## ∞-dim Lie algebras & higher spins

- A "Cartan" approach to higher-spin gauge theories:  $\bigcirc$ 
	- *1987*: proposal for a **higher-spin algebra** in AdS4 Fradkin, Vasiliev
	- *1990*: procedure to implement its **gauging** → Vasiliev's equations Vasiliev
	- 2003: higher-spin algebras and interacting e.o.m. in AdS<sub>D</sub> Eastwood; Vasiliev
- Other recent (and less recent) developments  $\bigcirc$ 
	- 3D HS algebras  $\rightarrow$  Chern-Simons gauge theories (& matter couplings) Blencowe (1989); Porkushkin, Vasiliev (1999) & many others…
	- HS algebras for mixed symmetry and partially-massless fields

Boulanger, Skvortsov (2011); Joung, Mkrtchyan (2016)

# Higher spins & (A)dS

- Why (massless) HS fields like (A)dS?  $\bigcirc$ 
	- Long-range HS interactions:
		- in flat-space  $\rightarrow$  trivial S-matrix Weinberg (1964)
		- in AdS  $\rightarrow$  free CFT boundary correlators  $\rightarrow$  "soluble" AdS/CFT Sezgin, Sundell (2002); Klebanov, Polyakov (2002); Maldacena, Zhiboedov (2011) et al.
- May Minkowski still play a role?  $\bigcirc$ 
	- Is String Theory a broken phase of a HS gauge theory?
	- Models with trivial S-matrix, but non-trivial interactions (*& symmetries*)?

Skvortsov, Tran, Tsulaia (2018); A.C., Francia, Heissenberg (2017)

Outlook: *"non-AdS" holography with higher spins*  $\bigcirc$ 

see e.g. Ponomarev (2021)

## Higher-spin algebras **beling a symmetric tensor** of  $\blacksquare$  $\overline{\phantom{a}}$  the formation  $\overline{\phantom{a}}$ '*µ*1*···µ<sup>s</sup>*  $T$  is the field content of  $S$  content of  $\mathcal{S}$

- Key ingredient in building HS theories and studying HS holography notes the background covariant derivative and indices enclosed between parentheses are symmetric in building ind the ones and studying individially
- **What is a HS algebra?** *Lie algebra on traceless Killing tensors*  $T$   $\blacksquare$   $\blacksquare$  denote  $\blacksquare$  denote roomulation  $\blacksquare$   $\$ **e what is a HS aigebra?** Lie aigebra on traceless Killing tensors
- Poincaré & (A)dS algebras: isometries of the vacuum • Poincaré & (A)dS algebras: isometries of the vacuum Interactions typically bring deformations *O*(') of the free gauge transformation (2.1).

HS "isometries" of the vacuum "isometries" of the vacuum be doubly traceless, but this does not bring any new condition on the gauge parameter. The gauge parameter  $\alpha$ HS "isometries" of the vacuum Still, preserving the vacuum solution '*µ*1*···µ<sup>s</sup>* = 0 only requires that the gauge parameters

• Fronsdal's gauge transf.:  $\delta\varphi_{\mu_1\cdots\mu_s} = \bar{\nabla}_{(\mu_1} \epsilon_{\mu_2\cdots\mu_s)} + \mathcal{O}(\varphi)$ **Interactions of the free gauge transformations**  $\overline{\nabla}_{\theta}$  **of the free gauge transformation (2.1).**  $\mathcal{O}_{\mathcal{P}, \mu_1 \cdots \mu_s} = \mathcal{O}_{\mathcal{P}, \mu_1 \cdots \mu_s} = \mathcal{O}_{\mathcal{P}, \mu_1 \cdots \mu_s} \mathcal{O}_{\mathcal{P}, \mu_2 \cdots \mu_s} + \mathcal{O}_{\mathcal{P}, \mu_1 \cdots \mu_s}$ <sup>r</sup>¯(*µ*<sup>1</sup> ✏*µ*2*···µs*) = 0 *.* (2.2) killing Gauge transformations generated by traceless Killing tensors can thus be interpreted as

*s*1

- Vacuum-preserving symm.:  $\bar{v}$ uum-preserving symm.:  $\bar{\nabla}_{(\mu_1} \epsilon_{\mu_2 \cdots \mu_s)} = 0$ global symmetries for particles of any spin. In Minkowski spin. In Minkowski space, the general solution of th  $\begin{matrix} \mathcal{L} \\ \mathcal{L} \end{matrix}$  is one that  $\begin{matrix} \mathcal{L} \\ \mathcal{L} \end{matrix}$  of the vacuum section  $\mathcal{L}$ 
	-

• Solution (in Minkowski): 
$$
\epsilon_{\mu_1\cdots\mu_{s-1}} = \sum_{k=0}^{s-1} M_{\mu_1\cdots\mu_{s-1}|\nu_1\cdots\nu_k} x^{\nu_1}\cdots x^{\nu_k}
$$

Andrea Campoleoni - UMONS	$\epsilon_{\mu_1\cdots\mu_{s-3}\lambda}^{\lambda} = 0$
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Andrea Campoleoni - UMONS ✏*µ*1*···µs*<sup>1</sup> satisfy<br>Satisfy '*µ*1*···µ<sup>s</sup>*

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## **Figher-spin algebras Formulae used in Keynote Formulae used in Keynote in Keynot**

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Vector space of traceless Killing tensors: • Vector space of tracele



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## Notable so(2,D−1) Inönü-Wigner contractions



## Notable so(2,D−1) Inönü-Wigner contractions



# Notable so(2,D−1) Inönü-Wigner contractions



# What about higher-spin algebras?

# Goals & strategy/hypotheses

- **Goal:** classify Lie algebras defined on the vector space V  $\bigcirc$ (traceless Killing tensors) that
	- 1. contain a Poincaré subalgebra, **iso(1,D−1)**
	- 2. contain a conformal Galilei subalgebra, **gca**<sub>D−1</sub>
	- **…**and discuss their properties
- **Strategy:** look for *coset algebras*, obtained by quotienting out an ideal from the universal enveloping algebras of iso(1,D−1) or gcaD−<sup>1</sup> (bonus: "good" Lorentz transf. for free) Eastwood (2002)

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partial classification, still partial classered<br>with interesting examples!

## HS algebras in AdS<sub>D</sub>

Conformal HS algebras in D−1 dimensions

### Coset construction of HS algebras *M<sup>µ</sup>* ≥ *M<sup>µ</sup>|‹* ≥  $P$ *C*<sub>1</sub>  $\overline{P}$  of HS algebras *Mµ‹* ≥ *Mµ‹|–* ≥ *Mµ‹|–—* ≥ *<u>Jotion</u>* of  $HC$  algobrac Coset construction of HS alge

 $\text{SO}(2, D-1)$  algebra:  $[J_{AB}, J_{CD}] = \tilde{\eta}_{AC} J_{BD} - \tilde{\eta}_{BC} J_{AD} - \tilde{\eta}_{AD} J_{BC} + \tilde{\eta}_{BD} J_{AC}$  $\frac{1}{\sqrt{2}}$  $\left[\sigma_{CD}\right]=\tilde{\eta}_{AC}\,J_{BD}-\tilde{\eta}_{BC}\,J_{AD}-\tilde{\eta}_{AD}\,J_{BC}$  - $\mathcal{O}(\mathcal{O}(\bigcap_{i=1}^n A_i) \cap \mathcal{O}(\mathcal{O}(\bigcap_{i=1}^n A_i))$  in  $\mathcal{O}(\mathcal{O}(\bigcap_{i=1}^n A_i))$  $\sum_{i=1}^{n}$  $[J_{AB}\, ,\, J_{CD}]=\tilde{\eta}_{AC}\,J_{BD}-\tilde{\eta}_{BC}\,J_{AD}-\tilde{\eta}_{AD}\,J_{BC}+\tilde{\eta}_{BD}\,J_{AC}$ 

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The isometries of the AdS*<sup>D</sup>* background are given by the so(2*, D* 1) algebra

· Quadratic products of the generators

$$
J_{A(B} \odot J_{C)D}
$$
 - traces  $\sim$   $\boxed{\phantom{0}}$   $C_2 \equiv \frac{1}{2} J_{AB} \odot J^{BA} \sim \bullet$ 

$$
\mathcal{I}_{AB} \equiv J_{C(A} \odot J_B)^C - \frac{2}{D+1} \tilde{\eta}_{AB} C_2 \sim \Box \qquad \mathcal{I}_{ABCD} \equiv J_{[AB} \odot J_{CD]} \sim \Box
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\mathfrak{hs}_D = \frac{\mathcal{U}(\mathfrak{so}(2, D-1))}{\langle \mathcal{I}_{AB} \oplus \mathcal{I}_{ABCD} \rangle} \Rightarrow C_2 \sim -\frac{(D+1)(D-3)}{4}id
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Andrea Campoleoni - UMONS  $\alpha$  shall of the symmetric altogether. The first part of the ideal product altogether. The ideal part of the ide hica campoleoni children products of social traces from products of social traces from products of social traces

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lazeolla, Sundel (2008)  
Andrea Campoleoni - UMONS

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#### Special cases: D=3 *{Lm,L*¯*n}* <sup>+</sup> *{Ln,L*¯*m}* ⇠ <sup>0</sup> *,* (2.33a) *{Lm,L*¯*n} {Ln,L*¯*m}* ⇠ <sup>0</sup> *,* (2.33b) with no restrictions on *m, n* and where we have omitted the anticommutator since the two the cases:  $D=3$ in three dimensions, it is enough to consider the dimensions, it is enough to consider the UEA of two copies o<br>In the UEA of two copies of two copies of two copies of the UEA of two copies of two copies of two copies of t On the other hand, contrary to the generic case, in three dimensions we do not need the vector space of global symmetries of global symmetries of massless particles. Actually, its dualisation gi<br>The vector symmetries of massless particles in the vector gives of massless particles in the vector symmetries *W* ⌘ <sup>8</sup> "*ABCDIABCD* <sup>=</sup> *<sup>j</sup>ap<sup>a</sup>* <sup>=</sup> *mnLmP<sup>n</sup>* <sup>=</sup> *<sup>L</sup>*<sup>2</sup> *<sup>L</sup>*¯<sup>2</sup> *.* (2.35) W\_3D *V* ƒ *•* ü ü ü ü *···*

so(2,2) algebra:  $[\mathcal{L}_m, \mathcal{L}_n] = (m-n)\mathcal{L}_{m+n}$ ,  $[\bar{\mathcal{L}}_m, \bar{\mathcal{L}}_n] = (m-n)\bar{\mathcal{L}}_{m+n}$ ,  $[\mathcal{L}_m, \bar{\mathcal{L}}_n] = 0$ sl(2*,* R) (see, e.g., [15] and the review [97]): factoring out *IAB* sets all mixed products to alge the other independent quadratic Casimir of so(2*,* 2):  $\bullet$  so(2,2) algebra:  $[\mathcal{L}_m, \mathcal{L}_n] = (m-n)\mathcal{L}_{m+n}$ ,  $[\bar{\mathcal{L}}_m, \bar{\mathcal{L}}_n] = (m-n)\bar{\mathcal{L}}_{m+n}$ ,  $[\mathcal{L}_m, \bar{\mathcal{L}}_n]$ of *<sup>C</sup>*<sup>2</sup> only. Substituting the result in the expansion of *<sup>W</sup>*<sup>2</sup> = 4! *<sup>I</sup>ABCDIABCD* one obtains  $\left[2m, \sum_{n=1}^{\infty}$   $\left[2m, \sum_{n=1}^{\infty}$   $\right]$ 

*W* ⌘

\n- Ideal to be factored out: 
$$
\int \mathcal{I}_{AB}
$$
\n

be factored out: 
$$
\begin{pmatrix} \mathcal{I}_{AB} \sim 0 & \Rightarrow & \mathcal{L}_m \bar{\mathcal{L}}_n \sim 0 \\ 0 & \Rightarrow & \mathcal{L}_m \bar{\mathcal{L}}_n \sim 0 \end{pmatrix}
$$

with no restrictions on an and where we have only no metal where we have only no restrictions of the two since the two since

to factor out the element *IABCD*. One can indeed dualise it into a singlet, that would fit in

 $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$  is the Casimir of  $\alpha$ (2) algebras  $\alpha$   $W = \frac{1}{8}$   $\epsilon$  and  $\alpha$   $L$   $ABCD$   $D$   $D$ No need to factor out  $W \equiv \frac{1}{2} \varepsilon^{ABCD} \mathcal{T}_{ABCD}$  but  $\left[W^2 \sim \frac{1}{2} (C_2)^2\right]$  $\sim$  4 in the dimensions, it is enough to consider the dimensions, it is enough to consider the UEA of the UEA No need to factor out  $W\equiv\frac{1}{8}\,\varepsilon^{ABCD}\mathcal{I}_{ABCD}\,$  but 1  $\frac{1}{8} \varepsilon^{ABCD} \mathcal{I}_{ABCD}$  but  $W^2 \sim \frac{1}{4} (C_2)^2$ 1  $\frac{1}{4}$   $(C_2)$ 2 **.** *.* **(2.36) relatively and**  $\alpha$ Imposing *IABCD* = <sup>3</sup> "*ABCDW* ⇠ 0 as in section 2.2.1 thus implies *C*<sup>2</sup> ⇠ 0 consistently • No need to factor out  $W \equiv \frac{1}{2} \varepsilon^{ABCD} \mathcal{I}_{ABCD}$  but  $\parallel W^2 \sim \frac{1}{4} (C_2)^2$ only with the weaker condition (2.36) that leaves the quadratic Casimir *C*<sup>2</sup> free. For the hs*<sup>D</sup>* =  $\frac{1}{8} \varepsilon^{ABCD} \mathcal{I}_{ABCD}$  $\overline{8}^{\text{c}}$  *IABCD*  $\overline{6}$   $\overline{6}$  $\sim \frac{1}{4} (C_2)^2$ 4

latter, one can eventually require

ے<br>1انہ  $C_2$  :

Still, better to get rid of 
$$
C_2
$$
: 
$$
\left(C_2 = 2\left(\mathcal{L}^2 + \bar{\mathcal{L}}^2\right) \sim \frac{\lambda^2 - 1}{2}id\right)
$$

recover that employed in the study of ultra-relativistic limits (see section 3). The quadratic

Andrea Campoleoni - UMONS decomposition of products of so(2*,* 2) generators. It carries nine independent components,

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with conformal dimensions *<sup>h</sup>* <sup>=</sup> *<sup>h</sup>*¯ <sup>=</sup> *<sup>h</sup><sup>±</sup>* <sup>=</sup>

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\n

latter, one can eventually require

ameter family of HS algebras  $t_{\text{total}}$  stacks we space of global symmetries of massless particles.  $\frac{1}{2}$  independent  $\frac{1}{2}$   $\frac{2}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$  in the parameter is *Iorising*  $I_{ABCD}$  *gives a one-parameter family of HS algebras* with the *D-dimensional result (2.20). However, when*  $\alpha$  *and*  $\alpha$  *and*  $\alpha$  *and*  $\alpha$  *we have the option to work the option to w*  $\frac{1}{2}$  $\mathbf{r}$ algahrac an extended algebra dubbed "large AdS higher-spin algebra" [51]. Similar extended algebras also appear in • Not factorising  $\mathcal{I}_{ABCD}$  gives a one-parameter family of HS algebras *C*<br>*C* factorising *⊥*  $\mathsf{i}\mathsf{v}$ a one parameter family of HS algebrac

to factor out the element *IABCD*. One can indeed dualise it into a singlet, that would fit in

$$
\mathfrak{hs}_3[\lambda]=id\oplus W\oplus \mathfrak{hs}[\lambda]\oplus \mathfrak{hs}[\lambda]\quad\text{ with }\quad 1\oplus \mathfrak{hs}[\lambda]=\frac{\mathcal{U}(\mathfrak{sl}(2,\mathbb{R}))}{\left\langle \mathcal{C}_2-\frac{\lambda^2-1}{4}\mathbb{1}\right\rangle}
$$

Andrea Campoleoni - UMONS decomposition of products of so(2*,* 2) generators. It carries nine independent components, ampoleoni - UMONS where *<sup>C</sup>*<sup>2</sup> denotes the sl(2*,* <sup>R</sup>) Casimir operator (say *<sup>L</sup>*<sup>2</sup> or *<sup>L</sup>*¯2). When <sup>=</sup> *<sup>N</sup>* <sup>2</sup> <sup>N</sup> its eigenvalue corresponds to that of a finite-dimensional interactional interactional irreducible representation and a<br>The further product representation and a further product representation and a further product representation a

## $S$ pecial cases: D = 3 Imposing *IABCD* = 1 <sup>3</sup> "*ABCDW* ⇠ 0 as in section 2.2.1 thus implies *C*<sup>2</sup> ⇠ 0 consistently *C*<sup>2</sup> © hs3[] = *id* ⌘ hs[] hs[] *,* (2.41) where  $\frac{1}{2}$  is defined as where *<sup>C</sup>*<sup>2</sup> denotes the sl(2*,* <sup>R</sup>) Casimir operator (say *<sup>L</sup>*<sup>2</sup> or *<sup>L</sup>*¯2). When <sup>=</sup> *<sup>N</sup>* <sup>2</sup> <sup>N</sup> its eigen-*Mµ‹* ≥ *Mµ‹|–* ≥ *Mµ‹|–—* ≥

where the first expression is again considered for *m n* and the last expression for *m>n*. ldeal to be factored out: of *<sup>C</sup>*<sup>2</sup> only. Substituting the result in the expansion of *<sup>W</sup>*<sup>2</sup> = 4! *<sup>I</sup>ABCDIABCD* one obtains ■ Ideal to be factored out: value corresponds to that of a finite-dimensional irreducible representation and a further

U<del>(sof(2</del>)))

$$
\mathcal{I}_{AB} \sim 0 \quad \Rightarrow \quad \mathcal{L}_m \bar{\mathcal{L}}_n \sim 0 \qquad \qquad C_2 \sim \frac{\lambda^2 - 1}{2} \, id \qquad \qquad W^2 \sim \frac{1}{4} \, (C_2)^2
$$

(**D + 1)**<br>(*D + 1)*(*D + 1)*(*D + 1)* 

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- $\bullet$  Are we evaluating  $U(\text{so}(2,2))$  on which module? Imposing *IABCD* = <sup>3</sup> "*ABCDW* ⇠ 0 as in section 2.2.1 thus implies *C*<sup>2</sup> ⇠ 0 consistently Are we evaluating  $U(\text{so}(2,2))$  on which module?  $\sigma$  are described in the dimensions fields in the dimensions  $\sigma$ • Are we evaluating  $U$ (so(2,2)) on which modul becomes particularly neat. The absence of mixed products of *<sup>L</sup>* and *<sup>L</sup>*¯ means that one has  $SU(2,2)$  on writer module:
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\mathcal{L}_m = \begin{pmatrix} l_m & 0 \\ 0 & 0 \end{pmatrix}, \quad \bar{\mathcal{L}}_m = \begin{pmatrix} 0 & 0 \\ 0 & \bar{l}_m \end{pmatrix} \quad \text{with } l_m \text{ } N \times N \text{ irrep of } \mathfrak{sl}(2, \mathbb{R})
$$

$$
\Rightarrow \quad C_2 = \frac{N^2 - 1}{2} \begin{pmatrix} \mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix}, \quad W = \frac{N^2 - 1}{4} \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}
$$

Andrea Campoleoni - UMONS of *<sup>C</sup>*<sup>2</sup> only. Substituting the result in the expansion of *<sup>W</sup>*<sup>2</sup> = 4! *<sup>I</sup>ABCDIABCD* one obtains  $\overline{\mathsf{u}}$  –  $\overline{\mathsf{u}}$ Andrea Campoleoni oni<br>∪ *⁄*<br>2 ∪ ∪ ∪ ∪ ∪ ∪ 2

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$$
\n
$$
\implies C_2 = \frac{N^2 - 1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad W = \frac{N^2 - 1}{4} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
$$
\nCasimir operator not proportional to the identity

#### Special cases: D=5 the set of generators and this is achieved by building the ideal out of a *I* = *IAB IABCD* **comprehensive** complex algebra,  $\Box$  $\mathbf{c}$ 1 2 *J* = 5

 $\bullet$  Again, no need to factor out  $n$ , no need to factor out  $\mathcal{I}_{ABCD}$  $\mathbf{b} = \mathbf{b} \cdot \mathbf{b}$ where *<sup>I</sup>* <sup>=</sup> *<sup>I</sup>AB <sup>I</sup>* hs*<sup>D</sup>* = **10 need to facto** <sup>È</sup>*IAB* <sup>ü</sup> *<sup>I</sup>ABCD*<sup>Í</sup> <sup>∆</sup> *<sup>C</sup>*<sup>2</sup> ≥ ≠

Ideal:

• Ideal:  
\n
$$
\mathcal{I}_{AB} \equiv J_{C(A}J_{B})^{C} - \frac{1}{3}\tilde{\eta}_{AB}C_{2},
$$
\n
$$
\mathcal{I}_{ABCD}^{\lambda} \equiv J_{[AB}J_{CD]} - i\frac{\lambda}{6}\varepsilon_{ABCDEF}J^{EF}
$$
\nmixing terms with different # of 7<sub>AB</sub>

4

h*I*i then factors out all traces from products of so(2*, D* 1) generators, while the second • One parameter family of HS algebras **Boulanger, Skvortsov (2011)** *S* zamily of HS algebras

Boulanger, Skvortsov (2011)

$$
\mathfrak{hs}_{5}[\lambda] = \frac{\mathcal{U}(\mathfrak{so}(2,4))}{\langle \mathcal{I}_{AB} \oplus \mathcal{I}_{ABDC}^{\lambda} \rangle} \quad \Rightarrow \quad C_2 \sim 3\left(\lambda^2 - 1\right) id
$$

Andrea Campoleoni - UMONS Andrea Campoleoni - UMONS *<sup>C</sup>*<sup>2</sup> <sup>+</sup> (*<sup>D</sup>* + 1)(*<sup>D</sup>* 3)

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\n
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# Warming up in 3D



### Carrollian and Galilean 3D HS algebras **U***Z* ∑ ∑ ∆ *I* (*D* + 1)(*D* ≠ 3)  $\overline{a}$ *Milian and Galilea* [*P– , <sup>J</sup> fl* (*µJ‹*)*fl* <sup>≠</sup> <sup>2</sup> *<sup>D</sup>÷µ‹ <sup>J</sup>* <sup>2</sup> ] = *{J–*(*µ,P‹*)*}* + *···*

 $iso(1,2)$  and gca<sub>2</sub> are isomorphic *<sup>L</sup>*<sup>2</sup> <sup>+</sup> *<sup>L</sup>*¯<sup>2</sup>  $\bigcirc$ *⁄*<sup>1</sup> 2)  $\overline{2}$ *and* gca<sub>2</sub> are isomor

Bagchi, Gopakumar, Mandal, Miwa (2009)

a hs[ $\lambda$ ]  $\oplus$  hs[ $\lambda$ ] algebra: *D* bs[ $\lambda$ ] algebra:

*<sup>⁄</sup>*<sup>2</sup> <sup>≠</sup> <sup>1</sup> *id* Schaller; Fradkin, Linetsky; Pope, Romans, Shen (1990)ו וכ<br>. **Bergshoeff, Blencowe, Stelle; Bordemann, Hoppe,** 

$$
P_m^{(s)} \equiv \epsilon \left( \mathcal{L}_m^{(s)} - \bar{\mathcal{L}}_m^{(s)} \right), \quad L_m^{(s)} \equiv \mathcal{L}_m^{(s)} + \bar{\mathcal{L}}_m^{(s)}
$$

$$
\begin{cases}\n\mathcal{L}_{\pm(s-1)}^{(s)} \equiv (\mathcal{L}_{\pm 1})^{s-1} \\
\mathcal{L}_{m\mp 1}^{(s)} \equiv \frac{\mp 1}{s \pm m-1} [\mathcal{L}_{\mp 1}, \mathcal{L}_m^{(s)}]\n\end{cases}
$$

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$$

$$
\left[P_m^{(s)}, P_n^{(t)}\right] = \epsilon^2 \sum_{\substack{u=|s-t|+2\\s+t+u \text{ even}}}^{s+t-2} g_{s+t-u}^{st}(m, n; \lambda) L_{m+n}^{(u)},
$$

$$
\[L_m^{(s)}, P_n^{(t)}\] = \sum_{\substack{u=|s-t|+2\\s+t+u \text{ even}}}^{s+t-2} g_{s+t-u}^{st}(m, n; \lambda) P_{m+n}^{(u)},\]
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#### Carrollian and Galilean 3D HS algebras *s*+*t*+*u* even Ë È *L*(*s*) *<sup>m</sup> , L*(*t*) = *s*+ ÿ **Carrollian and Galilean 3D HS algebras**  $\overline{\phantom{a}}$ *<sup>s</sup>*+*t*≠*<sup>u</sup>*(*m, n*; *<sup>⁄</sup>*)*L*(*u*) imply a non-zero contribution of higher spins to the gravitation of higher spins to the gravitational energy-m<br>In the gravitation of higher spins to the gravitation of the gravitation of the gravitation of the gravitation

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*<sup>m</sup>*+*<sup>n</sup>* (3)

**• Carrollian (aka flat!) & Galilean limits defined by** È*IAB* ü *I<sup>⁄</sup> ABDC*<sup>Í</sup> <sup>∆</sup> *<sup>C</sup>*<sup>2</sup> <sup>≥</sup> <sup>3</sup> limits d Carrollian (aka flat!) & Galilean limits defined by  $\epsilon \to 0$ ⇣  $(Aka$  *flat!*)  $\overline{\mathbf{S}}$  $\mathbf{k}$  Galilean limits defined by  $\epsilon \to 0$ 

$$
P_m^{(s)} \equiv \epsilon \left( \mathcal{L}_m^{(s)} - \bar{\mathcal{L}}_m^{(s)} \right), \quad L_m^{(s)} \equiv \mathcal{L}_m^{(s)} + \bar{\mathcal{L}}_m^{(s)}
$$
Fareghbal, Grumiller, Rosseel (2013);  
Gonzalez, Matulich, Pino, Troncoso (2)

 $\begin{bmatrix} a & b & c \end{bmatrix}$  in the  $\begin{bmatrix} a & b & c \end{bmatrix}$  original  $\begin{bmatrix} a & b & d \end{bmatrix}$  and  $\begin{bmatrix} a & b & d \end{bmatrix}$  and  $\begin{bmatrix} a & b & d \end{bmatrix}$  Gonzalez, Matulich, Pino, Troncoso (2013); Blencowe (1989); Afshar, Bagchi, Ammon, Grumiller, Prohazka, Riegler, Wutte (2017)

$$
\left[P_m^{(s)}, P_n^{(t)}\right] = 0 \qquad \text{iks} \llbracket \lambda \rrbracket \text{ algebra}
$$

$$
\[L_m^{(s)}, P_n^{(t)}\] = \sum_{\substack{u=|s-t|+2\\s+t+u \text{ even}}}^{s+t-2} g_{s+t-u}^{st}(m, n; \lambda) P_{m+n}^{(u)},
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$$

 $s+t+u$  even

$$
\begin{array}{c}\n\lim_{m+n \to 0} \\
\left[\begin{array}{c}\n\text{[P,P]} \approx \\
\text{[L,P]} = \text{P} \\
\text{[L,L]} = \text{L}\n\end{array}\right]\n\end{array}
$$

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## Coset construction from *U*(iso(1,2))  $\sqrt{2}$  $\overline{a}$ *<u>Coset construction from <i>LI*(iso(1.2)</u> (with the conventions (2.28)) and correspond to the independent vectors in an irreducible  $(1,2)$ and obtain all other components of *<sup>L</sup>*(*s*)

• 
$$
\mathfrak{hs}[\lambda]
$$
 generators:  $\mathcal{L}^{(s)}_{\pm(s-1)} \equiv (\mathcal{L}_{\pm})^{s-1}$  &  $\mathcal{L}^{(s)}_{m\mp 1} \equiv \frac{\mp 1}{s \pm m - 1} [\mathcal{L}_{\mp}, \mathcal{L}^{(s)}_{m}]$ 

*<sup>m</sup>* by acting with the operators *L*<sup>+</sup> or *L* as

*We* wish to get [P,P] = 0, [L,P] = P and [L,L] = L *U*(*so*(2*,* 2)) *⁄* œ N  $\mathsf{L},\mathsf{F}$ *u*, [L,P] ≃ P and [l *<sup>m</sup>* ] *,* (3.2) L^(s)  $et$   $IP$  $P$ *<sup>C</sup>*<sup>2</sup> ⌘ *<sup>L</sup>*0*L*<sup>0</sup> <sup>1</sup>

> Andrea Campoleoni - UMONS *n*<br>*u d d i d v*

### Coset construction from *U*(iso(1,2))  $\sqrt{2}$  $\overline{a}$ *<u>Coset construction from <i>LI*(iso(1.2)</u> (with the conventions (2.28)) and correspond to the independent vectors in an irreducible  $(1,2)$ and obtain all other components of *<sup>L</sup>*(*s*) Coset construction from *Hiso(1.2)* to recover the algebra (3.7) as a quotient of *<sup>U</sup>*(iso(1*,* 2)) by building the *<sup>P</sup>*(*s*) **Iction** fro Coset construction from  $U($  iso(1,2)). The construction from  $U($  iso(1,2),

• 
$$
\mathfrak{hs}[\lambda]
$$
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*We* wish to get [P,P] = 0, [L,P] = P and [L,L] = L *U*(*so*(2*,* 2)) *⁄* œ N  $\mathsf{L},\mathsf{F}$ *u*, [L,P] ≃ P and [l *<sup>m</sup>* ] *,* (3.2) L^(s)  $et$   $IP$  $P$ *<sup>C</sup>*<sup>2</sup> ⌘ *<sup>L</sup>*0*L*<sup>0</sup> <sup>1</sup>  $M_0$  wich to ant  $[DD]_{\infty}$   $\cap$   $[1, D]_{\infty}$   $D$  and  $[1, 1]_{\infty}$  $\sim$   $\sim$   $\sim$  $\Gamma$  commutations  $\Gamma$  $M$ e We wish to get  $[PP] \sim 0$   $[1 \ P] \sim P$  and  $[1 \ 1 \ 1 \sim 1$  $\bullet$  We wish to get  $[P, P] \cong O$ ,  $[L, P] \cong P$  and *<sup>m</sup>* corresponding to generalisation of Lorentz

Which option do you choose?  $M<sub>b</sub>$  Which antion do you choose? sector, that automatically gives the correct adjoint action for the Lorentz subalgebra:  $\overline{\phantom{a}}$  are the satisfied by defining by defining by defining  $\overline{\phantom{a}}$  $\bullet \;$  Which option do you choose?  $\hspace{0.1em}$ 

we denote by *P<sup>m</sup>* and *L<sup>m</sup>* the generators of the latter:

**A** 
$$
P_{\pm(s-1)}^{(s)} \equiv (P_{\pm})^{s-1}
$$
 &  $L_{\pm(s-1)}^{(s)} \equiv (s-1) (P_{\pm})^{s-2} L_{\pm}$ 

*gst*

and similarly for the barrel similar lying the barrel sector, with the barrel sector, with the barrel sector,<br>, with the barrel sector, with the barrel sector, with the barrel sector, with the barrel sector, with the bar

*<sup>s</sup>*+*tu*(*m, n*; )*L*(*u*)

the adjoint action of *P<sup>m</sup>* is consistent with the definitions

For instance, for *s* = 3, one obtains

realised as products of Lorentz generators as in eqs. (3.1) and (3.2), that is to consider

*<sup>m</sup>*+*<sup>n</sup> ,* (3.4) hs[lambda]

*<sup>m</sup>* by acting with the operators *L*<sup>+</sup> or *L* as

Higher-spin generators We now consider the following Ansatz for the higher-translation

*<sup>m</sup>* as products of

**B** 
$$
L_{\pm(s-1)}^{(s)} \equiv (L_{\pm})^{s-1} \& P_{\pm(s-1)}^{(s)} \equiv (L_{\pm})^{s-2} P_{\pm}
$$

.<br>Xarikatif dhexat

 $(other component)$  $[L_{\pm}$ ,  $]$ )  $[L_{\pm}$ ,  $]$  $\mathbf{f}(\mathsf{L}_{\mathsf{L}}|\mathsf{L}_{\mathsf{L}})$  (*L*<sub>+, .</sub> ])  $\frac{1}{2}$  **P**  $(\text{other components fixed by } \text{P}(\text{1})$ of the operator *P<sup>±</sup>* is irrelevant since [*L±, P±*]=0. The other components are then fixed (other components fixed by  $[L_{\pm,}]$ )

i

=

Andrea Campoleoni - UMONS *n*<br>*u d d i d v* Andrea Campoleoni - UMONS *Lea* Campoleoni - UMONS by the known action of Lorentz transformations on the *P*(*s*)

*<sup>m</sup>* © *<sup>L</sup>*(*s*)

The commutators of the commutators<br>The commutators of the commutators

*<sup>m</sup>* <sup>+</sup> *<sup>L</sup>*¯(*s*)

*m*

*<sup>L</sup>*(*s*)

*<sup>m</sup> , <sup>L</sup>*(*t*)

h

### Coset construction from *U*(iso(1,2))  $\sqrt{2}$  $\overline{a}$ *<u>Coset construction from <i>LI*(iso(1.2)</u> (with the conventions (2.28)) and correspond to the independent vectors in an irreducible  $(1,2)$ and obtain all other components of *<sup>L</sup>*(*s*) Coset construction from *Hiso(1.2)* to recover the algebra (3.7) as a quotient of *<sup>U</sup>*(iso(1*,* 2)) by building the *<sup>P</sup>*(*s*) **Iction** fro Coset construction from  $U($  iso(1,2)). The construction from  $U($  iso(1,2),

• 
$$
\mathfrak{hs}[\lambda] \text{ generators: } \mathcal{L}^{(s)}_{\pm(s-1)} \equiv (\mathcal{L}_{\pm})^{s-1} \& \mathcal{L}^{(s)}_{m\mp 1} \equiv \frac{\mp 1}{s \pm m - 1} [\mathcal{L}_{\mp}, \mathcal{L}^{(s)}_{m}]
$$

*<sup>m</sup>* by acting with the operators *L*<sup>+</sup> or *L* as

Higher-spin generators We now consider the following Ansatz for the higher-translation

*<sup>m</sup>* as products of

*We* wish to get [P,P] = 0, [L,P] = P and [L,L] = L *U*(*so*(2*,* 2)) *⁄* œ N  $\mathsf{L},\mathsf{F}$ *u*, [L,P] ≃ P and [l *<sup>m</sup>* ] *,* (3.2) L^(s)  $et$   $IP$  $P$  $M_0$  wich to ant  $[DD]_{\infty}$   $\cap$   $[1, D]_{\infty}$   $D$  and  $[1, 1]_{\infty}$  $\sim$   $\sim$   $\sim$  $\Gamma$  commutations  $\Gamma$  $M$ e We wish to get  $[PP] \sim 0$   $[1 \ P] \sim P$  and  $[1 \ 1 \ 1 \sim 1$  $\bullet$  We wish to get  $[P, P] \cong O$ ,  $[L, P] \cong P$  and *<sup>m</sup>* corresponding to generalisation of Lorentz

we denote by *P<sup>m</sup>* and *L<sup>m</sup>* the generators of the latter:

\n- \n**Which option do you choose?**\n
	\n- A 
	$$
	P_{\pm(s-1)}^{(s)} \equiv (P_{\pm})^{s-1}
	$$
	\n- B  $L_{\pm(s-1)}^{(s)} \equiv (L_{\pm})^{s-1}$
	\n- Can only get  $i \text{hs} \lfloor \infty$ ]
	\n\n
\n- \n**6**\n
	\n- $L_{\pm(s-1)}^{(s)} \equiv (L_{\pm})^{s-1}$
	\n- $L_{\pm(s-1)}^{(s)} \equiv (L_{\pm})^{s-2}$
	\n- Coher components fixed by  $[L_{\pm,1}]$
	\n\n
\n- \n**6**\n
	\n- $L_{\pm(s-1)}^{(s)} \equiv (L_{\pm})^{s-1}$
	\n- $L_{\pm(s-1)}^{(s)} \equiv (L_{\pm})^{s-2}$
	\n\n
\n
\n\n

Andrea Campoleoni - UMONS *n*<br>*u d d i d v* Andrea Campoleoni - UMONS *Lea* Campoleoni - UMONS by the known action of Lorentz transformations on the *P*(*s*)

#### Coset construction from  $U(iso(1,2))$  see also Ammon, Pannier, realised as products of Lorentz generations as in equations as in equations as in equations as in equations as<br>The interesting as in equations as in equation Casimir operator. *P <i>P* i = ((*<sup>s</sup>* 1) *<sup>m</sup> <sup>n</sup>*) *<sup>P</sup>*(*s*) *m*+*n* = h *iso*(1 i Notice also that the relations (3.19) and (3.23) imply **1** See also Ammon, Pannier, *W*<sup>2</sup> ⇠ *<sup>L</sup>*2*P*<sup>2</sup> ⇠ 0 *.* (3.25)  $L$  *Coset construction from*  $U($ *iso(1.2))* see also Ammy Riegler (2009)

the adjoint action of *P<sup>m</sup>* is consistent with the definitions

• HS generators: 
$$
L_{\pm(s-1)}^{(s)} \equiv (L_{\pm})^{s-1}
$$
 &  $P_{\pm(s-1)}^{(s)} \equiv (L_{\pm})^{s-2} P_{\pm}$  etc.

Consistency conditions to recover the ihs[λ] commutators: For instance, for *s* = 3 we find The expression for *P*(*s*) *<sup>±</sup>*(*s*1) follows from the adjoint action of *<sup>P</sup>*<sup>0</sup> on *<sup>L</sup>*(*s*) of the operator *P<sup>±</sup>* is irrelevant since [*L±, P±*]=0. The other components are then fixed sistency  $\overline{a}$  $\overline{m}$ *L*<br>*Litions* to i<br>M *D* ver the ihs[λ] commutators: *Pmn* ⌘ *PmP<sup>n</sup>* ⇠ 0 *,* (3.26a) ideal-3D\_1  $\alpha$  *Ponsistency conditions to recover the ihs[* $\lambda$ *] commutators:* 

These relations give relations give rise to a first set of consistency consistency conditions, since the two commutators  $\mathcal{C}$ 



 $\int \mathcal{L}^2 \mathcal{L}^2 \mathcal{L}^2 \mathcal{L}^2 \mathcal{L}^2 = \mathcal{L}(\mathfrak{iso}(1,2))/\langle \mathcal{I} \rangle$  $U \cap W \cap W$ ,  $V \cap W$ ⇥ *L*2*, P<sup>m</sup>* ⇤ ⇠ 0 *.* (3.20) [L^2,P]  $\int (i\mathfrak{so}(1,2))/\langle \mathcal{T}\rangle$ [*Lk,Pmn*]=(*k m*)*P*(*m*+*k*)*<sup>n</sup>* + (*k n*)*Pm*(*n*+*k*) *,* (3.27b)  $id \oplus W \oplus \mathfrak{i}\mathfrak{h}\mathfrak{s}_3[\lambda] = \mathcal{U}(\mathfrak{iso}(1,2))/\left\langle \mathcal{I} \right\rangle$ 

Taking advantage of the relation (3.19) to get *P*(3)

#### Coset construction from  $U(iso(1,2))$  see also Ammon, Pannier, realised as products of Lorentz generations as in equations as in equations as in equations as in equations as<br>The interesting as in equations as in equation Casimir operator. *P <i>P* i = ((*<sup>s</sup>* 1) *<sup>m</sup> <sup>n</sup>*) *<sup>P</sup>*(*s*) *m*+*n* = h *iso*(1 i Notice also that the relations (3.19) and (3.23) imply **1** See also Ammon, Pannier, *W*<sup>2</sup> ⇠ *<sup>L</sup>*2*P*<sup>2</sup> ⇠ 0 *.* (3.25) Riegler (2009)

the adjoint action of *P<sup>m</sup>* is consistent with the definitions

- HS generators:  $L_{+}^{(s)}$  $\frac{1}{2}(s-1) \equiv (L_{\pm})$  $\int s-1$  &  $P_{+}^{(s)}$  $\mathcal{L}(s-1)$  $\overline{L}_{\perp}$ *s ± m* 1 h  $P^{(s)}_{\pm (s-1)} \equiv \left( L_\pm \right)^{s-2} P_\pm \,\,\,$  etc.  $P_{\pm (s-1)}^{(s)} \equiv (L_{\pm})^{s-2} P_{\pm}$  etc. *<sup>m</sup>*⌥<sup>1</sup> ⌘  $m_{\text{max}}(s)$  and  $r(s)$  $\mu_{\pm(s-1)} - \mu_{\pm(s-1)}$ ) *L±P*<sup>0</sup> ⇠ *P±L*<sup>0</sup> *,* (3.18a)  $iota$  HS generators:  $L_{\pm (s-1)}^{(s)} \equiv (L_\pm)^{s-1}$  &  $F$ 
	- The expression for *P*(*s*) *<sup>±</sup>*(*s*1) follows from the adjoint action of *<sup>P</sup>*<sup>0</sup> on *<sup>L</sup>*(*s*) *Pmn* ⌘ *PmP<sup>n</sup>* ⇠ 0 *,* (3.26a) ideal-3D\_1 For instance, for *s* = 3 we find sistency  $\overline{a}$ *L*<br>*Litions* to i<br>M  $\overline{m}$ *D* ver the ihs[λ] commutators: Consistency conditions to recover the ihs[λ] commutators:  $\bigcirc$ of the operator *P<sup>±</sup>* is irrelevant since [*L±, P±*]=0. The other components are then fixed

These relations give relations give rise to a first set of consistency consistency conditions, since the two commutators  $\mathcal{C}$ 



 $\overline{a}$  esentation are we evaluating  $\bigcap$ (*L* //iso(1 2))? *P* $\mathcal{L}$ (*iso*(1,2 • On which representation are we evaluating *U*(iso(1,2): = ((*<sup>s</sup>* 1) *<sup>m</sup> <sup>n</sup>*) *<sup>P</sup>*(*s*) of *U*(iso(1*,* 2)). Using eq. (3.19), one can also check that  $\frac{1}{2}$ *L*2*, P<sup>m</sup>* . (3.20) [L<sup>2</sup>, P<sub>2</sub>,  $\frac{1}{2}$  *I*  $\frac{1}{2}$   $\frac{1}{2}$ [*Lk,Pmn*]=(*k m*)*P*(*m*+*k*)*<sup>n</sup>* + (*k n*)*Pm*(*n*+*k*) *,* (3.27b) On which representation are we evaluating *U*(iso(1,2))?

Taking advantage of the relation (3.19) to get *P*(3)

#### Coset construction from  $U(iso(1,2))$  see also Ammon, Pannier, realised as products of Lorentz generations as in equations as in equations as in equations as in equations as<br>The interesting as in equations as in equation Casimir operator. *P <i>P* i = ((*<sup>s</sup>* 1) *<sup>m</sup> <sup>n</sup>*) *<sup>P</sup>*(*s*) *m*+*n* = h *iso*(1 i Notice also that the relations (3.19) and (3.23) imply **1** See also Ammon, Pannier, *W*<sup>2</sup> ⇠ *<sup>L</sup>*2*P*<sup>2</sup> ⇠ 0 *.* (3.25)  $C$ oset constructi **tion from** *U***(iso(1,2)**) See also Ammo 1 **higher** (2009)  $\sim$  as well as  $\sim$ <u><u>**A**</u></u>  $\frac{1}{2}$  $\mathbf{1}$ <u>List</u> +2 *,* (3.38a) *Poset construction from <i>U*(iso(1.2)) See als h  $\overline{1}$ 3 Riegler (2009)

*L*(3)

<sup>1</sup> *, L*(4)

the adjoint action of *P<sup>m</sup>* is consistent with the definitions

(<sup>2</sup> 9)*L*(3)

<sup>=</sup> 9*L*(5)

• HS generators: 
$$
L_{\pm(s-1)}^{(s)} \equiv (L_{\pm})^{s-1}
$$
 &  $P_{\pm(s-1)}^{(s)} \equiv (L_{\pm})^{s-2} P_{\pm}$  etc.

Consistency conditions to recover the ihs[λ] commutators: For instance, for *s* = 3 we find 1  $\frac{1}{2}$  to recover the ihs[ $\lambda$ ] commutators: of the operator *P<sup>±</sup>* is irrelevant since [*L±, P±*]=0. The other components are then fixed In conclusion, the product of any two translation generators must vanish: sistency  $\overline{a}$  $\overline{m}$ *L*<br>*Litions* to i<br>M • Consistency conditions to recover the ihs[λ] commutators: *Pmn* ⌘ *PmP<sup>n</sup>* ⇠ 0 *,* (3.26a) ideal-3D\_1 stency conditions to **r**  $\overline{\phantom{a}}$ s to recover the ihs[ $\lambda$ ] commutators: **• Consistency conditions to recove** 

These relations give relations give rise to a first set of consistency consistency conditions, since the two commutators  $\mathcal{C}$ 

Taking advantage of the relation (3.19) to get *P*(3)

$$
P_m P_n \sim 0 \qquad L_m P_n \sim P_m L_n \qquad L^2 - \frac{\lambda^2 - 1}{4} \, id \sim 0
$$
   
 **Poincaré ideal**

 $\overline{a}$  esentation are we evaluating  $\bigcap$ (*L* //iso(1 2))? *P* $\mathcal{L}$ (*iso*(1,2 • On which representation are we evaluating *U*(iso(1,2): = ((*<sup>s</sup>* 1) *<sup>m</sup> <sup>n</sup>*) *<sup>P</sup>*(*s*) of *U*(iso(1*,* 2)). Using eq. (3.19), one can also check that ⇥ *L*2*, P<sup>m</sup>* ⇤  $\frac{1}{2}$  *I*  $\frac{1}{2}$   $\frac{1}{2}$ • On which representation are we evaluating *U*(iso(1,2))? *reser l<sup>m</sup>* 0 *, we eval*  $\frac{1}{2}$ iting L  $(ISO(1,2))^2$  $\overline{M}$ 0 *l<sup>m</sup> l l l* icol<sup>-</sup>  $2))$ ?

belongs to the higher-translation sector. When developing [*P*(*s*)

 $\frac{1}{2}$ 

$$
L_m = \begin{pmatrix} l_m & 0 \\ 0 & l_m \end{pmatrix}, \quad P_m = \begin{pmatrix} 0 & l_m \\ 0 & 0 \end{pmatrix} \quad \text{with } l_m \ N \times N \text{ irrep of } \mathfrak{so}(1, 2) \simeq \mathfrak{sl}(2, \mathbb{R})
$$

$$
\Rightarrow \qquad L^2 = \begin{pmatrix} l^2 & 0 \\ 0 & l^2 \end{pmatrix} = \frac{N^2 - 1}{4} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad W = \begin{pmatrix} 0 & l^2 \\ 0 & 0 \end{pmatrix} = \frac{N^2 - 1}{4} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}
$$
Andrea Campoleoni - UMONS

Furthermore, we already showed that *L*<sup>2</sup> is central in the quotient and in section 3.1.2 we

so that also the conditions (3.26c) and (3.24) are satisfied. Notice that *W* is manifestly

⇠ 0 *.* (3.25)

## From *U*(so(2,D)) to *U*(iso(1,2)) **1 a** 0 **1** *J*(ISO( 1,∠)) *u*=*|s*≠*t|*+2 *s y j u <sup>s</sup>*+*t*≠*<sup>u</sup>*(*m, n*; *<sup>⁄</sup>*)*L*(*u*)

$$
\bullet \text{ so (2,2) ideal:} \qquad \mathcal{I}_{AB} \sim 0 \ \Rightarrow \ \mathcal{L}_m \bar{\mathcal{L}}_n \sim 0 \ \Rightarrow \begin{cases} P_m P_n - L_m L_n \sim 0 \\ L_m P_n - P_m L_n \sim 0 \end{cases}
$$

$$
C_2 = L^2 + P^2 \sim 2 L^2 \sim \frac{\lambda^2 - 1}{2} id
$$

Introducing the contraction parameter via  $P_m \to \epsilon^{-1} P_m$ *C*<sup>2</sup> = *L*<sup>2</sup> + *P*<sup>2</sup> <sup>≥</sup> <sup>2</sup>*L*<sup>2</sup> ≥ 2 *id s*+*t*+*u* even

$$
\epsilon^{-2} P_m P_n - L_m L_n \sim 0
$$
\n
$$
\epsilon^{-1} (L_m P_n - P_m L_n) \sim 0 \qquad \Longrightarrow \qquad L_m P_n - P_m L_n \sim 0
$$
\n
$$
L^2 - \frac{\lambda^2 - 1}{4} id \sim 0
$$
\n
$$
L^2 - \frac{\lambda^2 - 1}{4} id \sim 0
$$
\n
$$
L^2 - \frac{\lambda^2 - 1}{4} id \sim 0
$$

$$
\Rightarrow \qquad L_m P_n - P_m L_n \sim 0
$$
\n
$$
L^2 - \frac{\lambda^2 - 1}{4} id \sim 0
$$

 $Poincaré ideal$ 

*PmP<sup>n</sup>* ≠ *LmL<sup>n</sup>* ≥ 0 *,* (4)

≥

2

## From *U*(so(2,D)) to *U*(iso(1,2)) **1 a** 0 **1** *J*(ISO( 1,∠)) *u*=*|s*≠*t|*+2 *s y j u <sup>s</sup>*+*t*≠*<sup>u</sup>*(*m, n*; *<sup>⁄</sup>*)*L*(*u*)

$$
\bullet \text{ so (2,2) ideal:} \qquad \mathcal{I}_{AB} \sim 0 \ \Rightarrow \ \mathcal{L}_m \bar{\mathcal{L}}_n \sim 0 \ \Rightarrow \begin{cases} P_m P_n - L_m L_n \sim 0 \\ L_m P_n - P_m L_n \sim 0 \end{cases}
$$

$$
C_2 = L^2 + P^2 \sim 2 L^2 \sim \frac{\lambda^2 - 1}{2} id
$$

Introducing the contraction parameter via  $P_m \to \epsilon^{-1} P_m$ *C*<sup>2</sup> = *L*<sup>2</sup> + *P*<sup>2</sup> <sup>≥</sup> <sup>2</sup>*L*<sup>2</sup> ≥ 2 *id s*+*t*+*u* even

$$
\epsilon^{-2} P_m P_n - L_m L_n \sim 0
$$
\n
$$
\epsilon^{-1} (L_m P_n - P_m L_n) \sim 0 \qquad \Longrightarrow \qquad L_m P_n - P_m L_n \sim 0
$$
\n
$$
L^2 - \frac{\lambda^2 - 1}{4} id \sim 0 \qquad \epsilon \to 0
$$
\n
$$
L^2 - \frac{\lambda^2 - 1}{4} id \sim 0
$$

$$
\Rightarrow \qquad L_m P_n - P_m L_n \sim 0
$$
\n
$$
L^2 - \frac{\lambda^2 - 1}{4} id \sim 0
$$

 $Poincaré ideal$ 

≥

2

 $\bullet$  The limit of the AdS<sub>3</sub> ideal is still an ideal

Factoring it out from the Poincaré universal enveloping algebra gives a higher-spin algebra with the same spectrum as the  $AdS<sub>3</sub>$  one

## Carrollian conformal HS algebras

(in any dimensions)



# From *U*(so(2,D−1)) to *U*(iso(1,D−1))

Now reverse the logic: look at how the contraction affects the  $\bigcirc$ so(2,D−1) ideal to *define* the iso(1,D−1) coset

## From *U*(so(2,D−1)) to *U*(iso(1,D−1))  $From II (2D_1) to II (2D_2)$ We recall that the algebra so(2*, D* 1) reads

• Now reverse the logic: look at how the contraction affects the so(2,D−1) ideal to *define* the iso(1,D−1) coset *ABB*  $SO(2,D-1)$  ideal to *define* the iso(1,D-1) coset

 $[J_{AB}\,,\,J_{CD}]=\tilde{\eta}_{AC}\,J_{BD}-\tilde{\eta}_{AD}\,J_{BC}-\tilde{\eta}_{BC}\,J_{AD}+\tilde{\eta}_{BD}\,J_{AC}$ with *A, B* 2 *{*0*,...,D}* and ⌘˜ = diag(*,* +*,...,* +*,* ). Within the *JAB* one can select the

$$
\left(\begin{array}{ccc} \mathcal{P}_a\equiv \epsilon\,J_{aD}\,, & & \mathcal{J}_{ab}\equiv J_{ab} \end{array}\right)
$$

$$
[\mathcal{J}_{ab}, \mathcal{J}_{cd}] = \eta_{ac} \mathcal{J}_{bd} - \eta_{ad} \mathcal{J}_{bc} - \eta_{bc} \mathcal{J}_{ad} + \eta_{bd} \mathcal{J}_{ac},
$$
  

$$
[\mathcal{J}_{ab}, \mathcal{P}_c] = \eta_{ac} \mathcal{P}_b - \eta_{bc} \mathcal{P}_a,
$$
  

$$
[\mathcal{P}_a, \mathcal{P}_b] = -\epsilon^2 \mathcal{J}_{ab},
$$

Next step: branching so(2,D-1) → so(1,D-1) of the ideal *P*  $\Rightarrow$  *P* • Next step: branching  $so(2, D-1) \rightarrow so(1, D-1)$  of the ideal  $\bullet$  Novt stop: branshing  $\circ$  0(2  $\Box$  -1)  $\rightarrow$   $\circ$  0(1  $\Box$  -1) of the ideal  $\sim$  real step. Dialioning su(2,D<sup>1</sup> dimensions we refer to all

Andrea Campoleoni - UMONS to the generators *P<sup>a</sup>* and *Jab*, so that ✏ is a dimensionless parameter. The Poincaré algebra Andrea Gampoleonii - Ol**vichvo** 

#### From *U*(so(2,D−1)) to *U*(iso(1,D−1)) *L*(*s*) *<sup>m</sup> , L*(*t*) *n* = ÿ *u*=*|s*≠*t|*+2 *s*+*t*+*u* even  $F$ *<sup>⁄</sup>*<sup>2</sup> <sup>≠</sup> <sup>1</sup> 2  $From II (2012 D_1) to II (2011 D_1)$ *<sup>J</sup>* <sup>2</sup> *<sup>D</sup>* <sup>1</sup> 2 <u>. I))</u> **PUTH U(SU(4,D-1)) to U(150(1,D-1))** product *JA*(*<sup>B</sup> JC*)*D*, while those in (4.9) correspond to the branching of *J*[*AB JCD*]. Notice  $\frac{1}{2}$  is the symmetric indices in  $\frac{1}{2}$  *p*  $\frac{1}{2}$  *c*  $\$ one vector, and the vector of the vector  $\sim$  $\Box$  divided  $\Box$  for the generators can be conveniently conveniently conveniently conveniently conveniently conveniently convenient and  $\Box$  $p(-1)$  to  $U(iso(1,D-1))$

Branching so(2,D−1) → so(1,D−1) of the ideal  $W(x) = \frac{1}{2} \int_{0}^{x} \frac{1}{2$ ■ **Branching so(z,D-1)** → so(1,D-1) or th  $mg \text{ so}(2, D-1) \rightarrow \text{ so}(1, D-1)$ *⁄*<sup>2</sup> ≠ 1<br>2 ± 1  $\overline{a}$  $\rightarrow$  so(1,D-1) of the idea *D* 2 + *D* 2 Branching  $so(2, D-1) \rightarrow so(1, D-1)$  of the ideal

$$
\mathcal{I}_{AB} \sim 0 \Rightarrow \qquad \qquad \mathcal{I}_{AB} \sim 0 \Rightarrow \qquad \qquad \mathcal{I}_{AB} \sim 0
$$
\n
$$
\mathcal{I}_{AB} \sim 0 \Rightarrow \qquad \qquad \mathcal{I}_{AB} \sim 0
$$
\n
$$
\mathcal{I}_{AB} \sim 0 \Rightarrow \qquad \qquad \mathcal{I}_{ab} \equiv \{\mathcal{P}^b, \mathcal{J}_{ba}\} \sim 0
$$
\n
$$
\mathcal{I}_{ABCD} \sim 0 \Rightarrow \qquad \qquad \mathcal{I}_{ab} \equiv \{\mathcal{I}_{ab} \sim 0\}
$$
\n
$$
\mathcal{I}_{ABCD} \sim 0 \Rightarrow \qquad \qquad \mathcal{I}_{ab} \equiv \{\mathcal{I}_{ab} \sim 0\}
$$
\n
$$
\mathcal{I}_{ABCD} \sim 0 \Rightarrow \qquad \qquad \mathcal{I}_{ab} \equiv \{\mathcal{I}_{ab} \sim 0\}
$$
\n
$$
\mathcal{I
$$

*, J<sup>µ</sup>*

⇠ 0 *,* (4.9b) ideal\_D\_carrollian:2

### Coset construction from *U*(iso(1,D−1)) *D* 2 (*<sup>D</sup>* 2)(*<sup>D</sup>* 1)(2⌘*µ*⌫⌘⇢ ⌘*µ*⇢⌘⌫ ⌘*µ*⌘⌫⇢)*<sup>J</sup>* <sup>2</sup>  $\overline{\mathbf{B}}$ (*<sup>D</sup>* 2)(*<sup>D</sup>* 1)(2⌘*µ*⌫⌘⇢ ⌘*µ*⇢⌘⌫ ⌘*µ*⌘⌫⇢)*<sup>J</sup>* <sup>2</sup> *D* 2 ⌘(*µ*h⇢*{J*⌫)*,<sup>J</sup>* i*}* <sup>2</sup> (*<sup>D</sup>* 2)(*<sup>D</sup>* 1)(2⌘*µ*⌫⌘⇢ ⌘*µ*⇢⌘⌫ ⌘*µ*⌘⌫⇢)*<sup>J</sup>* <sup>2</sup> *,* on from *U*  $\mathsf{in}$  $(1\ \mathsf{D})$  $SO(1, D-1))$ *i*  $\binom{1}{2}$

have the symmetries of the symmetries [*P– , <sup>J</sup> fl*  $\frac{1}{2}$ *b*<sup>+</sup>/<sub>1</sub> ide

• iso(1,D-1) ideal  
\n
$$
\mathcal{I}_a \equiv \{ \mathcal{P}^b, \mathcal{J}_{ba} \} \sim 0
$$
\n
$$
\mathcal{I}_{abc} \equiv \{ \mathcal{J}_{[ab}, \mathcal{P}_c] \} \sim 0
$$
\n
$$
\mathcal{I}_{abcd} \equiv \{ \mathcal{J}_{[ab}, \mathcal{J}_{cd]} \} \sim 0
$$
\n
$$
\mathcal{I}_{abcd} \equiv \{ \mathcal{J}_{[ab}, \mathcal{J}_{cd]} \} \sim 0
$$
\n
$$
\mathcal{J}^2 + \frac{(D-1)(D-3)}{4} \, id \sim 0
$$

scalar singleton proposed in [121]. Compared to the three-dimensional case, the eigenvalue

Combining eqs. (4.14b) and (4.15) one can eventually recast these expressions as

· Leftover quadratic combinations, i.e. spin-3 generators:  $\bullet$  Leftover quadratic combinations, i.e. spin-3 genera  $\alpha$  recovered that we recover the condition  $\alpha$  in  $\alpha$  already manifestic manifest in  $\alpha$ Lettover quadratic combinations, i.e. spin-3 generators: eftover quadratic combinations, i.e. spin-3 generators: also completions, i.e. opin o gonoratore.

 $\frac{1}{2}$ 

• *Mµ*⌫*|*⇢

 $\frac{1}{2}$ 

• *Kµ*⌫*|*⇢

$$
\mathcal{S}_{ab} \equiv \{ \mathcal{J}^c_{(a}, \mathcal{J}_{b)c} \} - \text{tr.} \quad \simeq \boxed{\phantom{a}}
$$

$$
\mathcal{K}_{ab|cd} \equiv \{ \mathcal{P}_{(a}, \mathcal{J}_{b)c} \} - \text{tr.} \simeq
$$

Andrea Campoleoni - UMONS Note that, at this stage, we may choose to write *Qµ*⌫ as the traceless part of either of the generator or its Young projection since the two are in one-to-one correspondence The generators of this algebra can still be labelled as the *<sup>Z</sup>*(*s,t*) of the AdS*<sup>D</sup>* one and spin-*<sup>s</sup>*

$$
\mathcal{S}_{ab} \equiv \{ \mathcal{J}^c_{(a}, \mathcal{J}_{b)c} \} - \mathrm{tr.} \quad \simeq \Box \qquad \mathcal{M}_{ab|c} \equiv \{ \mathcal{J}_{a(c}, \mathcal{J}_{d)b} \} - \mathrm{tr.} \quad \simeq \Box
$$

$$
\mathcal{K}_{ab|cd} \equiv \{ \mathcal{P}_{(a}, \mathcal{J}_{b)c} \} - \text{tr.} \approx \boxed{\qquad \qquad \text{if } \mathfrak{sp} \equiv \mathcal{U}(\mathfrak{iso}(1, D-1))/\langle \mathcal{I}_c \rangle \}
$$
\nAndrea Campoleoni - UMONS

✏*a*1*···aD*3*bcd<sup>J</sup> bcP<sup>d</sup>* (4.17)

### Some commutators… action of *P<sup>a</sup>* on the allowed products of *J* 's, with the prescription that *{J*[*ab,Pc*]*}* ⇠ 0 *and commutators ...* generators nor extra consistency conditions because *PaP<sup>b</sup>* ⇠ 0.  $A$ and *{P<sup>b</sup> ,Jba}* ⇠ 0. Computing an additional commutator with *P<sup>a</sup>* does not produce new

All generators transform as Lorentz tensors Since the generators of ihs*<sup>D</sup>* are realised as products of Poincaré generators, they all transforms as Lorentz tensors. For instance, for *s* = 3 one has Since the generators transform as Lorentz tensors of Poincaré generators of Poincaré generators, they are realised as products of Poincaré generators, they allow a poincaré generators, they allow a poincaré generators, the transforms as Lorentz tensors. For instance, for *s* = 3 one has

$$
[\mathcal{J}_{ab}, \mathcal{S}_{cd}] = \eta_{ac} \mathcal{S}_{bd} + \eta_{ad} \mathcal{S}_{bc} - \eta_{bc} \mathcal{S}_{ad} - \eta_{bd} \mathcal{S}_{ac} ,
$$
  

$$
[\mathcal{J}_{ab}, \mathcal{M}_{cd]e}] = 2 \eta_{a(c} \mathcal{M}_{d)b|e} + \eta_{ae} \mathcal{M}_{cd|b} - 2 \eta_{b(c} \mathcal{M}_{d)a|e} - \eta_{be} \mathcal{M}_{cd|a} ,
$$
  

$$
[\mathcal{J}_{ab}, \mathcal{K}_{cd|ef}] = 2 (\eta_{a(c} \mathcal{K}_{d)b|ef} + \eta_{a(e} \mathcal{K}_{f)b|cd} - \eta_{b(c} \mathcal{K}_{d)a|ef} - \eta_{b(e} \mathcal{K}_{f)a|cd})
$$

• Commutators with translations:  $\alpha$ On the other hand, their commutators with translations take a more "exotic" form:

$$
[\mathcal{P}_a, \mathcal{S}_{bc}] = -2 \mathcal{M}_{bc|a},
$$
  
\n
$$
[\mathcal{P}_a, \mathcal{M}_{bc|d}] = 0,
$$
  
\n
$$
[\mathcal{P}_a, \mathcal{K}_{bc|de}] = -\eta_{ab} \mathcal{M}_{de|c} - \eta_{ac} \mathcal{M}_{de|b} - \eta_{ad} \mathcal{M}_{bc|e} - \eta_{ae} \mathcal{M}_{bc|d}
$$
  
\n
$$
-\frac{2}{D-2} (\eta_{d(b} \mathcal{M}_{c)e|a} + \eta_{e(b} \mathcal{M}_{c)d|a} - \eta_{bc} \mathcal{M}_{de|a} - \eta_{de} \mathcal{M}_{bc|a})
$$

### Some commutators… action of *P<sup>a</sup>* on the allowed products of *J* 's, with the prescription that *{J*[*ab,Pc*]*}* ⇠ 0 *and commutators ...* generators nor extra consistency conditions because *PaP<sup>b</sup>* ⇠ 0.  $A$ and *{P<sup>b</sup> ,Jba}* ⇠ 0. Computing an additional commutator with *P<sup>a</sup>* does not produce new

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$$
[\mathcal{J}_{ab}, \mathcal{S}_{cd}] = \eta_{ac} \mathcal{S}_{bd} + \eta_{ad} \mathcal{S}_{bc} - \eta_{bc} \mathcal{S}_{ad} - \eta_{bd} \mathcal{S}_{ac} ,
$$
  

$$
[\mathcal{J}_{ab}, \mathcal{M}_{cd|e}] = 2 \eta_{a(c} \mathcal{M}_{d)b|e} + \eta_{ae} \mathcal{M}_{cd|b} - 2 \eta_{b(c} \mathcal{M}_{d)a|e} - \eta_{be} \mathcal{M}_{cd|a} ,
$$
  

$$
[\mathcal{J}_{ab}, \mathcal{K}_{cd|ef}] = 2 (\eta_{a(c} \mathcal{K}_{d)b|ef} + \eta_{a(e} \mathcal{K}_{f)b|cd} - \eta_{b(c} \mathcal{K}_{d)a|ef} - \eta_{b(e} \mathcal{K}_{f)a|cd})
$$

• Commutators with translations:  $\alpha$ On the other hand, their commutators with translations take a more "exotic" form:

$$
[\mathcal{P}_a, \mathcal{S}_{bc}] = -2 M_{bc|a},
$$
\n
$$
[\mathcal{P}_a, \mathcal{M}_{bc|d}] = 0,
$$
\n
$$
[\mathcal{P}_a, \mathcal{K}_{bc|de}] = -\eta_{ab} M_{de|c} - \eta_{ac} M_{de|b} - \eta_{ad} M_{bc|e} - \eta_{ae} M_{bc|d}
$$
\n
$$
-\frac{2}{D-2} (\eta_{d(b} M_{c)e|a} + \eta_{e(b} M_{c)d|a} - \eta_{bc} M_{de|a} - \eta_{de} M_{bc|a})
$$
\nThe linearised curvatures do not reproduce those of Fradkin and Vasiliev

#### Structure of the algebra Structure of the algebra as can be worked out directly from the branching rules of two-row Young directly from the branching rules of two<br>The branching rules of two-row Young diagrams of two-row Young diagrams of two-row Young diagrams of two-row <u>ul lile diyenid birin bir</u> <sup>3</sup> *<sup>J</sup>abI<sup>b</sup>* <sup>+</sup> 3 3 to the identity so as to avoid multiplicities in the spectrum. Its eigenvalue is then fixed by In conclusion, if one wants to build a higher-spin extension of the Poincaré algebra with  $P$  or the Poincaré algebra with  $\alpha$ <sup>3</sup> *<sup>J</sup>abI<sup>b</sup>* <sup>+</sup> <u>*<u>D</u>*  $\frac{1}{2}$   $\frac$  $S_{t}$ **JU 2018 IS 2 IS 1990 LAND FIXED FIXED. MOREOVER, BOTH THE QUADRATIC CASE OF AN UP 2018**  $t_{\rm eff}$  to avoid multiplicities in the spectrum. Its eigenvalue is the spectrum. It see fixed by  $t_{\rm eff}$ a central element thanks to the previous conditions. It is the previous conditions. It is thus natural to set i  $\mathsf{ot}$  the aldebra in the spectrum. Its eigenvalue is the spectrum. Its eigenvalue is then fixed by  $\mathsf{c}_i$ (*µJ‹*)*fl* <sup>≠</sup> <sup>2</sup>

Higher-spin generators so(2*, D* 1) into so(1*, D* 1): for *s* 2, the spectrum of higher-spin gauge fields is spanned  $\bullet$  Higher-spin generators [*Pa, Sbc*] = 2*Mbc|<sup>a</sup> ,* (4.20a) [*Pa, Sbc*] = 2*Mbc|<sup>a</sup> ,* (4.20a) the Eastwood-Vasiliev spectrum (2.222) as a quotient of its UEA by a quotient of its UEA by a two-sided ideal,  $\overline{\phantom{a}}$ ✏*a*1*···aD*3*bcd<sup>J</sup> bcP<sup>d</sup>* (4.17)

$$
\mathcal{Z}^{s,t} \equiv \boxed{\frac{s-1}{s-t-1}} \quad \text{with } t \in \{0, \dots, s-1\} \qquad \text{•} \quad t \text{ odd: one } P
$$

Once again, there are multiple equivalent ways in which to write the higher-spin gen-

2

 $\overline{y}$ 

[*Pa,Mbc|d*]=0 *,* (4.20b)  $\mathcal{L}$ 

<sup>3</sup> *<sup>J</sup>abI<sup>b</sup>* <sup>+</sup>

*D* 3

*<sup>J</sup>* <sup>2</sup> <sup>+</sup> (*<sup>D</sup>* 1)(*<sup>D</sup>* 3)

*Sab* + ✏

*<sup>I</sup><sup>a</sup>* <sup>=</sup> <sup>4</sup>

<sup>4</sup> *id*◆

2*Qab*

 $\frac{4}{3}$ 

◆

*<sup>D</sup>* ⌘*ab* ✓

*<sup>J</sup>* <sup>2</sup> <sup>+</sup> (*<sup>D</sup>* 1)(*<sup>D</sup>* 3)

 $\bullet$  todd: one  $P$ *<sup>D</sup>* ⌘*ab* ✓ *<sup>J</sup>* <sup>2</sup> *<sup>D</sup>* <sup>1</sup>

<sup>2</sup> [*P<sup>a</sup> , <sup>I</sup>b*] =

*s*≠1

2

This structure generalises to any value of *s* according to the following schematic rules:

On the other hand, their commutators with translations take a more "exotic" form:

2

✓

*P<sup>a</sup> .* (4.25)

<sup>0</sup> ⇠ *<sup>I</sup>abc<sup>J</sup> bc* <sup>+</sup>

*<sup>I</sup><sup>a</sup>* <sup>=</sup> <sup>4</sup>

*D* 3

0

Commutators with P ideal), but this will not be true anymore in flat space.  $\bullet$  Commutators with P *a* Commutators with P  $\overline{\phantom{a}}$ 

For D=4 see also Fradkin, Vasiliev (1987)  $\left\| \begin{array}{c} p, \mathcal{Z} \end{array} \right\|$ 

\n- Commutators with P
\n- For D=4 see also  
Fradkin, Vasiliev (1987)
\n- $$
[P, Z^{(s,t)}] \propto Z^{(s,t-1)} + \eta Z^{(s,t+1)}
$$
 for *t* even for *t* odd for *t* odd
\n

where  $\frac{1}{2}$  is the international denotes a sum of the international denotes as  $\frac{1}{2}$  is the indices as  $\frac{1}{2}$ 

even number of *P*'s, while those with odd *t* can be written as products of an odd number

 $\mathfrak{u}\mathfrak{b}\mathfrak{s}_D$  as Inönü-Wigner contraction of  $\begin{array}{c}\n\downarrow \Lambda \end{array}$  $\bullet\;\;$ i $\mathfrak{hs}_D$  as Inönü-Wigner contraction of  $\mathfrak{hs}_D$ 

Andrea Campoleoni - UMONS  $Z^{(s,t)} \to e^{-t} Z^{(s,t)}$  for todd *abc|def* <sup>=</sup> *Jd*(*<sup>a</sup> <sup>J</sup>b|e<sup>|</sup> <sup>J</sup>c*)*<sup>f</sup>* <sup>+</sup> *···* ' *,* (4.22a)  $\nu$ ith  $a^2 + b^2 + 2$  *de*  $b^2 - 2$  *de*  $a^2 - 2$   $a^2 - 2$ *Z*(4*,*2) *abc|<sup>d</sup>* <sup>=</sup> *<sup>J</sup> <sup>e</sup>* (*<sup>a</sup> Jb|e<sup>|</sup> Jc*)*<sup>d</sup>* + *···* ' *,* (4.22c) *Z*(4*,*3) *a*  $\mathcal{L}^{(0,0)}$ (*<sup>a</sup> Jb|e<sup>|</sup> Pc*) + *···* ' *,* (4.22d)  $s_1 + s_2 - s_3 \mod 2 = 0$  $V$ ltn  $t_1 + t_2 - t_3 \mod 2 = 0$  $\sqrt{4}$  $\overline{a}$   $\overline{b}$   $\overline{c}$   $\overline{c}$   $\overline{d}$   $\overline{d}$   $\overline{d}$ (*<sup>a</sup> Jb|e<sup>|</sup> Jc*)*<sup>d</sup>* + *···* ' *,* (4.22c) *Z*(4*,*3) *abc* <sup>=</sup> *<sup>J</sup> <sup>e</sup>* (*<sup>a</sup> Jb|e<sup>|</sup> Pc*) + *···* ' *,* (4.22d)  $\left[ \mathcal{Z}^{(s_1,t_1)}, \mathcal{Z}^{(s_2,t_2)} \right] \propto \sum \mathcal{Z}^{(s_3,t_3)}$ *, Z*(*x*)  $\left( \begin{array}{c} 0 & \to \infty \\ -\left( a & t \right) \end{array} \right)$ consequence of the spin addition rules guaranteed by the UEA construction rules guaranteed by the UEA construction *s*3*,t*<sup>3</sup> with the second with the second s  $\Rightarrow$   $\int z^{(s,t)} \rightarrow e^{-1} z^{(s,t)}$  $\left[\mathcal{Z}(s_1,t_1) \mathcal{Z}(s_2,t_2)\right] \propto \sum \mathcal{Z}(s_3,t_3)$  with  $s_1+s_2-s_3 \mod 2=0$  $\begin{array}{lll}\n\mathbf{z} & \mathbf{z} \\
 & \mathbf{z}_{3,t_3}\n\end{array}$  with  $t_1 + t_2 - t_3 \mod 2 = 0$  $c_1 + c_2$  c<sub>0</sub> the sequence  $\frac{1}{2}$ have *t*1+*t*<sup>2</sup> mod 2 = *t*<sup>3</sup> mod 2, since the terms with even *t* can be written as products of an  $e^{\bullet}$   $\mathcal{Z}^{(s,t)} \to \epsilon^{-1} \mathcal{Z}^{(s,t)}$  for todd  $\parallel$ of *P*'s and the number of *P*'s is conserved modulo 2 both by the Lie bracket and by the  $t_1 + t_2 - t_3 \mod 2 = 0.$ decomposition is *s*<sup>3</sup> = *s*<sup>1</sup> + *s*<sup>2</sup> 2, the one with lowest spin is *s*<sup>3</sup> = *|s*<sup>1</sup> *s*2*|* + 2 (this is a  $\Rightarrow$   $\left( \mathcal{Z}^{(s,t)} \to \epsilon^{-1} \mathcal{Z}^{(s,t)} \text{ for } t \text{ odd } \right)$  $\frac{1}{2}$   $\frac{1}{2}$ 

## **P**  $R$  Classification of consistent ideals *<sup>P</sup>*↵*,<sup>J</sup>* ⇢ (*µJ*⌫)⇢ <sup>2</sup> *Jassification of consistent ideals* where a vertical bar is used to separate groups of indices. The groups of indices. The generators  $\overline{\mathcal{M}}$ where a vertical bar is used to separate groups of indices. The generators *Q*, *M* and *K* [*P, <sup>Z</sup>s,t*] <sup>Ã</sup> *<sup>Z</sup>s,t*+1 (12) SSIII CALIOI I OI CONSISTEMENT IN THE ADAPTER CONSTRUCTION IN THE ADAPTEMENT CONSTRUCTION IN THE ADAPTEMENT CO

■ Can one build other conformal Carrollian HS algebras from *U*(iso(1,D−1))? **n** *id* ≥ 0 (6) <sup>=</sup> *Mµ*⌫*|*↵ *<sup>D</sup>* <sup>2</sup> **• Can one build other conformal Carrollian HS algebras from** the following combinations: the following combinations: the following combinations: the following combinations *<sup>J</sup>* <sup>2</sup> *<sup>D</sup>* <sup>1</sup>

*D* 2

• Portion of the ideal we need to quotient out:

*IABCD* ≥ 0 ∆ Candidate spin-3 generators: Since *<sup>P</sup>*<sup>2</sup> is a Casimir of the Poincaré algebra, it forms an ideal in itself that we can factorise (we will see in a moment what is the value of this Casimir). Among the remaining elements of the list, only *{Jµ*(⌫*,J*⇢)*}* tr. has the correct projection to fit the role of spin 3 higherrotations in the Eastwood-Vasiliev spectrum so we keep it. Similarly only fits the role of the mixed-symmetry "hook" type spin 3 generator. The delicate point is that both *{Pµ,P*⌫*}*tr. and *{J* ⇢ *<sup>µ</sup>,J*⌫⇢*}*tr. have the desired symmetric traceless projection to *, {P*(*µ,P*⌫)*}* tr. *,* n *P ,J<sup>µ</sup>* o *, P*(*µ,J*⌫)⇢ tr. *, {J*[*µ*⌫*,P*⇢]*} , <sup>J</sup>* <sup>2</sup> ⌘ *<sup>J</sup>µ*⌫*<sup>J</sup>* ⌫*<sup>µ</sup> , {J* ⇢ (*µ,J*⌫)⇢*}* tr. *, {J*(*µ*h⇢*,J*⌫)i*}* tr. *, {J*[*µ*⌫*,J*⇢]*} ,* (4.8a) ≃ <sup>=</sup> *<sup>M</sup>*⌫*µ|*⇢, *<sup>M</sup>*(*µ*⌫*|*⇢) = 0 and *<sup>M</sup> |*⇢ = 0; = *K*⌫*µ|*⇢ = *Kµ*⌫*|*⇢, *K*(*µ*⌫*|*⇢) <sup>=</sup> *<sup>K</sup>µ*(⌫*|*⇢) = 0 and *<sup>K</sup> |*⇢ = *Kµ*⌫*|* or in other terms they are irreducible representations of so(1*, D* 1) corresponding to the *Q* = *, M* = *, K* = *,* (4.12) ≃ • *Mµ*⌫*|*⇢ <sup>=</sup> *<sup>M</sup>*⌫*µ|*⇢, *<sup>M</sup>*(*µ*⌫*|*⇢) = 0 and *<sup>M</sup>* • *Kµ*⌫*|*⇢ = *K*⌫*µ|*⇢ = *Kµ*⌫*|*⇢, *K*(*µ*⌫*|*⇢) <sup>=</sup> *<sup>K</sup>µ*(⌫*|*⇢) = 0 and *<sup>K</sup>* or in other terms they are irreducible representations of so(1*, D* 1) corresponding to the Young diagrams *Q* = recall the 3D poll! *Sab* + ✏ <sup>2</sup> *<sup>Q</sup>ab* ⇠ <sup>0</sup> *,* (4.10c) ✏ <sup>1</sup> *{J*[*ab ,Pc*]*}* ⇠ <sup>0</sup> *,* (4.10d) *{J*[*ab , Jcd*]*}* ⇠ 0 *,* (4.10e) *<sup>C</sup>*<sup>2</sup> ⌘ *<sup>J</sup>* <sup>2</sup> <sup>+</sup> ✏ (*D* + 1)(*D* 3) <sup>4</sup> *id .* (4.10f)

## **P**  $R$  Classification of consistent ideals *<sup>P</sup>*↵*,<sup>J</sup>* ⇢ (*µJ*⌫)⇢ <sup>2</sup> *Jassification of consistent ideals* where a vertical bar is used to separate groups of indices. The groups of indices. The generators  $\overline{\mathcal{M}}$ where a vertical bar is used to separate groups of indices. The generators *Q*, *M* and *K* [*P, <sup>Z</sup>s,t*] <sup>Ã</sup> *<sup>Z</sup>s,t*+1 (12) SSIII CALIOI I OI CONSISTEMENT IN THE ADAPTER CONSTRUCTION IN THE ADAPTEMENT CONSTRUCTION IN THE ADAPTEMENT CO

■ Can one build other conformal Carrollian HS algebras from *U*(iso(1,D−1))? **n** *id* ≥ 0 (6) <sup>=</sup> *Mµ*⌫*|*↵ *<sup>D</sup>* <sup>2</sup> **• Can one build other conformal Carrollian HS algebras from** the following combinations: the following combinations: the following combinations: the following combinations *<sup>J</sup>* <sup>2</sup> *<sup>D</sup>* <sup>1</sup>

*D* 2

• Portion of the ideal we need to quotient out: dear we need to quotient out.  $\frac{1}{2}$ 

$$
\mathcal{I}_{ABCD} \sim 0 \Rightarrow \qquad \qquad \left\{ \begin{array}{l} \overline{\epsilon^{-1} \{ \mathcal{J}_{[ab}, \mathcal{P}_{c]} \} \sim 0} \\ \overline{\{ \mathcal{J}_{[ab}, \mathcal{J}_{cd]} \} \sim 0} \end{array} \right\}
$$
\n• Candidate spin-3 generators:

\n
$$
\{ \mathcal{P}_{\mu}, \mathcal{P}_{\nu} \} - \text{tr.} \approx \boxed{\phantom{\frac{1}{2}}\qquad \{ \mathcal{J}^{\rho}_{(\mu}, \mathcal{J}_{\nu)\rho} \} - \text{tr.} \approx \boxed{\phantom{\frac{1}{2}}\qquad \{ \mathcal{J}^{\rho}_{(\mu)}, \mathcal{J}_{\nu)\rho} \} - \text{tr.} \approx \boxed{\frac{1}{2}} \quad \text{for all } \rho \in \mathbb{R} \}
$$

Show one use P<sub>P</sub>P<sub>v</sub> as spin-3 generator? where  $\frac{1}{2}$  means we are the traceless projection of the previous  $\frac{1}{2}$ written in the symmetric convention. In the following, we shall use all use  $P$  are shall use alternatively the name of  $\alpha$ of the generator or its Young projection since the two are in one-to-one-to-one-to-one-to-one-to-one-to-one-to-o of the generator or its Young projection since the two are in one-to-one correspondence Can one use  $P_{\mu}P_{\nu}$  as spin-3 generator?

$$
[\mathcal{P}_\alpha\,,\,\mathcal{J}^\rho{}_{(\mu}\mathcal{J}_{\nu)\rho}-\frac{2}{D}\eta_{\mu\nu}\,\mathcal{J}^2]=\{\mathcal{J}_{\alpha(\mu},\mathcal{P}_{\nu)}\}+\cdots
$$

Andrea Campoleoni - UMONS  $\frac{1}{2}$ Andrea Campoleoni - UMONS commutator *{·, ·}* automatically projects in the symmetric part of the indices. Some linear  $\Lambda$ *{Pµ,P*⌫*}* or *{J* ⇢ *area Campoleoni - UIVICINS* are in the two expressions are identified in the internal of the internal control of the intern *<i>P*  $\overline{M}$   $\overline{M}$ 

#### **P**  $R$  Classification of consistent ideals *<sup>P</sup>*↵*,<sup>J</sup>* ⇢ (*µJ*⌫)⇢ <sup>2</sup> *<u>Ition</u>* **of consistent ideals** where a vertical bar is used to separate groups of indices. The groups of indices. The generators  $\overline{\mathcal{M}}$ where a vertical bar is used to separate groups of indices. The generators *Q*, *M* and *K* [*P, <sup>Z</sup>s,t*] <sup>Ã</sup> *<sup>Z</sup>s,t*+1 (12) **n**<sub>2</sub>  $\overline{a}$  *k*<sub>2</sub> *j*<sub>2</sub> *j <u>ISLE</u>*  $\mathbf{A}$   $\mathbf{B}$   $\mathbf{B$  $\mathbf{b}$  because the anticommutator anticommutation  $\mathbf{b}$ SSIII CALIOI I OI CONSISTEMENT IN THE ADAPTER CONSTRUCTION IN THE ADAPTEMENT CONSTRUCTION IN THE ADAPTEMENT CO

■ Can one build other conformal Carrollian HS algebras from *U*(iso(1,D−1))? **n** *id* ≥ 0 (6) <sup>=</sup> *Mµ*⌫*|*↵ *<sup>D</sup>* <sup>2</sup> **• Can one build other conformal Carrollian HS algebras from**  $\overline{1}$   $\overline{C}$ 2 *D*<br>Del CO ⌘(*µ*h⇢*{J*⌫)*,<sup>J</sup>* i*}* <sup>2</sup>  $\Box$  *D* **2013 D** 2023 **Example Carrollian HS alg**<br>
⇒ *<i>⁄* ± 2 ± 10 *id* the following combinations: the following combinations: the following combinations: the following combinations *<sup>J</sup>* <sup>2</sup> *<sup>D</sup>* <sup>1</sup>

*D* 2

*D* 2

*, K* =

• Portion of the ideal we need to quotient out: *P D D <i>P Z D <i>P Z P P <i>P P P <i>P P*  $\frac{1}{2}$ 

$$
\mathcal{I}_{ABCD} \sim 0 \Rightarrow \qquad \qquad \left\{ \begin{array}{l} \epsilon^{-1} \left\{ \mathcal{J}_{[ab}, \mathcal{P}_{c]} \right\} \sim 0 \\ \left\{ \mathcal{J}_{[ab}, \mathcal{J}_{cd]} \right\} \sim 0 \end{array} \right\}
$$
\ncandidate spin-3 generators:

\n
$$
\{ \mathcal{P}_{\mu}, \mathcal{P}_{\nu} \} - \text{tr.} \approx \boxed{\phantom{0}} \qquad \{ \mathcal{J}^{\rho}{}_{(\mu}, \mathcal{J}_{\nu)\rho} \} - \text{tr.} \approx \boxed{\phantom{0}} \qquad \qquad
$$

Show one use P<sub>P</sub>P<sub>v</sub> as spin-3 generator? written in the symmetric convention. In the following, we shall use all use  $P$  are shall use alternatively the name of  $\alpha$ Can one use P<sub>u</sub>P<sub>v</sub> as spin-3 generator?

$$
[\mathcal{P}_{\alpha}, \mathcal{J}^{\rho}(\mu \mathcal{J}_{\nu}) = \mathcal{J}^2] = \{ \mathcal{J}_{\alpha(\mu}, \mathcal{P}_{\nu)} \} + \cdots \implies
$$

Andrea Campoleoni - UMONS  $\frac{1}{2}$ Andrea Campoleoni - UMONS commutator *{·, ·}* automatically projects in the symmetric part of the indices. Some linear *{Pµ,P*⌫*}* or *{J* ⇢ *area Campoleoni - UIVICINS* are in the two expressions are identified in the internal of the internal control of the intern *<i>P*  $\overline{M}$   $\overline{M}$ written in the symmetric convention. In the symmetric convention  $\alpha$  is alternatively the name of  $\alpha$ 

## Galilean conformal HS algebras

(in any dimensions)



## From *U*(so(2,D−1)) to *U*(iso(1,D−1))  $F_{\text{mean}}$ *J* $\mu_{\text{mean}}$ /O, D,  $A_{\text{M}}$ ,  $A_{\text{M}}$  (*Although*  $A_{\text{M}}$ **in**  $P(SO(Z, D-1))$  to  $P(SO(1, D-1))$ changes expression as well, as  $m$   $l/(so(2, D-1))$  to  $l/(iso(1, D-1))$

■ Same approach as for Carroll, but with a new splitting of so(2,D-1) Same approach as for Carroll, but with a new splitting of so(2, D-1) terms of Galilean generators. In order to set the ground for the Inönü-Wigner contraction, where *Jij* generates spatial rotations; *Ti,* generate spatial translations, *Ti,*<sup>0</sup> generate spawhere *Jij* generates spatial rotations; *Ti,* generate spatial translations, *Ti,*<sup>0</sup> generate spa- $\int f(x)dx = \int f(x)g(x)dx$  and  $\int f(x)g(x)dx$  is the *induction*  $\int f(x)g(x)dx$  (1) the *n*  $f(x)g(x)dx$  is the *n*  $f(x)g(x)dx$  of  $f(x)g(x)dx$  (1)  $\sigma$ *jdine approach as for Garron, but with a new spilling or so(2,D=1)* 

 $[J_{AB}\,,\,J_{CD}]=\tilde{\eta}_{AC}\,J_{BD}-\tilde{\eta}_{AD}\,J_{BC}-\tilde{\eta}_{BC}\,J_{AD}+\tilde{\eta}_{BD}\,J_{AC}$ [*P– , <sup>J</sup> fl*  $\int \frac{\gamma - \eta_{BC} J_{A}}{J}$  $\tilde{\eta}_{AC}$   $J_{BD} - \tilde{\eta}_{AD}$   $J_{BC} - \tilde{\eta}_{BC}$   $J_{AD} + \tilde{\eta}_{BD}$   $J_{AC}$ generates the translations,  $\mu_{\text{D}}$  generates the zeroth component co  $AC \, \theta BD \quad \text{YAD} \, \theta BC \quad \text{YBC} \, \theta AD + \text{YBD} \, \theta AC$  $\begin{array}{ccc} \n\text{I} & \text{I} & \$  $\begin{aligned} \left[ J_{AB}\, ,\, J_{CD} \right] = \tilde{\eta}_{AC}\, J_{BD} - \tilde{\eta}_{AD}\, J_{BC} - \tilde{\eta}_{BC}\, J_{AD} + \tilde{\eta}_{BD}\, J_{AC} \end{aligned}$ 

$$
\begin{aligned}\n\overrightarrow{J_{ij}} \\
\overrightarrow{L}_{-} &= H, & \overrightarrow{L}_{0} &= D, & \overrightarrow{L}_{+} &= K, \\
\overrightarrow{T_{i,-}} &= P_{i}, & \overrightarrow{T_{i,0}} &= B_{i}, & \overrightarrow{T_{i,+}} &= K_{i}\n\end{aligned}\n\qquad \longrightarrow \quad \mathfrak{sl}(2,\mathbb{R})
$$

 $[J_{ij}, \bar{L}_m] = 0$   $[J_{ij}, T_{k,m}] = \delta_{ik} T_{j,m} - \delta_{jk} T_{i,m}$   $[\bar{L}_m, T_{i,n}] = (m - n) T_{i,m+n}$  $G = \frac{1}{\sqrt{2}}$  $\left[J_{ij},L_{m}\right]=0$   $\left[J_{ij},L_{k,m}\right]=0$ ik $L_{j,m}-0$ jk $L_{i,m}$   $\left[\mu_{m},L_{i,n}\right]=\left(\mu_{m}-\mu_{m}\right)$ 

$$
[T_{i,m}, T_{j,n}] = \delta_{ij}(m-n)\bar{L}_{m+n} + \gamma_{mn}J_{ij}
$$

Andrea Campoleoni - UMONS adjoint representation of the group of Euclidean rotations so(*<sup>D</sup>* 2), while *<sup>L</sup>*¯*<sup>m</sup>* lives in the andrea Campoleoni **OF ADSPRESENTING THE NON-DEGENERATE part of the non-degenerate part of the metric in the Galilean limit and a OF ADS** bas an inference in terms of the rigid isometry algebra of the rigid isometry algebra of AdS2 sitting isometry drea Campol

4.1.1 From *U*(so(2*, D* 1)) to *U*(iso(1*, D* 1))

## From *U*(so(2,D−1)) to *U*(iso(1,D−1))  $F_{\text{mean}}$ *J* $\mu_{\text{mean}}$ /O, D,  $A_{\text{M}}$ ,  $A_{\text{M}}$  (*Although*  $A_{\text{M}}$ **in**  $P(SO(Z, D-1))$  to  $P(SO(1, D-1))$ changes expression as well, as  $m$   $l/(so(2, D-1))$  to  $l/(iso(1, D-1))$  $m \frac{1}{(2012)}$   $m$   $1)$  to  $1$   $l$ ien(1  $m$   $1)$ )

■ Same approach as for Carroll, but with a new splitting of so(2,D-1) Same approach as for Carroll, but with a new splitting of so(2, D-1) terms of Galilean generators. In order to set the ground for the Inönü-Wigner contraction, where *Jij* generates spatial rotations; *Ti,* generate spatial translations, *Ti,*<sup>0</sup> generate spawhere *Jij* generates spatial rotations; *Ti,* generate spatial translations, *Ti,*<sup>0</sup> generate spa- $\int f(x)dx = \int f(x)g(x)dx$  and  $\int f(x)g(x)dx$  is the *induction*  $\int f(x)g(x)dx$  (1) the *n*  $f(x)g(x)dx$  is the *n*  $f(x)g(x)dx$  of  $f(x)g(x)dx$  (1)  $\sigma$ *jdine approach as for Garron, but with a new spilling or so(2,D=1)*  $\frac{1}{2}$  *J j z for Carroll* but with a now enlitting of eq(2 D  $\rightarrow$  1) [*Ti,m, Tj,n*] = *ij* (*<sup>m</sup> <sup>n</sup>*)*L*¯*m*+*<sup>n</sup>* <sup>+</sup> *mnJij .* (5.3c)  $\int f(x)dx$  annmach as for Carroll, but with a new splitting of so(2 D\_1) and approach as for Sarroll, but with a flow spitting of so(2,0  $\,$ 1)

 $[J_{AB}\,,\,J_{CD}]=\tilde{\eta}_{AC}\,J_{BD}-\tilde{\eta}_{AD}\,J_{BC}-\tilde{\eta}_{BC}\,J_{AD}+\tilde{\eta}_{BD}\,J_{AC}$ [*P– , <sup>J</sup> fl*  $\int \frac{\gamma - \eta_{BC} J_{A}}{J}$  $\tilde{\eta}_{AC}$   $J_{BD} - \tilde{\eta}_{AD}$   $J_{BC} - \tilde{\eta}_{BC}$   $J_{AD} + \tilde{\eta}_{BD}$   $J_{AC}$ generates the translations,  $\mu_{\text{D}}$  generates the zeroth component co  $AC \, \theta BD \quad \text{YAD} \, \theta BC \quad \text{YBC} \, \theta AD + \text{YBD} \, \theta AC$  $\begin{array}{ccc} \n\text{I} & \text{I} & \$  $\begin{aligned} \left[ J_{AB}\, ,\, J_{CD} \right] = \tilde{\eta}_{AC}\, J_{BD} - \tilde{\eta}_{AD}\, J_{BC} - \tilde{\eta}_{BC}\, J_{AD} + \tilde{\eta}_{BD}\, J_{AC} \end{aligned}$ where is the sl(2*,* R) Killing metric and the Kronecker symbol in *D* 2 dimensions where is the sl(2*,* R) Killing metric and the Kronecker symbol in *D* 2 dimensions

$$
\begin{aligned}\n\begin{bmatrix}\nJ_{ij} \\
\bar{L}_{-} &= H, & \bar{L}_{0} &= D, & \bar{L}_{+} &= K, \\
T_{i,-} &= P_{i}, & T_{i,0} &= B_{i}, & T_{i,+} &= K_{i}\n\end{bmatrix} \xrightarrow{\mathfrak{s}(\mathbf{D}-2)} \mathfrak{s}(\mathbf{2}, \mathbb{R}) \\
\begin{bmatrix}\nJ_{ij}, \bar{L}_{m}\n\end{bmatrix} &= 0 \qquad [J_{ij}, T_{k,m}] = \delta_{ik} T_{j,m} - \delta_{jk} T_{i,m} \qquad [\bar{L}_{m}, T_{i,n}] = (m-n) T_{i,m+n} \\
\begin{bmatrix}\nT_{i,m}, T_{j,n}\n\end{bmatrix} &= \delta_{ij}(m-n) \bar{L}_{m+n} + \gamma_{mn} J_{ij}\n\end{aligned}
$$

so(*<sup>D</sup>* 2) irreps. as well as their integer<sup>16</sup> spin under sl(2*,* <sup>R</sup>) that we group in a doublet.

Contraction:  $T_{i,m}\to \epsilon^{-1}T_{i,m}$  with  $\mathcal{P} = \{ \mathcal{P}^{\mathcal{P}} \colon \mathcal{P}^{\mathcal{P}} \colon \mathcal{P}^{\mathcal{P}} \}$  and  $\mathcal{P}^{\mathcal{P}}$  and  $\mathcal{P}^{\mathcal{P}}$  (2000) **is the sluid metric metric metric and**  $\mathbb{R}$  $T_a \rightarrow \epsilon^{-1}T_a$ , with  $\epsilon \rightarrow 0$ , Besseli Construes (2000)  $\begin{bmatrix} \mathcal{L}(m) & \mathcal{L}(2, m) \\ \mathcal{L}(m) & \mathcal{L}(m) \end{bmatrix}$  Bayoni, Objakuniai (2003)  $T \sim \epsilon^{-1}T$  with  $\epsilon \cdot 0$ ,  $\epsilon$   $\sim$   $\epsilon$  $\mu(i,m \rightarrow \epsilon \quad \mu(i,m \text{ V)$  is the second of the subalgebra  $\mu(i,m \rightarrow \epsilon \quad \mu(i,m \text{ V))}$ **Contraction:**  $T_{i,m} \to \epsilon^{-1} T_{i,m}$  with  $\epsilon \to 0$  **Bagchi, Gopakumar (2009)** adjoint representation of the group of Euclidean rotations so(*<sup>D</sup>* 2), while *<sup>L</sup>*¯*<sup>m</sup>* lives in the  $\mathcal{L}_{\mathcal{I}}$  and we will represent the spin 2 generators and we will represent them by the m **Contraction:**  $T_{i,m} \to \epsilon^{-1} T_{i,m}$  with  $\epsilon \to 0$  **Bagchi, Gopakumar (2009)** 

with  $\mathcal{L}_{\mathcal{A}}$  ,  $\mathcal{L}_{\mathcal{A}}$ 

Bagchi, Gopakumar (2009)

has an interesting interpretation in terms of the rigid isometry algebra of AdS<sup>2</sup> sitting inside

As explained in the review on relativistic higher-spin gauge algebras, the higher-spin gauge algebra , the higher-spin gauge

As explained in the review on relativistic higher-spin gauge algebras, the higher-spin gauge algebra  $\mathcal{A}$ 

has an interesting interpretation in terms of the rigid isometry algebra of AdS<sup>2</sup> sitting inside

### Andrea Campoleoni - UMONS adjoint representation of the group of Euclidean rotations so(*<sup>D</sup>* 2), while *<sup>L</sup>*¯*<sup>m</sup>* lives in the andrea Campoleoni **OF ADSPRESENTING THE NON-DEGENERATE part of the non-degenerate part of the metric in the Galilean limit and a OF ADS** bas an inference in terms of the rigid isometry algebra of the rigid isometry algebra of AdS2 sitting isometry drea Campoleoni - UMONS

playing the role of the metric tensor on the spatial indices. The generator *Jij* lives in the

4.1.1 From *U*(so(2*, D* 1)) to *U*(iso(1*, D* 1))

#### The so(2,D−1) ideal  $\mathsf{a} \in \mathsf{b}$  and  $\mathsf{a} \in \math$  $\mathsf{I} \cap \mathsf{B} \ \mathsf{SO}(\mathsf{Z},\mathsf{D}^{\perp} \mathsf{I}) \ \mathsf{I} \ \mathsf{O} \ \mathsf{C} \ \mathsf{I}$ *'* æ 0 **P**<sub>*n*</sub> *n P <i>n**i***<b>d** 1<br>1<br>1 2 *<i><u><b>Jeal*</u>

$$
\mathcal{I}_{AB} \sim 0 \qquad \qquad \mathcal{I}_{ABCD} \sim 0 \qquad \qquad C_2 \sim -\frac{(D+1)(D-3)}{4} \ id \qquad \text{or} \dots
$$

$$
\gamma^{mn} \{T_{i,m}, T_{j,n}\} - J_{k(i}J_{j)}{}^{k} - \frac{2}{D-2} \delta_{ij} (T^{2} - J^{2}) \sim 0,
$$
  
\n
$$
\delta^{ij} \{T_{i,m}, T_{j,n}\} - \{\bar{L}_{m}, \bar{L}_{n}\} - \frac{2}{3} \gamma_{mn} (T^{2} - \bar{L}^{2}) \sim 0,
$$
  
\n
$$
6J^{2} - 2(D-2)\bar{L}^{2} - (D-5)T^{2} \sim 0,
$$
  
\n
$$
\{J_{i}{}^{j}, T_{j,m}\} + \gamma^{kn}(m-n) \{\bar{L}_{k}, T_{i,m+n}\} \sim 0,
$$
  
\n
$$
\{J_{[ij}, T_{k],m}\} \sim 0,
$$
  
\n
$$
\gamma^{mn} \{\bar{L}_{m}, T_{i,n}\} \sim 0,
$$
  
\n
$$
2 \{T_{[i,m}, T_{j],n}\} + (m-n) \{J_{ij}, \bar{L}_{m+n}\} \sim 0,
$$
  
\n
$$
J_{[ij}J_{kl]} \sim 0,
$$
  
\n
$$
C_{2} \equiv J^{2} + \bar{L}^{2} + T^{2} \sim -\frac{(D+1)(D-3)}{2}id
$$

## The gca<sub>D-1</sub> ideal and Galilean HS algebras *i*deal and Galilean HS algebras

$$
\gamma^{mn} \{T_{i,m}, T_{j,n}\} - \frac{2}{D-2} \delta_{ij} T^2 \sim 0,
$$
  

$$
\delta^{ij} \{T_{i,m}, T_{j,n}\} - \frac{2}{3} \gamma_{mn} T^2 \sim 0,
$$
  

$$
J^2 - \bar{L}^2 \sim -\frac{(D-3)(D-5)}{4} id,
$$
  

$$
\{J_i^j, T_{j,m}\} + \gamma^{kn}(m-n) \{\bar{L}_k, T_{i,m+n}\} \sim 0,
$$
  

$$
\{J_{[ij}, T_{k],m}\} \sim 0,
$$
  

$$
\gamma^{mn} \{\bar{L}_m, T_{i,n}\} \sim 0,
$$
  

$$
\{T_{[i,m}, T_{j],n}\} \sim 0,
$$
  

$$
J_{[ij} J_{kl]} \sim 0,
$$
  

$$
T^2 \sim 0.
$$

 $\bullet$  Galilean conformal HS algebra:  $\parallel$  gh $\mathfrak{s}_D \equiv \mathcal{U}(\mathfrak{gca}_{D-1})/\langle \mathcal{I}_{\mathfrak{g}} \rangle$   $\parallel$ 

the suitable power of  $\sigma$  to cancel divergences, we have a smooth limit limit  $\sigma$  smooth limit limit  $\sigma$ 

\n- Galilean conformal HS algebra: 
$$
\boxed{\mathfrak{g} \mathfrak{h} \mathfrak{s}_D \equiv \mathcal{U}(\mathfrak{g} \mathfrak{c} \mathfrak{a}_{D-1}) / \langle \mathcal{I}_{\mathfrak{g}} \rangle}
$$
\n

Ξ

*<sup>J</sup>*[*ij , Tk*]*,m* ⇠ <sup>0</sup> *,* (5.18e) gal\_D\_singleton\_ideal:5

Andrea Campoleoni - UMONS are untouched in the limit, whereas the limit of counterparts are in the limit of the limit of  $\sigma$ 

## Carrollian and Galilean HS algebras in D=5 tollian and Calilean LIC algebras in D<sub>-</sub>F previous combinations so as to move from the so(3) subalgebra spanned by the *Jij* to a

sl(2*,* R) subalgebra. This will allow us to present the relativistic conformal algebra in a more

- In D=5 we start from a one-parameter family of algebras  $\bigcirc$ be collected in the tensor *Tm,n* with *m* 2 *{*1*,* 0*,* 1*}* defined as
	- Carrollian contraction: only one extra non-isomorphic algebra obtained in the limit  $\lambda \rightarrow 0$ *n*, *P*<sub>1</sub>, *P*<sub>2</sub>  $\alpha$ , *P*<sup>2</sup>, *P*<sub>2</sub>  $\alpha$ , (*n*)*, p*<sup>2</sup>, (*n*)*,* (*n*)*,* (*n*)*,* (*n*) *Tm,*<sup>0</sup> = (*iB*<sup>2</sup> + *B*3*, B*1*, iB*<sup>2</sup> *B*3)*,* (5.25b)
- Galilean contraction: a 3D like structure emerges... *Tm,*+1 = (*iK*<sup>2</sup> + *K*3*, K*1*, iK*<sup>2</sup> *K*3)*,* (5.25c)

 $L_m = \{J_{31} + i J_{12}\,,\, i J_{23}\,,\, J_{31} - i J_{12}\}$ 

where *mn* is the sl(2*,* R) Killing metric (2.31).

$$
[L_m, L_n] = (m - n) L_{m+n},
$$
  
\n
$$
[L_m, \bar{L}_n] = (m - n) \bar{L}_{m+n},
$$
  
\n
$$
[L_m, T_{n,k}] = (m - n) T_{m+n,k},
$$
  
\n
$$
[\bar{L}_m, T_{k,n}] = (m - n) T_{k,m+n},
$$
  
\n
$$
[L_m, T_{k,n}] = (m - n) T_{k,m+n},
$$
  
\n
$$
[\bar{L}_m, L_m] = (m - n) T_{k,m+n},
$$
  
\n
$$
[\bar{L}_m, L_m] = 0,
$$

of the real form of *D*<sup>3</sup> obtained via the changes of basis (5.24) and (5.25a)) read

*Tm,*<sup>1</sup> = (*iP*<sup>2</sup> + *P*3*, P*1*, iP*<sup>2</sup> *P*3)*,* (5.25a)

...but only one extra non-isomorphic algebra results from the  $coset$  *construction* 

Andrea Campoleoni - UMONS various components. In this basis the commutation relations of so(2*,* 4) (or, more precisely, *kl {Tm,k, Tn,l} {Lm, Ln}* <sup>2</sup>

## Other flat/Carrollian conformal

HS algebras





## "Geometric" algebras for Killing tensors?  $\mathbf{b}$  a natural generalisation and  $\mathbf{c}$  algebra. For Killing tensors, a natural  $\mathbf{c}$ bracket is provided by the School statement of School and References the School St

Why cannot we use the following bracket? Schouten (1940)  $\bigcirc$ 

• 
$$
[v, w]^{\mu_1 \cdots \mu_{p+q-1}} \equiv \frac{(p+q-1)!}{p!q!} \left( p v^{\alpha(\mu_1 \cdots \partial_\alpha w^{\cdots \mu_{p+q-1}} - q w^{\alpha(\mu_1 \cdots \partial_\alpha v^{\cdots \mu_{p+q-1}})} \right)
$$

two symmetric contravariant tensors *v* (of rank *p*) and *w* (of rank *q*) it yields the following

- for  $p=1$  and  $q=1$  it coincides with the Lie bracket. Killing tensors is again a Killing tensor. On the other hand, the bracket of two traceless
- the bracket of two Killing tensors is a Killing tensor
	- the bracket of two traceless tensors isn't traceless

### "Geometric" algebras for Killing tensors?  $\mathbf{b}$  a natural generalisation and  $\mathbf{c}$  algebra. For Killing tensors, a natural  $\mathbf{c}$ bracket is provided by the School statement of School and References the School St  $2"$  algebras for Killing tensors? Minkowski space equipped with the Schouten bracket.

Why cannot we use the following bracket? Schouten (1940)

• 
$$
[v, w]^{\mu_1 \cdots \mu_{p+q-1}} \equiv \frac{(p+q-1)!}{p!q!} \left( p v^{\alpha(\mu_1 \cdots \partial_\alpha w^{\cdots \mu_{p+q-1}} - q w^{\alpha(\mu_1 \cdots \partial_\alpha v^{\cdots \mu_{p+q-1}})} \right)
$$

two symmetric contravariant tensors *v* (of rank *p*) and *w* (of rank *q*) it yields the following

- for  $p=1$  and  $q=1$  it coincides with the Lie bracket Killing tensors is again a Killing tensor. On the other hand, the bracket of two traceless (computed via their Lie brackets). To proceed, we recall that all Killing tensors of constant-
- the bracket of two Killing tensors is a Killing tensor Lie derivative of a traceless Killing tensor along a Killing vector is again a traceless Killing *<sup>m</sup>±*1) *<sup>µ</sup>*1*···µs*<sup>1</sup> ⌘ **sors** is a *L*<br>I∠∶II:∞ ∞, +, *±*1 h i*µ*1*···µs*<sup>1</sup>
- the bracket of two traceless tensors isn't traceless  $t$  of ture trees less topeors is p<sup>1</sup>t trees less product and the traceless is *s* ⌥ *m* 1 <u>The tensories of the watercolour tensors tensors for the well-colours.</u>

• **Exception in D=3:** 
$$
(P_{\pm(s-1)}^{(s)})^{\mu_1 \cdots \mu_{s-1}} \equiv \frac{(s-1)!}{(2\sqrt{2})^{s-2}} (P_{\pm 1})^{\mu_1} \cdots (P_{\pm 1})^{\mu_{s-1}},
$$
  
AC, Henneaux (2014)  

$$
(L_{\pm(s-1)}^{(s)})^{\mu_1 \cdots \mu_{s-1}} \equiv (s-1) \frac{(s-1)!}{(2\sqrt{2})^{s-2}} (P_{\pm 1})^{(\mu_1} \cdots (P_{\pm 1})^{\mu_{s-2}} (L_{\pm 1})^{\mu_{s-1}}.
$$

$$
\left[ [L_m^{(3)}, P_n^{(3)}]^{\mu\nu\rho} = (m - n) \left( 2 \left( P_{m+n}^{(4)} \right)^{\mu\nu\rho} - \frac{2m^2 + 2n^2 - mn - 8}{20} \eta^{(\mu\nu} (P_{m+n})^{\rho)} \right) \right]
$$

 $T_{\text{max}}$  the state ment, one can state  $\frac{1}{2}$ 

<sup>12</sup>When *D >* 3 the right-hand side of (3.61) can also be decomposed in traceless components, but in

Andrea Campoleoni - UMONS general they do not satisfy the Killing equation (2.2) even when both *v* and *w* are traceless Killing tensors. ihs[∞] ! <sup>13</sup>We normalised the Killing tensors in (3.63) so as to simplify the comparison with the structure constants  $\mathcal{I}$ <sup>35</sup> (5*m*<sup>3</sup> *<sup>n</sup>*<sup>3</sup> <sup>5</sup>*m*2*<sup>n</sup>* + 3*mn*<sup>2</sup> <sup>17</sup>*<sup>m</sup>* + 9*n*) ⌘(*µ*⌫(*P*(3)

#### "Geometric" algebras for Killing tensors? *m n* satisfying where in the first two definitions we only the terms of  $\mathbf S$  are necessary to implementations that are necessary to implementations of  $\mathbf S$ a traceless projection in the Latin indices (as in eq. (6.4c)) and that can be read of  $\alpha$ a traceless projection in the Latin in the Latin indices (as in  $\Omega$ from eqs. (4.7) and (4.8). These rank-two tensors are also traceless in the Greek indices (⌘*µ*⌫(*Qab*)*µ*⌫ = 0 etc.) and they actually form a basis for the subspace of *traceless* Killing

Can we do something similar in any dimensions? *<sup>µ</sup>*(*s*1) = 0 *,* (6.3) thing similar in any dimensions?  $\blacksquare$ tries of a massless spin-three field and they correspond to the tensors introduced in *D* = 3 tries of a massless spin-three field and they correspond to the tensors introduced in *D* = 3



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$$
\mathcal{I}_{abc}^{\quad \mu\nu} \equiv 2 \mathcal{J}_{[ab}^{\quad \left(\mu \mathcal{P}_c\right]^{\nu)}} = 0 \,, \quad \mathcal{I}_{abcd}^{\mu\nu} \equiv 2 \mathcal{J}_{[ab}^{\quad \left(\mu \mathcal{J}_{cd\right]}^{\quad \nu)}} = 0
$$

The key observation to proceed is the key observation to proceed in the identity: its School behaves like the i

to the specific realisation of the Poincaré algebra introduced in (6.2), one has

Andrea Campoleoni - UMONS Andrea Campoleoni - UMONS *the same as the same as the algebras A* 

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$$
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\n

## Schouten bracket algebra as HS algebra? A series of higher-derivative equations of motion involving a traceful tensor '*µ*(*s*) that are

- invariant under gauge transformations Double interpretation for the Schouten bracket algebra  $\bigcirc$ '*µ*(*s*) = @*µ*✏*µ*(*s*1) (6.23)
	- Rigid symmetries for unconstrained Fronsdal transformations
	- Inönü-Wigner contraction of the rigid symmetries of partiallymassless fields  $\overline{127}$ -virgher contraction or the right symmetries or partiallywhere we use the same conventions as in equations as in equations of motion for motion for motion for motion for
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 $P$  Francia; Joung, Mkrtchyan (2012 Francia; Joung, Mkrtchyan (2012)

• Higher-derivative theories:  $\mathbb{R}^n$  is the Figure system (2.51).

$$
\mathcal{R}^{[\frac{s}{2}]}_{\mu(s)} = 0 \quad \text{for } s \text{ even},
$$
  
Higher-derivative theories:  

$$
\partial \cdot \mathcal{R}^{[\frac{s-1}{2}]}_{\mu(s)} = 0 \quad \text{for } s \text{ odd},
$$

• Partially-massless-like eom: AC, Francia, Heissenberg (2020)

AC, Francia, Heissenberg (2020)

$$
\Box \, \varphi_{\mu(s)} - \frac{s(D+2s-4)}{(t+1)(D+2s-t-4)} \left( \partial_\mu \partial \cdot \varphi_{\mu(s-1)} - \frac{s-1}{D+2(s-2)} \, g_{\mu\mu} \partial \cdot \partial \cdot \varphi_{\mu(s-2)} \right) = 0
$$

[97], while for the other values of *t* one gets more complicated spectra. In particular, for

### Andrea Campoleoni - UMONS '*µ*(*s*) = *s*! (*<sup>s</sup> <sup>t</sup>* 1)! @*<sup>µ</sup> ···* @*µ*✏*µ*(*st*1) *,* with ✏*µ*(*st*3)

# Summary & overview

- One can build non-Abelian HS algebras including subalgebras h=iso(1,D−1) or h=gca<sub>D−1</sub> (with the same spectrum as in AdS)
- "Good" Lorentz commutators guaranteed in UEA constructions
- Atypical commutators with translations (counterpart of the absence of "naive" minimal coupling?)

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- Atypical commutators with translations (counterpart of the absence of "naive" minimal coupling?)



- Asymptotic symmetries?
- Modules associated to our algebras?
- Linearised curvatures?
- Recovering the algebras in interacting theories?