Novel perspectives in string phenomenology



- 1989 · · · Minimal Standard Heterotic String Models · · ·
- 2003 \cdots Classification of fermionic $Z_2 \times Z_2$ orbifolds \cdots
- 2019 · · · 10D tachyonic vacua \rightarrow phenomenology?

AEF, EPJC 79 (2019) 703; AEF, B Percival, V Matyas, EPJC 80 (2020) 337; + in preparation

Okinawa Institute of Science and Technology, Zoom, 1 June 2020

Fermionic $Z_2 \times Z_2$ orbifolds

'Phenomenology of the Standard Model and Unification'

- Minimal Superstring Standard Model NPB 335 (1990) 347
- \bullet Top quark mass \sim 175–180GeV
- Generation mass hierarchy
- CKM mixing
- Stringy seesaw mechanism
- Gauge coupling unification
- Proton stability
- Squark degeneracy
- Moduli fixing
- Classification

(with Nanopoulos & Yuan) PLB 274 (1992) 47 NPB 407 (1993) 57 NPB 416 (1994) 63 (with Halyo) PLB 307 (1993) 311 (with Halyo) NPB 457 (1995) 409 (with Dienes) NPB 428 (1994) 111 NPB 526 (1998) 21 (with Pati) NPB 728 (2005) 83 2003 _ . . .

(with Kounnas, Rizos & ... Harries, Percival)

Other approaches

<u>Geometrical</u> Greene, Kirklin, Miron, Ross (1987) Donagi, Ovrut, Pantev, Waldram (1999) Blumenhagen, Moster, Reinbacher, Weigand (2006) Heckman, Vafa (2008)

<u>Orbifolds</u>

Gepner (1987) Schellekens, Yankielowicz (1989) Gato–Rivera, Schellekens (2009)

Orientifolds

Cvetic, Shiu, Uranga (2001) Ibanez, Marchesano, Rabadan (2001) Kiristis, Schellekens, Tsulaia (2008) Point, String, Membrane



Free Fermionic Construction

<u>Left-Movers</u>: $\psi_{1,2}^{\mu}$, χ_i , y_i , ω_i $(i = 1, \cdots, 6)$ <u>Right-Movers</u>

$$\begin{split} \bar{\phi}_{A=1,\cdots,44} = \begin{cases} \bar{y}_i \ , \ \bar{\omega}_i & i = 1, \cdots, 6 \\\\ \bar{\eta}_i & U(1)_i & i = 1, 2, 3 \\\\ \bar{\psi}_{1,\cdots,5} & SO(10) \\\\ \bar{\phi}_{1,\cdots,8} & SO(16) \end{cases} \\ V \longrightarrow V & f \longrightarrow -e^{i\pi\alpha(f)}f \\ Z &= \sum_{\substack{all \ spin \\ structures}} c\binom{\vec{\alpha}}{\vec{\beta}} \ Z\binom{\vec{\alpha}}{\vec{\beta}} \\\\ \text{Models} \longleftrightarrow \text{Basis vectors} + \text{ one-loop phases} \end{cases}$$

<u>The NAHE set</u>: { 1, S, b_1 , b_2 , b_3 } $N = 4 \rightarrow 2$ 1 1 vacua $Z_2 \times Z_2$ orbifold compactification \implies Gauge group $SO(10) \times SO(6)^{1,2,3} \times E_8$ beyond the NAHE set Add $\{\alpha, \beta, \gamma\}$ e.g. FNY model number of generations is reduced to three $SO(10) \longrightarrow SU(3) \times SU(2) \times U(1)_{T_{3_R}} \times U(1)_{B-L}$ $U(1)_Y = \frac{1}{2}(B-L) + T_{3_R} \in SO(10) !$ $SO(6)^{1,2,3} \longrightarrow U(1)^{1,2,3} \times U(1)^{1,2,3}$

(Modern School)

Basis vectors:

$$\begin{split} 1 &= \{\psi^{\mu}, \ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\} \\ S &= \{\psi^{\mu}, \chi^{1,\dots,6}\}, \\ z_1 &= \{\bar{\phi}^{1,\dots,4}\}, \\ z_2 &= \{\bar{\phi}^{5,\dots,8}\}, \\ e_i &= \{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\}, \ i = 1, \dots, 6, \\ b_1 &= \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, \\ b_2 &= \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}, \\ \lambda &= 2 \rightarrow N = 1 \\ \alpha &= \{\bar{\psi}^{4,5}, \bar{\phi}^{1,2}\} \\ \beta &= \{\bar{\psi}^{1,\dots,5} \equiv \frac{1}{2}, \dots\} \\ \end{split}$$

Independent phases $c \begin{bmatrix} vi \\ v_j \end{bmatrix} = \exp[i\pi(v_i|v_j)]$: upper block

A priori 66 independent coefficients $\rightarrow 2^{66}$ distinct vacua

Pati-Salam class: with Assel, Christodoulides, Kounnas, Rizos

RESULTS: of random search of over 10¹¹ vacua



Number of 3-generation models versus total number of exotic multiplets

flipped SU(5) class: with Sonmez, Rizos

RESULTS: of random search of over 10^{12} vacua



Number of exophobic models versus the number of generations

<u>Standard-like Model class:</u> with Sonmez, Rizos NPB 927 (2018) 1 <u>RESULTS: random search of over 10^{11} vacua \Rightarrow few 3 gen models</u>

- Adaptation of the methodology:
- Two stage process;
- Random fertile SO(10) models; Fertility conditions
- Complete SLM classification of fertile cores.

 10^7 Three generation SLMs with standard light and heavy Higgs spectrum

Left-Right Symmetric class:with Harries, Rizos NPB 936 (2018) 472with fertility conditions:with Percival, Rizos NPB 953 (2020) 114969

Spinor-vector duality:

Invariance under exchange of $\#(16 + \overline{16}) < - > \#(10)$



Symmetric under exchange of rows and columns

 E_6 : 27 = 16 + 10 + 1 $\overline{27} = \overline{16} + 10 + 1$ Self-dual: $\#(16 + \overline{16}) = \#(10)$ without E_6 symmetry

Spinor–Vector Duality from a Novel Basis

$S = \{\psi^{\mu}, \chi^{1, \dots, 6}\},\$	
$z_1 = \{\bar{\phi}^{1,\dots,4}\},$	
$z_2 = \{\bar{\phi}^{5,\dots,8}\},$	
$z_3 = \{\bar{\psi}^{1,\dots,4}\},$	
$z_4 = \{ \bar{\eta}^{0,,3} \},$	$ar{\eta}^0~\equiv~ar{\psi}^5$
$e_i = \{y^i, \omega^i \bar{y}^i, \bar{\omega}^i\}, \ i = 1, \dots, 6,$	N=4 Vacua
$1 = S + \sum e_i + z_1 + z_2 + z_3 + z_4$	
$b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots}\}$	$^{,5}\}, \qquad N=4 \to N=2$
Vector bosons: NS, $z_{1,2,3,4}$, $z_i + z_j$	$NS \rightarrow SO(8)^4$
$SO(12)$ -GUT \rightarrow from enhancement	

Duality picture is facilitated

Spinor \longleftrightarrow Vector map $\longrightarrow B \iff B + z_4$

 z_4 spectral flow operator (with ..., Tsulaia, NPB 848 (2011) 332)

SO(12) enhancement $\longrightarrow B \iff B + z_3$

A convenient basis to study dualities; modular properties

 $\rightarrow 2D \rightarrow 24$ dimensional lattices

GUT structure is obscured

NON–SUSY String Phenomenology:

$$\begin{array}{ll} \underline{ Starting \ with:} \\ using \ the \ level-one \ SO(2n) \ characters \end{array}$$

$$O_{2n} = \frac{1}{2} \left(\frac{\theta_3^n}{\eta^n} + \frac{\theta_4^n}{\eta^n} \right), \qquad V_{2n} = \frac{1}{2} \left(\frac{\theta_3^n}{\eta^n} - \frac{\theta_4^n}{\eta^n} \right), \\ S_{2n} = \frac{1}{2} \left(\frac{\theta_2^n}{\eta^n} + i^{-n} \frac{\theta_1^n}{\eta^n} \right), \qquad C_{2n} = \frac{1}{2} \left(\frac{\theta_2^n}{\eta^n} - i^{-n} \frac{\theta_1^n}{\eta^n} \right).$$

where

$$\theta_3 \equiv Z_f \begin{pmatrix} 0\\ 0 \end{pmatrix} \qquad \theta_4 \equiv Z_f \begin{pmatrix} 0\\ 1 \end{pmatrix} \qquad \theta_2 \equiv Z_f \begin{pmatrix} 1\\ 0 \end{pmatrix} \qquad \theta_1 \equiv Z_f \begin{pmatrix} 1\\ 1 \end{pmatrix}$$

Apply $g = (-1)^{F + F_{z_1} + F_{z_2}}$

$$Z_{10d}^{-} = \begin{bmatrix} V_8 \left(\overline{O}_{16} \overline{O}_{16} + \overline{S}_{16} \overline{S}_{16} \right) - S_8 \left(\overline{O}_{16} \overline{S}_{16} + \overline{S}_{16} \overline{O}_{16} \right) \\ + \underbrace{O_8 \left(\overline{C}_{16} \overline{V}_{16} + \overline{V}_{16} \overline{C}_{16} \right) - C_8 \left(\overline{C}_{16} \overline{C}_{16} + \overline{V}_{16} \overline{V}_{16} \right) \end{bmatrix}.$$

In fermionic language: { $\mathbf{1}$, z_1 , z_2 }

where
$$z_1 = \{\bar{\psi}^{1, \cdots, 5}, \bar{\eta}^{1, 2, 3}\}$$
; $z_2 = \{\bar{\phi}^{1, \cdots, 8}\} \Rightarrow S = 1 + z_1 + z_2$
 $c\binom{z_1}{z_2} = +1 \implies E_8 \times E_8$; $c\binom{z_1}{z_2} = -1 \implies SO(16) \times SO(16)$

non-SUSY string phenomenology

Alternatively: Apply
$$g = (-1)^{F+F_{z_1}}$$

$$Z_{10d}^{-} = \left(V_8 \overline{O}_{16} - S_8 \overline{S}_{16} + \underline{O_8 \overline{V}_{16}} - C_8 \overline{C}_{16} \right) \left(\overline{O}_{16} + \overline{S}_{16} \right),$$

 $O_8 \overline{V}_{16} \overline{O}_{16} \implies \text{tachyon}$

In fermionic language: $\{ \mathbf{1}, z_2 \} \implies \text{No } S$

Tachyon free models: $S \longleftrightarrow \tilde{S}$ -map

Modified NAHE $\longleftrightarrow \overline{NAHE}$

	ψ^{μ}	χ^{12}	χ^{34}	χ^{56}	$y^{3,,6}$	$\bar{y}^{3,,6}$	$y^{1,2},\omega^{5,6}$	$ar{y}^{1,2},ar{\omega}^{5,6}$	$\omega^{1,,4}$	$\bar{\omega}^{1,\dots,4}$	$ar{\psi}^{1,,5}$	$\bar{\eta}^1$	$ar{\eta}^2$	$ar{\eta}^3$	$ar{\phi}^{1,,8}$
1	1	1	1	1	1,,1	1,,1	1,,1	1,,1	1,,1	1,,1	1,,1	1	1	1	1,1,1,1,1,1,1,1
\tilde{S}	1	1	1	1	0,,0	0,,0	0,,0	0,,0	0,,0	0,,0	0,,0	0	0	0	1,1,1,1,0,0,0,0
b_1	1	1	0	0	1,,1	1,,1	0,,0	0,,0	0,,0	0,,0	1,,1	1	0	0	0,0,0,0,0,0,0,0
b_2	1	0	1	0	0,,0	0,,0	1,, 1	1,,1	0,,0	0,,0	1,,1	0	1	0	0,0,0,0,0,0,0,0
b_3	1	0	0	1	0,,0	0,,0	0,,0	0,,0	1,,1	1,,1	1,,1	0	0	1	0,0,0,0,0,0,0,0

Beyond the NAHE-set

	ψ^{μ}	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4 ar{y}^4$	$y^5 ar{y}^5$	$ar{y}^3ar{y}^6$	$y^1\omega^5$	$y^2 ar y^2$	$\omega^6 \bar{\omega}^6$	${}^{5} \bar{y}^{1} ar{\omega}^{5}$	$\omega^2 \omega^4$	$\omega^1 \bar{\omega}^1$	$\omega^3 \bar{\omega}^3$	$^3 \ \bar{\omega}^2 \bar{\omega}^4$	$ar{\psi}^{1,,5}$	$\bar{\eta}^1$	$ar{\eta}^2$	$ar{\eta}^3$	\bar{q}
α	0	0	0	0	1	0	0	1	0	0	1	1	0	0	1	1	11100	1	0	0	000
eta	0	0	0	0	0	0	1	1	1	0	0	1	0	1	0	1	11100	0	1	0	110
γ	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$0 \ 0 \ \frac{1}{2}$
				a			$\tilde{\gamma}$														

Up to the $S \longleftrightarrow S$ -map

Same model as published with

with Cleaver, Manno and Timirgazi in PRD78 (2008) 046009

Stable non–SUSY heterotic–string vacuum?

 $\begin{array}{l} \underline{\mathsf{Moduli}} \to \mathsf{WS \ Thirring \ interactions} \ (R - \frac{1}{R}) J_L^i(z) \bar{J}_R^j(\bar{z}) = (R - \frac{1}{R}) y^i \omega^i \bar{y}^j \bar{\omega}^j \\ \\ & \quad \text{To \ identify \ the \ untwisted \ moduli \ in \ the \ free \ fermionic \ models} \\ & \quad \rightarrow \ \text{find \ the \ operators \ of \ the \ form} \\ & \quad J_L^I(z) \bar{J}_R^J(\bar{z}) \end{array}$

that are allowed by the orbifold (fermionic) symmetry group $Z_2 \times Z_2$ { 1, S, z_1 , z_2 } + { b_1 , b_2 } $\rightarrow SO(4)^3 \times E_6 \times U(1)^2 \times E_8$

The Thirring interactions that remain invariant are

$$J_L^{1,2} \bar{J}_R^{1,2} \qquad ; \qquad J_L^{3,4} \bar{J}_R^{3,4} \qquad ; \qquad J_L^{5,6} \bar{J}_R^{5,6} \\ y^{1,2} \omega^{1,2} \bar{y}^{1,2} \bar{\omega}^{1,2} \qquad ; \qquad y^{3,4} \omega^{3,4} \bar{y}^{3,4} \bar{\omega}^{3,4} \qquad ; \qquad y^{5,6} \omega^{5,6} \bar{y}^{5,6} \bar{\omega}^{5,6}$$

These moduli are always present in symmetric $Z_2 \times Z_2$ orbifolds

in realistic models

$\{ 1 ,$	$S, z_1, z_2 \} \oplus$	$\{ b_1 \ , \ b_2 \ \}$	$\oplus \ \left\{ \ lpha \ , \ eta \ , \ \gamma ight\}$
	N = 4	N = 1	
	$E_8 \times E_8$	$Z_2 \times Z_2$	
new feature	Asymmetric orbi	fold	
the key focus	boundary condition	ons of the inte	ernal fermions
	$\{ y ,$	$\omega \mid \bar{y} , \bar{\omega} \}$	
WS fermions th	at have same B.C	. in all basis ve	ectors are paired

pairing of LR fermions \rightarrow Ising model \rightarrow symmetric real fermions pairing of LL & RR fermions \rightarrow complex fermions \rightarrow asymmetric STRING DERIVED STANDARD-LIKE MODEL (PLB278)

	ψ^{μ}	χ^{12}	χ^{34}	χ^{56}	$y^{3,,6}$	$\bar{y}^{3,,6}$	$y^{1,2},\omega^{5,6}$	$ar{y}^{1,2},ar{\omega}^{5,6}$	$\omega^{1,,4}$	$\bar{\omega}^{1,\dots,4}$	$ar{\psi}^{1,\dots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$ar{\phi}^{1,,8}$
1	1	1	1	1	1,,1	1,,1	1,,1	1,,1	1,,1	1,,1	1,,1	1	1	1	1,,1
S	1	1	1	1	0,,0	0,,0	0,,0	0,,0	0,,0	0,,0	0,,0	0	0	0	0,,0
b_1	1	1	0	0	1,,1	1,,1	0,,0	0,,0	0,,0	0,,0	1,,1	1	0	0	0,,0
b_2	1	0	1	0	0,,0	0,,0	1,,1	1,,1	0,,0	0,,0	1,,1	0	1	0	0,,0
b_3	1	0	0	1	0,,0	0,,0	0,,0	0,,0	1,,1	1,,1	1,,1	0	0	1	0,,0

	ψ^{μ}	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4 \bar{y}^4$	$y^5 ar{y}^5$	$ar{y}^3ar{y}^6$	$y^1\omega^5$	$y^2 \bar{y}^2$	$\omega^6 \bar{\omega}^6$	${ar y}^1ar \omega^5$	$\omega^2 \omega^4$	$\omega^1 \bar{\omega}^1$	$\omega^3 \bar{\omega}^3$	$^3 \ \bar{\omega}^2 \bar{\omega}^4$	$ar{\psi}^{1,,5}$	$\bar{\eta}^1$	$ar{\eta}^2$	$ar{\eta}^3$	
α	0	0	0	0	1	0	0	0	0	0	1	1	0	0	1	1	11100	0	0	0	11
β	0	0	0	0	0	0	1	1	1	0	0	0	0	1	0	1	11100	0	0	0	11
γ	0	0	0	0	0	1	0	1	0	1	0	1	1	0	0	0	$\frac{1}{2} \ \frac{1}{2} \ \frac{1}{2} \ \frac{1}{2} \ \frac{1}{2} \ \frac{1}{2} \ \frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$ 0 1

Asymmetric $BC \Rightarrow$ all untwisted moduli are projected out! all $y_i \omega_i \bar{y}_i \bar{\omega}_i$ are disallowed

can be translated to asymmetric bosonic identifications

$$X_L + X_R \rightarrow X_L - X_R$$

moduli fixed at enhanced symmetry point

Twisted moduli

(2,2) $b_j \oplus b_j + z_1 \rightarrow (16 \oplus 10 + 1) + 1 = 27 + 1 \Rightarrow \text{twisted moduli}$

(2,0)
$$b_j + b_j + 2\gamma \rightarrow 16_{SO(10)} + 16_{SO(16)}$$

all twisted moduli are projected out

MINIMAL DOUBLET HIGGS CONTENT (PRD78)

	ψ^{μ}	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4 \bar{y}^4$	$y^5 \bar{y}^5$	$\bar{y}^3 \bar{y}^6$	$y^1\omega^5$	$y^2 \bar{y}^2$	$\omega^6 \bar{\omega}^6$	${ar y}^1ar \omega^5$	$\omega^2 \omega^4$	$\omega^1 \bar{\omega}^1$	$\omega^3 \bar{\omega}^3$	$^3 \bar{\omega}^2 \bar{\omega}^4$	$ar{\psi}^{1,,5}$	$\bar{\eta}^1$	$ar{\eta}^2$	$\bar{\eta}^3$	Q
α	0	0	0	0	1	0	0	1	0	0	1	1	0	0	1	1	11100	1	0	0	110
β	0	0	0	0	0	0	1	1	1	0	0	1	0	1	0	1	$1\ 1\ 1\ 0\ 0$	0	1	0	000
γ	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$0 \ 0 \ \frac{1}{2}$

(With Cleaver, Manno and Timirgaziu)

 $\mathsf{SYMMETRIC} \leftrightarrow \mathsf{ASYMMETRIC}$

with respect to $b_1 \& b_2$

 $h_1,\ ar{h}_1,\ D_1,\ ar{D}_1$, $\ h_2,\ ar{h}_2,\ D_2,\ ar{D}_2$ are projected out

 $h_3, \ \overline{h}_3$ remain in the spectrum

 $\lambda_t Q_3 t_3^c \bar{h}_3$ with $\lambda_t O(1)$

No Phenomenologically viable flat directions

Cleaver, Faraggi, Manno, Timirgaziu, PRD 78 (2008) 046009 Classification of F and D flat directions in EMT reduced Higgs model No D flat direction which is F-flat up to order eight in the superpotential no stringent flat directions to all orders

Suggesting no supersymmetric flat directions in this model (class of models) implying no supersymmetric moduli

only remaining perturbative moduli is the dilaton

quasi-realistic model: SLM; 3 gen; SO(10) embed; Higgs & $\lambda_t \sim 1$; ... vanishing one-loop partition function, perturbatively broken SUSY Fixed geometrical, twisted and SUSY moduli

Basis vectors:

$$1 = \{\psi^{\mu}, \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}$$

- $\tilde{S} = \{\psi^{\mu}, \chi^{1,\dots,6} \mid \bar{\phi}^{3,\dots,6}\},\$
- $z_1 = \{\bar{\phi}^{1,\dots,4}\},\$
- $z_2 = \{\bar{\phi}^{5,\dots,8}\},\$
- $e_i = \{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\}, \ i = 1, \dots, 6,$ N = 4 Vacua

$$b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, \qquad N = 4 \to N = 2$$

$$b_2 = \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}, \qquad N = 2 \to N = 1$$

with Viktor Matyas and Ben Percival, work in progress

Partition functions and the cosmological constant

Full Partition Function for Free Fermionic models:

$$Z_{ToT} = \int_{\mathfrak{F}} \frac{d^2 \tau}{\tau_2^2} Z_B Z_F \equiv \Lambda$$

Integral over the inequivalent tori

• Fermionic contribution:

$$Z_F = \sum_{Sp.Str.} c \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \prod_f Z \begin{bmatrix} \alpha(f) \\ \beta(f) \end{bmatrix}$$
$$Z \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \sqrt{\frac{\theta_1}{\eta}}, \qquad Z \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \sqrt{\frac{\theta_2}{\eta}}, \qquad Z \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \sqrt{\frac{\theta_3}{\eta}}, \qquad Z \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \sqrt{\frac{\theta_4}{\eta}},$$

• Bososnic :

$$Z_B = \frac{1}{\tau_2} \frac{1}{\eta^2 \bar{\eta}^2}$$

from spacetime Bosons.

Evaluated using $q \equiv e^{2\pi i \tau}$ expansion

$$Z = \sum_{n.m} a_{mn} \int_{\mathfrak{F}} \frac{d^2 \tau}{\tau_2^3} q^m \bar{q}^n$$

$$\begin{cases} d\tau_1 & \longrightarrow analytic \\ d\tau_2 & \longrightarrow numeric \end{cases}$$

q – expansion of Z

$$I_{mn} = \begin{cases} \infty & \text{if } m+n < 0 \land m-n \notin \mathbb{Z} \setminus \{0\} \\ \text{Finite} & \text{Otherwise.} \end{cases}$$

- On-Shell Tachyons cause divergence
- Off-Shell Tachyons allowed (necessary)

Modular invariance $\longrightarrow m - n \in \mathbb{Z}$.

Allowed states

$$a_{mn} = \begin{pmatrix} 0 & 0 & a_{-\frac{1}{2}-\frac{1}{2}} & 0 & 0 & 0 & a_{-\frac{11}{22}} & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{-\frac{1}{4}-\frac{1}{4}} & 0 & 0 & 0 & a_{-\frac{13}{44}} & 0 & 0 \\ a_{0-1} & 0 & 0 & 0 & a_{00} & 0 & 0 & a_{01} & 0 \\ 0 & a_{1}-\frac{3}{4} & 0 & 0 & 0 & a_{\frac{11}{44}} & 0 & 0 & 0 & \cdots \\ 0 & 0 & a_{\frac{1}{2}-\frac{1}{2}} & 0 & 0 & 0 & a_{\frac{11}{22}} & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{\frac{3}{4}-\frac{1}{4}} & 0 & 0 & 0 & a_{\frac{33}{44}} & 0 & 0 \\ a_{1-1} & 0 & 0 & 0 & a_{10} & 0 & 0 & a_{11} & 0 \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots \end{pmatrix}$$

Coefficients $a_{mn} = N_b - N_f$ at specific mass level.

For SUSY Theories $a_{mn} = 0 \ \forall m, n$

Some interesting results

Distribution of Λ





- DATA \longrightarrow UNIFICATION
- STRINGS THEORY \longrightarrow GAUGE & GRAVITY UNIFICATION
- STRINGS PHENOMENOLOGY \longrightarrow AT ITS INFANCY STILL LEARNING HOW TO WALK
- 10D vacua without *S*–SUSY generator
- Role of non-geometric backgrounds ? Novel symmetries ?
- String Phenomenology \longrightarrow Physics of the third millennium

e.g. Aristarchus to Copernicus