z_i \to \frac{a z_i + b}{c z_i + d}$$

global conformal symmetry on CS^2

Why study CCFT ?



Outline of this Talk

- Celestial CFT basics: Conformal Primary wave functions, CCFT operators, Mellin amplitudes, correlators
- Operator Product expansion in CCFT
- Symmetries of CCFT and soft theorems
- Extended BMS in CCFT
- Supersymmetric Extended BMS:
  1. Supermultiplets of Conformal Primary Wavefunctions and CCFT operators
  2. OPEs of fermionic fields
  3. Fermionic Soft theorems
  4. OPEs of Super-BMS generators
  5. Super-BMS Algebra

CCFT: Massless particles on celestial sphere

$p^\mu = \omega\ q^\mu(z,\bar{z}) \qquad q^\mu = (1+|z|^2, z+\bar{z}, -i(z-\bar{z}), 1-|z|^2)$

In the massless case, transition from momentum space to conformal primary wavefunctions (CPW) with conformal dimension Δ is implemented by Mellin transform:

Pasterski, Shao (2017), also Banerjee (2018)

$$\tilde{\phi}(\Delta, z, \bar{z}; X,) = \int_0^\infty d\omega\ \omega^{\Delta-1}\phi(\omega, z, \bar{z}; X) \qquad \Delta = 1 + i\lambda,\ \lambda \in \mathbf{R}$$

described by
- the point $z \in CS^2$ at which it enters or exits the celestial sphere
- SL(2,C) Lorentz quantum numbers $(h, \bar{h})$

E.g.: scalar plane wave $\quad e^{\pm ip\cdot X}$

$$\varphi^{\pm}_{\Delta}(X, z, \bar{z}) = \int_0^\infty d\omega\ \omega^{\Delta-1}\ e^{\pm i\omega q_\mu X^\mu - \epsilon\omega} = \frac{(\mp i)^\Delta \Gamma(\Delta)}{(q(z,\bar{z})\cdot X \mp i\epsilon)^\Delta}$$

solves D=4 Klein-Gordon equation

gauge boson: $\quad \epsilon_\mu e^{ip\cdot X}$

$$V^{\Delta,J}_\mu(X^\mu, z, \bar{z}) \equiv (\partial_\ell q_\mu)\int_0^\infty d\omega\ \omega^{\Delta-1} e^{\mp i\omega q\cdot X - \epsilon\omega} \qquad (\ell = z, \bar{z}; J = \pm 1),$$

solves Maxwell in D=4

CCFT: Particles <-> Operators

in momentum basis: plane waves with momentum $p = \omega q(z)$

in conformal basis: Conformal Primary Wave functions Φ

$$\Phi_{h,\bar{h}}\left(\frac{az+b}{cz+d}, \frac{\bar{a}\bar{z}+\bar{b}}{\bar{c}\bar{z}+\bar{d}}\right) = (cz+d)^{2h}\,(\bar{c}\bar{z}+\bar{d})^{2\bar{h}}\,\Phi_{h,\bar{h}}(z,\bar{z})$$

Holography: CFT operator $\mathcal{O}_{h,\bar{h}}$

with:

$h + \bar{h} = \Delta$ dimension

$h - \bar{h} = J$ spin

$$(h,\bar{h}) = \frac{1}{2}(\Delta + J, \Delta - J)$$

CCFT: n-point amplitude on celestial sphere

$$\mathscr{A}(\{p_i, \epsilon_j\}) = i(2\pi)^4\ \delta^{(4)}\left(p_1 + p_2 - \sum_{k=3}^{n} p_k\right) A(\{p_i, \epsilon_j\})$$

with:

$$\langle ij\rangle = 2\ (\omega_i\omega_j)^{1/2}\ (z_i - z_j)$$
$$[ij] = 2\ (\omega_i\omega_j)^{1/2}\ (\bar{z}_i - \bar{z}_j)$$

$$\epsilon^\mu(q)_\pm = \frac{1}{\sqrt{2}}\begin{cases}\partial_z q^\mu = (\bar{z}, 1, -i, -\bar{z})\\ \partial_{\bar{z}} q^\mu = (z, 1, \ i, -z)\end{cases}$$

Celestial amplitudes $\widetilde{\mathscr{A}}$ of massless particles are obtained from momentum-space amplitudes $\mathscr{A}$ by Mellin transforms w.r.t. particle energies $\Delta_j = 1 + i\lambda_j$

$$\left\langle \prod_{k=1}^{n} \mathcal{O}_{\Delta_k, J_k}(z_k, \bar{z}_k)\right\rangle =$$

Cheung, de la Fuente, Sundrum (2016)
Pasterski, Shao Strominger (2017)

$$= \widetilde{\mathscr{A}}_{\{\Delta_k, J_k\}}(z_k, \bar{z}_k) = \left(\prod_{k=1}^{n}\int_0^\infty \omega_k^{\Delta_k - 1}\ d\omega_l\right)\ \delta^{(4)}(\omega_1 q_1 + \omega_2 q_2 - \sum_{m=3}^{n}\omega_m q_m) \times A(\omega_n, z_n, \bar{z}_n)$$

D=2 CFT correlators involve conformal wave packets

Modified basis with null infinity coordinate: Banerjee, Ghosh, Pandey, Saha (2019)

Gauge Amplitudes

example four-gluon amplitude:

$$\widetilde{\mathcal{A}}_4(-,-,+,+) = 8\pi\,\delta(r-\bar{r})\,\theta(r-1)\left(\prod_{i<j}^{4} z_{ij}^{\frac{h}{3}-h_i-h_j}\,\bar{z}_{ij}^{\frac{\bar{h}}{3}-\bar{h}_i-\bar{h}_j}\right)$$

$$\times\, r^{\frac{5}{3}}\,(r-1)^{\frac{2}{3}}\,\delta\left(-4+\sum_{i=1}^{4}\Delta_i\right)$$

$$r = \frac{z_{12}\,z_{34}}{z_{23}\,z_{41}}$$

conformal invariant cross-ratio on CS^2

Pasterski, Shao, Strominger (2017)

$$h_1 = \tfrac{i}{2}\lambda_1,\;\; h_2 = \tfrac{i}{2}\lambda_2,\;\; h_3 = 1+\tfrac{i}{2}\lambda_3\,,\, h_4 = 1+\tfrac{i}{2}\lambda_4$$

$$\bar{h}_1 = 1+\tfrac{i}{2}\lambda_1,\;\; \bar{h}_2 = 1+\tfrac{i}{2}\lambda_2,\;\; \bar{h}_3 = \tfrac{i}{2}\lambda_3\,,\, \bar{h}_4 = \tfrac{i}{2}\lambda_4$$

higher-point: involve Gaussian hypergeometric functions like string amplitudes

Schreiber, Volovich, Zlotnikov (2017)

Graviton Amplitudes

four-graviton amplitude:

$$\tilde{\mathscr{A}}_4(--,--,++,++) = 2\pi\, \delta(r-\bar{r})\, \theta(r-1) \left(\prod_{i<j}^{4} z_{ij}^{\frac{h}{3}-h_i-h_j}\; \bar{z}_{ij}^{\frac{\bar{h}}{3}-\bar{h}_i-\bar{h}_j} \right)$$

$$\times\, r^{\frac{11}{3}-\frac{\beta}{3}}\, (r-1)^{-\frac{1}{3}-\frac{\beta}{3}}\, \delta\!\left(-2+\sum_{i=1}^{4}\Delta_i\right)$$

S.Stieberger., Taylor (2018)

$$h_1 = -\frac{1}{2}+\frac{i}{2}\lambda_1,\;\; h_2 = -\frac{1}{2}+\frac{i}{2}\lambda_2,\;\; h_3 = \frac{3}{2}+\frac{i}{2}\lambda_3\;, h_4 = \frac{3}{2}+\frac{i}{2}\lambda_4$$

$$\bar{h}_1 = \frac{3}{2}+\frac{i}{2}\lambda_1,\;\; \bar{h}_2 = \frac{3}{2}+\frac{i}{2}\lambda_2,\;\; \bar{h}_3 = -\frac{1}{2}+\frac{i}{2}\lambda_3\;, \bar{h}_4 = -\frac{1}{2}+\frac{i}{2}\lambda_4 \qquad \beta = 2-\frac{1}{2}\sum_{i=1}^{4}\Delta_i$$

- *first calculation of graviton amplitude in the conformal basis*
- *important for the soft graviton theorems* $\Delta \to 1,0,\ldots$ *in celestial basis*

no holomorphic factorization (due to supertranslation operator P)

OPE in CCFT: Collinear singularities (1)

Gauge theory → collinear limits → singularities

$\overrightarrow{p_1} \parallel \overrightarrow{p_2}$

+ soft theorems determine almost uniquely scattering amplitudes

work by too many to cite properly

$$p^\mu = \omega q^\mu(z, \bar{z})$$

$z_1 \to z_2$

CCFT → Operators approach → singularities OPE

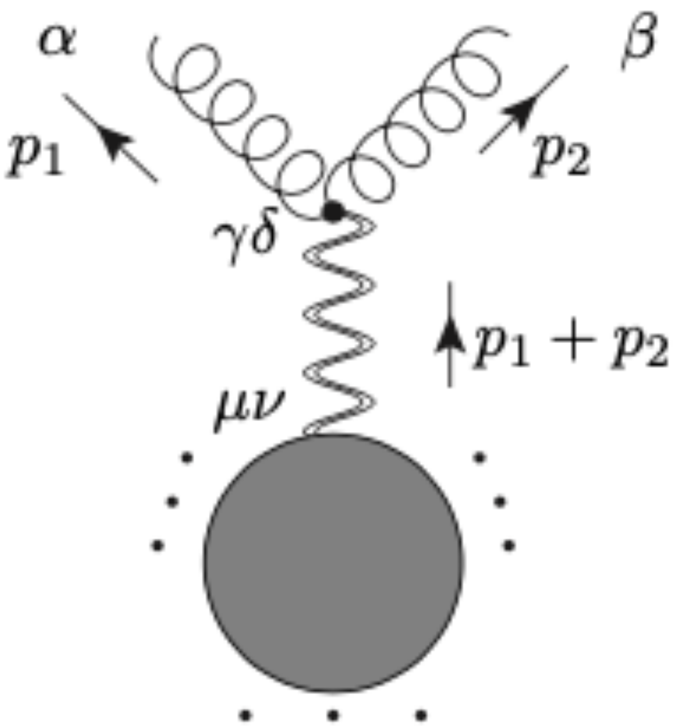
OPE for Conformal primaries

$$\mathcal{O}_i(z_i)\mathcal{O}_j(z_j) \sim \frac{C_{ijk}}{(z_i - z_j)^{h_i+h_j-h_k}}\mathcal{O}_k(z_j) + \ldots$$

in 2D: structure constants
+Virasoro (local conformal) symmetry → CFT correlators

OPE in CCFT: Collinear singularities (2)

EYM Feynman Diagram for collinear gauge boson singularity



$$S_g^{\mu\nu} = \epsilon_1^{\alpha}\epsilon_2^{\beta} D^{\mu\nu}{}_{\gamma\delta}(p_1+p_2) V^{\gamma\delta}_{\alpha,\beta}(p_1,p_2)$$

Fan, A.F. .Stieberger., Taylor, Zhu (2019)

gauge collinear

$$\mathcal{M}(1^+,2^-,3,\ldots,N) \sim \frac{1}{\bar{z}_{12}}\frac{\omega_1}{\omega_2\omega_P}\mathcal{M}(P^+,3,\ldots,N) + \frac{1}{z_{12}}\frac{\omega_2}{\omega_1\omega_P}\mathcal{M}(P^-,3,\ldots,N)$$
$$- \frac{z_{12}}{\bar{z}_{12}}\frac{\omega_1^2}{\omega_P^2}\mathcal{M}(P^{++},3,\ldots,N) - \frac{\bar{z}_{12}}{z_{12}}\frac{\omega_2^2}{\omega_P^2}\mathcal{M}(P^{--},3,\ldots,N)$$

$\omega_P = \omega_1 + \omega_2$

gravity collinear

Mellin transform → CCFT OPE!

OPE in CCFT (3)

Celestial conformal field theory (CCFT)

$$\mathcal{O}^a_{\Delta_1,-1}(z,\bar{z})\mathcal{O}^b_{\Delta_2,+1}(w,\bar{w}) = \frac{C_{(-,+)-}(\Delta_1,\Delta_2)}{z-w}\sum_c f^{abc}\ \mathcal{O}^c_{(\Delta_1+\Delta_2-1),-1}(w,\bar{w})$$
$$+\frac{C_{(-+)+}(\Delta_1,\Delta_2)}{\bar{z}-\bar{w}}\sum_c f^{abc}\ \mathcal{O}^c_{(\Delta_1+\Delta_2-1),+1}(w,\bar{w})$$
$$+\ C_{(-+)--}(\Delta_1,\Delta_2)\ \frac{\bar{z}-\bar{w}}{z-w}\ \delta^{ab}\ \mathcal{O}_{(\Delta_1+\Delta_2),-2}(w,\bar{w})$$
$$+\ C_{(-+)++}(\Delta_1,\Delta_2)\ \frac{z-w}{\bar{z}-\bar{w}}\ \delta^{ab}\ \mathcal{O}_{(\Delta_1+\Delta_2),+2}(w,\bar{w})\ +reg\,.$$

Derive from collinear limits of D=4 EYM amplitudes

Fan, A.F. St. Stieberger., Taylor, Zhu (2019)

↕ D=4 S-matrix constrains OPE or vice versa

Derive from first principles and consistency conditions

Pate, Raclariu, Strominger, Yuan (2019)

} extended BMS symmetry

Symmetries of CCFT and Soft theorems

At null infinity $\mathscr{I}^{\pm}$ more (hidden) symmetries present to constrain S-matrix

→ non-trivial consistency on amplitudes

$$z_i \to \frac{az_i + b}{cz_i + d}$$

$$SL(2,\mathbf{C})_{z_i}: \quad \widetilde{\mathscr{A}}_n(\{\Delta_i, J_i\}) \longrightarrow (cz_i + d)^{\Delta_i + J_i} (\bar{c}\bar{z}_i + \bar{d})^{\Delta_i - J_i} \widetilde{\mathscr{A}}_n(\{\Delta_i, J_i\})$$

$$P_{-1/2,-1/2} = e^{(\partial_h + \partial_{\bar{h}})/2} = P^0 + P^3$$

Stieberger, Taylor (2018)

$$P^{(j)}_{-1/2,-1/2}: \quad \widetilde{\mathscr{A}}_n(\{\Delta_j, J_i\}) \longrightarrow \widetilde{\mathscr{A}}_n(\{\Delta_j + 1, J_i\})$$

translation operator P^{μ} *shifts conformal dimension* Δ_j

$$\sum_i P^{(i)}_{-\frac{1}{2}\frac{-1}{2}} \widetilde{\mathscr{A}}_n(\{\Delta_j, J_i\}) = 0$$

Symmetries of CCFT and Soft theorems

In usual QFT soft theorems $\omega_s \to 0$ play an important role in consistency conditions on scattering amplitudes

$$\mathscr{M}_{n+1} \longrightarrow \left(\underbrace{\omega_s^{-1}\, S_G^{(0)}}_{} + \underbrace{\omega_s^{0}\, S_G^{(1)}}_{} + \underbrace{\omega_s\, S_G^{(2)}}_{} + \ldots \right) \mathscr{M}_n$$

on CS^2: poles at: $\Delta \to 1$ $\Delta \to 0$ $\Delta \to -1$

Weinberg (1965)
Cachazo, Strominger (2014)

$$\mathscr{A}_{n+1} \longrightarrow \left(\underbrace{\omega_s^{-1}\, S_{YM}^{(0)}}_{} + \underbrace{\omega_s^{0}\, S_{YM}^{(1)}}_{} + \ldots \right) \mathscr{A}_n$$

poles at: $\Delta \to 1$ $\Delta \to 0$

soft theorems imply
Ward identities for asymptotic symmetries

Strominger (2013)

Mellin amplitude Residues: $\Delta \to 0,1,\ldots$ (conformally soft operators)

Kapec, Mitra, Raclariu, Strominger (2016)
Donnay, Puhm, Strominger (2018)

Soft (Conformal) Theorems

gauge

$$\mathcal{A}_{+1}(\Delta_0, \Delta_1, \ldots, \Delta_N) \longrightarrow = \frac{1}{\Delta_0 - 1}\left(\frac{1}{z - z_1} - \frac{1}{z - z_N}\right)\mathcal{A}(\Delta_1 \cdots \Delta_N)$$

$$\mathcal{A}_{+1}(\Delta_0, \Delta_1, \ldots, \Delta_N) = \frac{1}{\Delta_0}\frac{1}{z - z_1}\left[(\bar{z} - \bar{z}_1)\partial_{\bar{z}_1} - 2\bar{h}_1 + 1\right]\mathcal{A}(\Delta_1 - 1 \cdots \Delta_N)$$

Taylor, AF, Adamo, Mason, Sharma/Guevara/ Puhm 2019

gravity

$$\mathcal{A}_{+2}(\Delta_0, \Delta_1, \ldots, \Delta_N) \longrightarrow \frac{1}{\Delta_0}\sum_{i=1}^{N}\frac{(\bar{z}_0 - \bar{z}_i)}{(z_0 - z_i)}\frac{(\xi - z_i)}{(\xi - z_0)}\left[(\bar{z}_0 - \bar{z}_i)\partial_{\bar{z}_i} - 2\bar{h}_i\right]\mathcal{A}(\Delta_1, \ldots, \Delta_N)$$

$$\mathcal{A}_{+2}(\Delta_0, \Delta_1, \ldots, \Delta_N) \longrightarrow \frac{1}{\Delta_0 - 1}\sum_{i=1}^{N}\frac{(\bar{z}_0 - \bar{z}_i)}{(z_0 - z_i)}\frac{(\xi - z_i)^2}{(\xi - z_0)^2}\mathcal{A}(\Delta_1, \ldots, \Delta_i + 1, \ldots, \Delta_N)$$

Ward identities and BMS symmetries:

CCFT description of soft operators

energy-momentum tensor T(z):

conformally soft-graviton
$\Delta \to 0$

$$T(z) := \tilde{\mathcal{O}}_{\Delta=2,J=+2}(z,\bar{z}) = \frac{3}{\pi}\int d^2w \, \frac{\mathcal{O}_{\Delta=0,J=-2}(w,\bar{w})}{(z-w)^4}$$

$(h, \bar{h}) = (2{,}0)$

Kapec, Mitra, Raclariu, Strominger (2016)
Cheung, de La Fuente, Sundrum (2016)

shadow transformation:

$$\tilde{\mathcal{O}}^a_{\tilde{\Delta},\tilde{J}}(z,\bar{z}) = \tilde{\mathcal{O}}^a_{2-\Delta,-J}(z,\bar{z}) = \frac{(\Delta+J-1)}{\pi}\int_{\mathbf{C}} \frac{d^2w}{(z-w)^{2-\Delta-J}(\bar{z}-\bar{w})^{2-\Delta+J}} \, \mathcal{O}^a_{\Delta,J}(w,\bar{w})$$

Ferrara, Grillo, Parisi, Gatto (1972)
Dolan, Osborn (2012)

a) Single Soft limit:

A.F., T.R. Taylor (2019)

$$\langle T(z) \prod_{i=1}^{n} O_{\Delta_i,J_i}(z_i,\bar{z}_i)\rangle = \sum_{i=1}^{n}\left(\frac{h_{O_i}}{(z-z_i)^2}+\frac{\partial_{z_i}}{z-z_i}\right)\langle\prod_{i=1}^{n} O_{\Delta_i,J_i}(z_i,\bar{z}_i)\rangle$$

OPE with Primaries: $T(z)\mathcal{O}^{h_i,\bar{h}_i}(w) \sim \frac{h_i}{(z-w)^2}\mathcal{O}^{h_i,\bar{h}_i}(w) + \frac{1}{z-w}\partial_w \mathcal{O}^{h_i,\bar{h}_i}(w)$

Gauge boson/ graviton CCFT operators → Virasoro primaries

b) Double soft limits

$$\left\langle T(w)T(z)\prod_{i=2}^{n} \mathcal{O}_{\Delta_i,J_i}(z_i,\bar{z}_i)\right\rangle \sim \left\langle\left(\frac{2}{(z-w)^2}T(w)+\frac{1}{z-w}\partial_w T(w)\right)\prod_{i=2}^{n} O_{\Delta_i,J_i}(z_i,\bar{z}_i)\right\rangle$$

OPE:

$$T(z)T(w) = \frac{2T(w)}{(z-w)^2} + \frac{\partial_w T(w)}{z-w} + \dots$$

$$T(w)\overline{T}(\bar{z}) = reg\,.$$

$c = 0$

A.F., Stieberger., Taylor, Zhu (2019)

(ii) supertranslation operator P(z):

conformally soft-graviton
$\Delta \to 1$

$$P(z,\bar{z}) := \partial_{\bar{z}}\mathcal{O}_{\Delta=1,J=+2}(z,\bar{z}) \qquad (h,\bar{h}) = (\tfrac{3}{2},\tfrac{1}{2})$$

$$\left\langle P(z_0)\prod_{j=1}^{n}\mathcal{O}_{\Delta_j,J_j}(z_j,\bar{z}_j)\right\rangle \sim \sum_{i=1}^{n}\frac{1}{z_0-z_i}\left\langle\prod_{n=1}^{n}\mathcal{O}_{\Delta_j,J_j}(z_j,\bar{z}_j)\right\rangle\Bigg|_{\Delta_i\to\Delta_i+1}$$

Adamo, Mason, Sharma/Guevara/ Puhm (2019)

$$P(z)\mathcal{O}_{\Delta,J}(w,\bar{w}) \sim \frac{1}{z-w}\,\mathcal{O}_{\Delta+1,J}(w,\bar{w}) + \ reg\,.$$

From double soft limits

OPEs:

$$T(z)P(w) = \frac{3}{2(z-w)^2}P(w) + \frac{1}{z-w}\partial_w P(w) + reg\,.$$

$$\bar{T}(\bar{z})P(w) = -\frac{\mathcal{O}_{1,+2}(w,\bar{w})}{(\bar{z}-\bar{w})^3} + \frac{1}{2(\bar{z}-\bar{w})^2}P(w) + \frac{1}{\bar{w}-\bar{z}}\partial_{\bar{w}}P(w) + \ reg\,.$$

Transforms as an antiholomorphic descendant

$$P(z)P(w) \sim reg\,.$$

In addition to Virasoro symmetry, we construct all supertranslation generators acting on primary fields

A.F., Stieberger, Taylor, Zhu (2019)

construct:

$$P_{n-\frac{1}{2},-\frac{1}{2}} = \frac{1}{i\pi(n+1)} \oint dw\, w^{n+1} \left[T(w), P_{-\frac{1}{2},-\frac{1}{2}}\right]$$

$$P_{n-\frac{1}{2},m-\frac{1}{2}} = \frac{1}{i\pi(m+1)} \oint d\bar{w}\, \bar{w}^{m+1} \left[\bar{T}(\bar{w}), P_{n-\frac{1}{2},-\frac{1}{2}}\right]$$

$$P_{-1/2,-1/2} = e^{(\partial_h + \partial_{\bar{h}})/2}$$

we find:

$$\left[P_{n-\frac{1}{2},m-\frac{1}{2}}, \mathcal{O}_{h,\bar{h}}(z,\bar{z})\right] = z^n \bar{z}^m \mathcal{O}_{h+\frac{1}{2},\bar{h}+\frac{1}{2}}(z,\bar{z})$$

$\rightarrow$ $P_{k,l}, \bar{P}_{k,l}$

$\rightarrow$ local (or extended) BMS algebra:

$$[P_{ij}, P_{k,l}] = 0\ ,$$

$$[L_n, P_{k,l}] = \left(\frac{1}{2}n - k\right) P_{n+k,l} + n(n^2-1)\ C_{n,k}\ ,$$

$$[\bar{L}_n, P_{k,l}] = \left(\frac{1}{2}n - l\right) P_{k,n+l} + n(n^2-1)\ \bar{C}_{n,l}\ .$$

$m,n \in \mathbf{Z},\ i,j,k,l \in \mathbf{Z}+\frac{1}{2}$

field dependent central charges =0

Bondi, Burg, Metzner (1962)
Sachs (1962)
Barnich Troessaert (2010)
Barnich (2017)

Conformal soft-theorems $\longleftrightarrow$ Ward identities $\longleftrightarrow$ extended BMS algebra

Extended BMS group on celestial sphere

global BMS symmetry on celestial sphere	local BMS symmetry on celestial sphere
<u>Lorentz group:</u> global conformal transformations on celestial sphere SL(2,C) $z \to \frac{az+b}{cz+d}$ $L_{-1} = \partial$ $L_0 = z\partial + h$ $L_1 = z^2\partial + 2hz$	local conformal transformations = superrotations T(z) $[L_m, L_n] = (m-n)\, L_{m+n}$ $[\bar{L}_m, \bar{L}_n] = (m-n)\, \bar{L}_{m+n}$
<u>global space-time translation:</u> Abelian subgroup of supertranslations $P_{-1/2,-1/2} = e^{(\partial_h+\partial_{\bar{h}})/2}$ $P_{1/2,1/2} = z\ e^{(\partial_h+\partial_{\bar{h}})/2}$ $P_{-1/2,1/2} = \bar{z}\ e^{(\partial_h+\partial_{\bar{h}})/2}$ $P_{-1/2,1/2} = \lvert z\rvert^2\ e^{(\partial_h+\partial_{\bar{h}})/2}$	local space-time translations =supertranslations P(z) $P_{n-\frac{1}{2},m-\frac{1}{2}}$ $n, m \in \mathbf{Z}$

→ Symmetries of the celestial OPEs and correlators

S-matrix (non-trivial consistency)

- *Extended Super BMS Algebra: to follow*

Supersymmetric Extended BMS

Supermultiplets of Conformal Primary Wavefunctions:

The Chiral multiplet

A.F. Stieberger, Taylor, Zhu, (2020)

Scalar CPW $\varphi^{\pm}_{\Delta}(X^\mu, z, \bar{z}) = \int_0^\infty d\omega\, \omega^{\Delta-1} e^{\pm i\omega q\cdot X - \epsilon\omega} = \frac{(\mp i)^\Delta \Gamma(\Delta)}{(-q\cdot X \mp i\epsilon)^\Delta}$

$(h, \bar{h}) = (\frac{\Delta}{2}, \frac{\Delta}{2})$

Fermion CPW $\psi^{\pm}_{\Delta,\alpha}(X, z, \bar{z}) = |q\rangle_\alpha \int d\omega\ \omega^{\Delta+\frac{1}{2}-1} e^{\pm i\omega q\cdot X - \epsilon\omega} = |q\rangle_\alpha\ \varphi^{\pm}_{\Delta+\frac{1}{2}}(X, z, \bar{z})$

$(h, \bar{h}) = (\frac{\Delta}{2} - \frac{1}{4}, \frac{\Delta}{2} + \frac{1}{4})$

Solve Weyl equation $\bar{\sigma}^\mu \partial_\mu \psi_\Delta = 0$

Dirac Spinors $\Psi^{\pm}_{\Delta, J=-\frac{1}{2}}(X, z, \bar{z}) = \begin{pmatrix} \psi^{\pm}_{\Delta,\alpha} \\ 0 \end{pmatrix}$, $\Psi^{\pm}_{\Delta, J=+\frac{1}{2}}(X, z, \bar{z}) = \begin{pmatrix} 0 \\ \bar{\chi}^{\pm\dot{\alpha}}_{\Delta} \end{pmatrix}$

Orthonormal under Dirac inner product

Quantum Fields 4D expanded in CPW

$$\varphi(X) = \int d^2z\ d(i\Delta) \left[a_{\Delta+}(z)\varphi^-_{\bar{\Delta}^*}(X,z) + a^\dagger_{\Delta-}(z)\varphi^+_{\Delta}(X,z)\right],$$

$$\psi_\alpha(X) = \int d^2z\ d(i\Delta) \left[b_{\Delta+}(z)\psi^-_{\bar{\Delta}^*,\alpha}(X,z) + b^\dagger_{\Delta-}(z)\psi^+_{\Delta,\alpha}(X,z)\right].$$

$$a_{\Delta\pm}(z) = \int_0^\infty d\omega\, \omega^{\Delta-1}\, a_\pm(\overrightarrow{p})\ , \qquad b_{\Delta\pm}(z) = \int_0^\infty d\omega\, \omega^{\Delta-1}\, b_\pm(\overrightarrow{p})$$

$$(h,\bar{h}) = (\frac{\Delta}{2}, \frac{\Delta}{2}) \qquad\qquad (h,\bar{h}) = (\frac{\Delta}{2} + \frac{1}{4}, \frac{\Delta}{2} - \frac{1}{4})$$

Oscillator Algebra

$$[a_{\Delta\pm}(z), a^\dagger_{\Delta'\pm}(z')] = 8\pi^4\delta(\Delta + (\Delta')^* - 2)\delta^{(2)}(z - z'),$$

$$\{b_{\lambda\pm}(z), b^\dagger_{\lambda'\pm}(z')\} = 8\pi^4\delta(\Delta + (\Delta')^* - 2)\delta^{(2)}(z - z').$$

On-shell Susy transformation

$$\delta_{\eta,\bar\eta}\varphi = [\langle \eta Q\rangle + [\bar\eta\bar Q], \varphi] = \sqrt{2}\eta\psi \qquad \delta_{\eta,\bar\eta}\psi = [\langle \eta Q\rangle + [\bar\eta\bar Q], \psi] = i\sqrt{2}\sigma^\mu\bar\eta\partial_\mu\varphi \ .$$

$\downarrow$

$$[\langle\eta Q\rangle, a_{\Delta+}] = \langle\eta q\rangle b_{(\Delta+\frac{1}{2})+} \ , \qquad [[\bar\eta\bar Q], a_{\Delta+}] = 0,$$

$$[\langle\eta Q\rangle, b_{\Delta+}] = 0 \ , \qquad [[\bar\eta\bar Q], b_{\Delta+}] = [\bar\eta\bar q]a_{(\Delta+\frac{1}{2})+} \ .$$

Holography

$$a_{\Delta+} \to \mathcal{O}_{h,\bar h}(z,\bar z) \ , \quad b_{\Delta,+} \to \mathcal{O}_{h+\frac{1}{4},\bar h-\frac{1}{4}}(z,\bar z) \ , \quad (h,\bar h) = \left(\frac{1+i\lambda}{2}, \frac{1+i\lambda}{2}\right) \ .$$

Susy transformation of CCFT Operators

$$[\langle \eta Q \rangle, \mathcal{O}_{\Delta, J^c}] = \langle \eta q \rangle \mathcal{O}_{(\Delta + \frac{1}{2}), J}$$

$$[[\bar{\eta} \bar{Q}], \mathcal{O}_{\Delta, J}] = [\bar{\eta} q] \mathcal{O}_{(\Delta + \frac{1}{2}), J^c}$$

$J^c = J - \dfrac{1}{2}$ *restricted by multiplet content*

supermutliplet	J	J^c
chiral	0, +1/2	-1/2,0
gauge	-1/2,+1	-1,+1/2
gravitational	-3/2, +2	-2, +3/2

OPEs of fermionic fields

Gaugino/Gravitino OPE:

- same helicity no collinear singularities
- but opposite helicity fuse to gluon or graviton

i.e. the two gravitino opposite helicity OPE becomes

$$\mathcal{O}_{\Delta_1,-\frac{3}{2}}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,+\frac{3}{2}}(z_2,\bar{z}_2) =$$

$$= \frac{z_{12}}{\bar{z}_{12}} B\left(\Delta_1 - \frac{1}{2}, \Delta_2 + \frac{5}{2}\right) \mathcal{O}_{\Delta_1+\Delta_2,+2}(z_2\bar{z}_2) + \frac{\bar{z}_{12}}{z_{12}} B\left(\Delta_1 + \frac{5}{2}, \Delta_2 - \frac{1}{2}\right) \mathcal{O}_{\Delta_1+\Delta_2,-2}(z_2\bar{z}_2) + \text{regular}\,.$$

- and similar OPEs for mixed boson fermion cases

Soft fermionic theorems

$$\mathcal{M}_{n+1} \longrightarrow \left(\underbrace{\omega_s^{-1/2}\, S_{\rm F}^{(0)}}_{\Delta \to 1/2} + \underbrace{\omega_s^{1/2}\, S_{\rm F}^{(1)}}_{\Delta \to -1/2} \ldots \right) \mathcal{M}_n$$

<u>on CS^2:</u> poles at: $\Delta \to 1/2$ $\Delta \to -1/2$

Operator $S_{\rm F}^{(i)}$ changes statistics of a single particle in $\mathcal{M}_n$

Dumitrescu, He Mitra, Strominger (2015)
Lysov (2015), Avery Schwab (2015)

CCFT Susy Ward identity

Leading soft

$$\left\langle \mathcal{O}_{\Delta,+\frac{3}{2}} \mathcal{O}_{\Delta_1,J_1} \cdots \mathcal{O}_{\Delta_i,J_i} \cdots \mathcal{O}_{\Delta_k,J_k} \mathcal{O}_{\Delta_{k+1},J^c_{k+1}} \cdots \mathcal{O}_{\Delta_N,J^c_N} \right\rangle$$

$$= \frac{1}{\Delta - \frac{1}{2}} \sum_{i=1}^{k} (-1)^{\sigma_i} \frac{\bar{z}_{si}}{z_{si}} \left\langle \mathcal{O}_{\Delta_1,J_1} \cdots \mathcal{O}_{\Delta_i+\frac{1}{2},J^c_i} \cdots \mathcal{O}_{\Delta_k,J_k} \mathcal{O}_{\Delta_{k+1},J^c_{k+1}} \cdots \mathcal{O}_{\Delta_N,J^c_N} \right\rangle$$

σ_i : keeps track of fermion ordering

CCFT Supersymmetry currents

use shadow transform

$$S(z) = \lim_{\Delta\to\frac{1}{2}} \frac{\Delta - \frac{1}{2}}{\pi} \int d^2z' \frac{\mathcal{O}_{\Delta,-\frac{3}{2}}(z',\bar{z}')}{(z-z')^3} \qquad (h,\bar{h}) = (\frac{3}{2},0)$$

$$\bar{S}(\bar{z}) = \lim_{\Delta\to\frac{1}{2}} \frac{\Delta - \frac{1}{2}}{\pi} \int d^2z' \frac{\mathcal{O}_{\Delta,+\frac{3}{2}}(z',\bar{z}')}{(\bar{z}-\bar{z}')^3} \qquad (h,\bar{h}) = (0,\frac{3}{2})$$

apply on leading soft gravitino theorem

$$S(z)\mathcal{O}_{\Delta,J^c}(w,\bar{w}) = \frac{1}{z-w}\mathcal{O}_{\Delta+\frac{1}{2},J}(w,\bar{w}) \ + \ regular$$

$$\bar{S}(\bar{z})\mathcal{O}_{\Delta,J}(w,\bar{w}) = \frac{1}{\bar{z}-\bar{w}}\mathcal{O}_{\Delta+\frac{1}{2},J^c}(w,\bar{w}) \ + \ regular$$

$S(z)$ generates Q and $\bar{S}(\bar{z})$ generates $\bar{Q}$ susy

$$Q_\eta = \oint \frac{\eta(z)dz}{2\pi i} S(z) \qquad \bar{Q}_{\bar{\eta}} = \oint \frac{\bar{\eta}(\bar{z})d\bar{z}}{2\pi i} \bar{S}(\bar{z})$$

Fermionic parameters

$$\eta(z) = \eta_0 + \eta_1 z$$

fall as $\frac{1}{z^3}$, $\frac{1}{\bar{z}^3}$ respectively at infinity

Ward identities: global symmetries on CCFT

$$\left\langle Q_\eta \mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_k,J_k}(z_k,\bar{z}_k)\mathcal{O}_{\Delta_{k+1},J^c_{k+1}}(z_{k+1},\bar{z}_{k+1})\cdots\mathcal{O}_{\Delta_N,J^c_N}(z_N,\bar{z}_N)\right\rangle = 0$$

$$\sum_{i=k+1}^{N}(-1)^{\sigma_i}\langle\eta q_i\rangle \times \left\langle \prod_{j=1}^{k}\mathcal{O}_{\Delta_j,J_j}(z_j,\bar{z}_j)\mathcal{O}_{\Delta_{k+1},J^c_{k+1}}(z_{k+1},\bar{z}_{k+1})\cdots\mathcal{O}_{\Delta_i+\frac{1}{2},J_i}(z_i,\bar{z}_i)\cdots\mathcal{O}_{\Delta_N,J^c_N}(z_N,\bar{z}_N)\right\rangle = 0$$

OPEs of Super-BMS generators

double soft limits of correlators → algebra

$$T(z)S(w) = \frac{3}{2}\frac{S(w)}{(z-w)^2} + \frac{\partial S(w)}{z-w} + \ regular$$

$$\bar{T}(\bar{z})\bar{S}(\bar{w}) = \frac{3}{2}\frac{\bar{S}(\bar{z})}{(\bar{z}-\bar{w})^2} + \frac{\bar{\partial}\bar{S}(\bar{w})}{\bar{z}-\bar{w}} + \ regular$$

$$S(z)P(w) \sim \ regular \qquad \bar{S}(\bar{z})P(w) \sim \ regular$$

$$S(z)S(w) \sim \ regular \qquad \bar{S}(\bar{z})\bar{S}(\bar{w}) \sim \ regular$$

Define composite operator: $\mathscr{P}(z,\bar{z}) =: S(z)\bar{S}(\bar{z}) + \bar{S}(\bar{z})S(z) :$

implies OPE $\quad \mathscr{P}(w,\bar{w})\mathcal{O}_{h,\bar{h}}(z,\bar{z}) = \frac{1}{w-z}\frac{1}{\bar{w}-\bar{z}}\mathcal{O}_{h+\frac{1}{2},\bar{h}+\frac{1}{2}}(z,\bar{z}) \ + \ regular$

Barnich (2017)

identify composite operator with supetranslation primary operator!!!

Super BMS algebra

Laurent expansion of fields

$$S(z) = \sum_{i \in \mathbb{Z}+\frac{1}{2}} \frac{G_i}{z^{i+\frac{3}{2}}} \,, \qquad G_i = \oint dz\, z^{i+1/2}\, S(z)$$

$$\bar{S}(\bar{z}) = \sum_{i \in \mathbb{Z}+\frac{1}{2}} \frac{\bar{G}_i}{\bar{z}^{i+\frac{3}{2}}} \,, \qquad \bar{G}_i = \oint d\bar{z}\, \bar{z}^{i+1/2}\, \bar{S}(\bar{z})$$

$$S(z)\mathcal{O}_{\Delta,J^c}(w,\bar{w}) = \frac{1}{z-w}\mathcal{O}_{\Delta+\frac{1}{2},J}(w,\bar{w}) \;+\; regular$$

$$\bar{S}(\bar{z})\mathcal{O}_{\Delta,J}(w,\bar{w}) = \frac{1}{\bar{z}-\bar{w}}\mathcal{O}_{\Delta+\frac{1}{2},J^c}(w,\bar{w}) \;+\; regular$$

OPEs with primaries imply

$$[G_i, \mathcal{O}_{\Delta,J^c}(w,\bar{w})] = w^{i+1/2}\, \mathcal{O}_{\Delta+\frac{1}{2},\,J}(w,\bar{w})$$

$$[\bar{G}_i, \mathcal{O}_{\Delta,J}(w,\bar{w})] = \bar{w}^{i+1/2}\, \mathcal{O}_{\Delta+\frac{1}{2},\,J^c}(w,\bar{w})$$

From action of the zero modes on Operators we identify

$$[\langle \eta Q \rangle, \mathcal{O}_{\Delta,J^c}] = \langle \eta q \rangle \mathcal{O}_{(\Delta+\frac{1}{2}),J}$$

$$[[\bar{\eta}\bar{Q}], \mathcal{O}_{\Delta,J}] = [\bar{\eta} q] \mathcal{O}_{(\Delta+\frac{1}{2}),J^c}$$

$$Q_1 \to G_{+1/2}\,, \qquad Q_2 \to G_{-1/2}\,,$$

$$\bar{Q}_{\dot{1}} \to \bar{G}_{+1/2}\,, \qquad \bar{Q}_{\dot{2}} \to \bar{G}_{-1/2}\,.$$

Super BMS algebra

Apply consecutively $G_n, \bar{G}_m$ on primaries

$$[\{G_i, \bar{G}_j\}, \mathcal{O}_{h,\bar{h}}(z,\bar{z})] = z^{i+\frac{1}{2}}\,\bar{z}^{j+\frac{1}{2}}\mathcal{O}_{h+\frac{1}{2},\bar{h}+\frac{1}{2}}(z,\bar{z}) = [P_{i,j}, \mathcal{O}_{h,\bar{h}}(z,\bar{z})] \qquad i,j \in \mathbb{Z} + \frac{1}{2}$$

in a similar manner as for the bosonic case we extract the Super-BMS algebra

$$\begin{aligned}
\{G_i, \bar{G}_j\} &= P_{i,j} \\
\{G_i, G_j\} = \{\bar{G}_i, \bar{G}_j\} &= 0 \\
[P_{k,l}, G_i] = [P_{k,l}, \bar{G}_j] &= 0 \\
\left[L_m, G_k\right] &= \left(\frac{1}{2}m - k\right)G_{m+k} \\
\left[\bar{L}_m, \bar{G}_l\right] &= \left(\frac{1}{2}m - l\right)\bar{G}_{m+l} \\
[L_m, \bar{G}_i] = [\bar{L}_m, G_i] &= 0
\end{aligned}$$

$m, n \in \mathbf{Z},\ i, j, k, l \in \mathbf{Z} + \frac{1}{2}$

Super BMS algebra: Comments

- We find infinite dimensional algebra $N=1$ extended SuperBMS
- Recently SuperBMS in 4D studied using Hamiltonian formulation. Found infinite number of fermionic charges [Fuentealba, Henneaux, Majumdar, Matulich, Neogi (2020)]
- Susy appears as a "square root" of supertranslations [Awada, Gibbons, Shaw (1986)]
- Similar algebras have appeared in 3D [Barnich, Donnay, Matulich, Troncoso (2014), Lodato Merbis (2016), Fuentealba, Matulich, Troncoso (2017)]
- <u>No "world-sheet" 2-dim Susy</u> → maybe realised in a special limit of CCFT?

Further Directions

- understand Virasoro central charge (-one-loop ?)
- establish double-copy structure (elaborate on gauge/gravity connections)

Casali and Puhm (2020)

- Conformal Bootstrap: determine spectrum and couplings of the theory
- understanding the nature of 2D CFT on celestial sphere would enable a holographic description of flat spacetime

THANK YOU!

EXTRAS

Can CCFT offer some new insights into gauge-gravity connections ?

related questions:

- celestial double-copy structure
- celestial KLT structure
- … ?

related recent work: Banerjee, Ghosh Paul (2020), Casali, Sharma, Phum (2020)

Sugawara construction:

$$T(w) = \frac{1}{2k + C_2} \lim_{z \to w} \left\{ \sum_a J^a(w) J^a(z) - \frac{k \dim(g)}{(w_1 - w_2)^2} \right\}$$

Sugawara (1968)

assumes Kac-Moody current algebra

(holomorphic) Kac-Moody current algebra:

gauge theory analog
of BMS transformations

$$j^a(z) = \mathcal{O}^a_{\Delta=1,J=+1}(z,\bar z)$$
$$\bar j^a(\bar z) = \mathcal{O}^a_{\Delta=1,J=-1}(z,\bar z)$$

soft particles

$$j^a(z) j^b(w) \sim \frac{f^{abc}\ \ j^c(w)}{z-w} + reg\,.$$

furthermore:

$$j^a(z)\bar j^b(\bar w) \sim \frac{f^{abc}\ \ \bar j^c(\bar w)}{z-w}$$

anti-holomorphic currents
transform in adjoint representation
of holomorphic Kac-Moody symmetry

$$\bar j^a(\bar z) j^b(w) \sim \frac{f^{abc}\ \ j^c(w)}{\bar z-\bar w}$$

follows from CCFT OPE

first look:

W. Fan, A.F
St. Stieberger., Taylor
(2020)

CCFT:

$$T^S(z) := \gamma \sum_a j^a(z) j^a(z) = \gamma \lim_{\Delta,\Delta'\to 1} \lim_{z'\to z} \sum_a \mathcal{O}^a_{\Delta,+1}(z,\bar z)\mathcal{O}^a_{\Delta',+1}(z',\bar z')$$

consider n-gluon MHV amplitude $A_n(-,-,+\dots,+)$
with insertion of pair of gauge currents

$$\lim_{z_j\to z_{n+1}} \langle \mathcal{O}^{a_1}_{\Delta_1 J_1}\dots j^{a_j}(z_j)\dots \mathcal{O}^{a_n}_{\Delta_n J_n} j^a(z_{n+1}) j^a(z_{n+1})\rangle$$

$$= \begin{cases} \tilde{C}_2(G)\left(\frac{1}{(z_j-z_{n+1})^2}+\frac{\partial_j}{(z_{n+1}-z_j)}\right)\langle \mathcal{O}^{a_1}_{\Delta_1 J_1}\dots j^{a_j}(z_j)\dots \mathcal{O}^{a_n}_{\Delta_n J_n}\rangle\ , & j=3,\dots,n \\ 0\ , & j=1,2 \end{cases}$$

follows from:

$$\lim_{z_{n+1}\to z_j} A_{n+2}(\{g^+_{n+2}, g_1,\dots,g_n, g^+_{n+1}\}) = -\frac{\tilde{C}_2(G)}{\omega_{n+1}\omega_{n+2}}\left(\frac{1}{(z_{n+1}-z_j)^2}+\frac{\tilde{\partial}_{z_j}}{z_{n+1}-z_j}\right)A_n(\{g_1,\dots,g_n\})$$

this Sugawara energy-momentum tensor

- only describes soft sector of the theory
- decouples negative helicity states
- only treats holomorphic sector

$$
\begin{aligned}
\mathcal{O}^a_{\Delta_2,+1}(w_2,\bar{w}_2)\mathcal{O}^b_{\Delta_1,-1}(z_1,\bar{z}_1) = &\frac{\Delta_1-1}{\Delta_2(\Delta_1+\Delta_2-2)}\sum_c \frac{\tilde{f}^{abc}}{w_2-z_1}\mathcal{O}^c_{(\Delta_1+\Delta_2-1),-1}(z_1,\bar{z}_1) \\
&-2\delta^{ab}\,\frac{\bar{w}_2-\bar{z}_1}{w_2-z_1}\,\frac{(\Delta_1-1)(\Delta_1+1)(\Delta_2-1)}{\Delta_2(\Delta_1+\Delta_2)(\Delta_1+\Delta_2-1)}\mathcal{O}_{(\Delta_1+\Delta_2),-2}(z_1,\bar{z}_1) \\
&+\tilde{f}^{abc}\,\Lambda(\Delta_1,\Delta_2)\,(\bar{w}_2-\bar{z}_1)\,\mathcal{O}^c_{\Delta_1+\Delta_2+1,-1}(z_1,\bar{z}_1)+ \\
&+\delta^{ab}\,M(\Delta_1,\Delta_2)\,(\bar{w}_2-\bar{z}_1)^2\,\mathcal{O}_{\Delta_1+\Delta_2+2,-2}(z_1,\bar{z}_1) \\
&+\sum_{k\geq 0}(w_2-z_1)^k\,\mathcal{C}_k(z_1,\bar{z}_1)
\end{aligned}
$$

→ celestial OPE constrained by BMS → celestial amplitudes

A double copy construction of the energy momentum tensor

consider OPE of two gluon operators of opposite helicity
and perform a shadow transformation:

$$\mathcal{O}^a_{\Delta_2,+1}(u,\bar{u})$$

$$\tilde{\mathcal{O}}^a_{2-\Delta_1,+1}(w,\bar{w}) \sim \int d^2z\ (z-w)^{-3}(\bar{z}-\bar{w})^{-1}\ \mathcal{O}^a_{\Delta_1,-1}(z,\bar{z})$$

$$T(w) = \frac{1}{2\dim(g)} \lim_{\Delta_1,\Delta_2\to 0} [\Delta_2(\Delta_1+\Delta_2)] \lim_{u\to w} \sum_a \mathcal{O}^a_{\Delta_2,+1}(u,\bar{u})\tilde{\mathcal{O}}^a_{2-\Delta_1,+1}(w,\bar{w})$$

$$\frac{1}{2k+C_2} \simeq \frac{1}{2\dim(g)},$$

$$k=0$$

- puts both soft and hard modes on equal footing:

$$T(z)\mathcal{O}_{\Delta,J}(w,\bar{w}) = \frac{h}{(z-w)^2}\mathcal{O}_{\Delta,J}(w,\bar{w}) + \frac{1}{z-w}\partial_w\mathcal{O}_{\Delta,J}(w,\bar{w}) + reg\,.$$

Fan, A.F., Stieberger., Taylor, (2020)